URSI GASS 2023, Sapporo, Japan, 19 - 26 August 2023

Statistical Characterization of Cavity Quality Factor due to Aperture Leakage

Shen Lin*(1), Yang Shao(1), Bisrat D. Addissie(2), Zachary B. Drikas(2) and Zhen Peng(1)
(1) Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA; e-mail: shenlin2@illinois.edu; yangshao@illinois.edu; zvpeng@illinois.edu
(2) Tactical Electronic Warfare Division, U.S. Naval Research Laboratory, Washington, DC 20375 USA; e-mail: bisrat.addissie@nrl.navy.mil; .zachary.drikas@nrl.navy.mil

Abstract

There has been a strong interest in statistically characterizing the cavity quality factor (Q-factor) for large, complex enclosures. While there are existing methods for analyzing the Q-factor statistics due to distributed losses, there is currently little discussion about the statistical cavity Q-factor caused by localized losses, such as aperture leakage and absorptive loading. This paper presents a physics-oriented, hybrid deterministic-stochastic model that calculates the probability distribution of cavity Q-factor. The research work is evaluated and validated through representative experiments.

1 Introduction

The cavity quality factor (Q-factor) is a fundamental parameter in analyzing the field properties of confined electromagnetic (EM) environments. Applications include mode-stirred reverberation chambers (MSRCs), intentional EMI (IEMI) to electronics housed inside metallic enclosures, and indoor wireless channels.

Historically in the study of cavity resonators, the modal Q-factor was introduced to account for dielectric and wall losses inside the cavity for a single eigenmode, defined by the ratio of stored energy to dissipated power, multiplied by the modal frequency [1]. Under high-frequency reverberation, the complex boundary of the enclosure can lead to high modal density and high modal overlap. As a result, the composite, effective Q-factor was introduced in the literature as an average value (frequency-averaged or stir-averaged) of the cavity quality factor [2, 3]. It is noted that the composite Q-factor varies smoothly with operating frequency. It does not characterize the quasi-random fluctuations between frequencies or cavity configurations.

To analyze stochastic EM fields in large enclosures, there has been a strong interest in characterizing the cavity quality factor in terms of a probability density function (PDF). For the mode-stirred reverberation chamber, it is shown that the PDF of Q-factor exhibits a Fisher-Snedecor F-distribution [4, 5], under the assumption of statistically homogeneous and isotropic cavity fields. Another noticeable work utilizing the volumetric number of independent field samples and the central limit theorem is presented in [6], which concludes that the Q-factor PDF is basically normal distribution in the overmoded regime of well-stirred reverberation chambers.

Whereas previous work has focused on the Q-factor statistics for cavities with homogeneous, distributed losses (i.e. uniform dielectric loss and cavity wall loss), there has been little discussion of the statistical cavity Q-factor due to localized losses (e.g. aperture leakage, absorptive loading). We solved this problem elegantly by using a newly developed stochastic Green's function approach [7, 8]. The statistical predictions are validated by numerical simulations and experimental results.

2 Methodology

2.1 Background and Problem Statement: The Q-factor of the enclosure is a key quality in calculating the statistical properties of internal EM fields in an electrically large cavity. Generally, the quality factor Q is defined as:

$$Q(\omega) = \omega \frac{U_s(\omega)}{P_d(\omega)} \tag{1}$$

where ω is the angular frequency, U_s is the steady-state energy stored in the cavity and P_d is the dissipated power. It is noted that the macroscopic definition of Q-factor as in [4–6] is considered here, instead of modal Q-factors for individual cavity eigenmodes. We are interested in studying the fluctuation of the Q-factor for an ensemble of statistically similar cavity environments, i.e. varying stir states in an MSRC, or changing the frequency within the coherence bandwidth.

Broadly speaking, the statistical fluctuation of the Q-factor depends on the nature of the loss mechanisms contributing to the dissipated power P_d. If the loss is distributed throughout the enclosure, the fluctuations in quality factor are reduced inversely proportional to the cavity volume. This was an assertion that was tested early in the development of what is known as "wave chaos" [9]. However, if the loss is localized in the cavity, the quality factor can fluctuate significantly, which is the primary interest of this paper.

Consider the case of aperture leakage, the design of the numerical experiment is illustrated in Fig. 1. A small electric dipole is placed inside the cavity as the transmitter (Tx). This Tx dipole is responsible for cavity excitation. Another set of receiving (Rx) electric dipoles is utilized as the probe devices to measure the received vectorial electric fields (E-fields), E^x, E^y, and E^z. Next, we introduce an artificial surface S_a over the aperture opening to calculate the surface currents. From the collected internal E-fields and aperture currents, we can obtain the stored energy U_s in the cavity and the dissipated power P_d through the aperture.

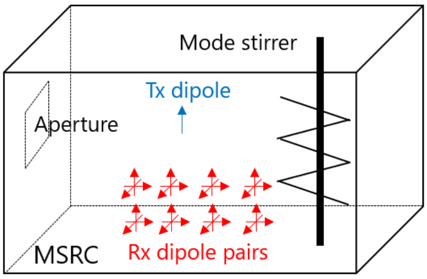


Figure 1. Illustration of the design of experiment.

It is clear that if first-principles modeling is used, one needs many full-wave simulations to obtain a statistical ensemble of U_s and P_d. The computation complexity is prohibitively expensive for electrically large cavities. In this work, we leverage a newly developed statistical wave model known as the stochastic dyadic Green's function (S-DGF), which may be viewed as an effective statistical solution to the vector wave equation in large, complex enclosures.

2.2 Introduction of Stochastic Dyadic Green's Function: The S-DGF is based on a statistical description of the eigenmodes of an enclosed EM environment. Starting from the 2nd order vector wave equation inside a 3D metallic cavity of volume V with distributed losses, the S-DGF can be constructed from the eigenfunction expansion:

$$\overline{\overline{\mathbf{G}}}_{\mathrm{S}}\left(\mathbf{r},\mathbf{r}'\right)=\sum_{i}\frac{\psi_i\left(\mathbf{r},k_i\right)\otimes\psi_i\left(\mathbf{r}',k_i\right)}{k^2-k_i^2-j\frac{k_i^2}{Q}} \tag{2}$$

where $\otimes$ indicates an outer product between two vectors. ψ_i and k_i are the i^{th} vector eigenfunction and eigenvalue of the cavity, and k is the wavenumber. In prior work [7], we prescribe substituting approximate, statistically defined eigenfunctions and eigenvalues. In particular, the eigenfunction statistics are derived from Berry's random wave model (RWM) [10], taking into account different orientations of polarization [8]. And eigenvalues statistics generated by Wigner's random matrix theory (RMT) [11].

After incorporating the RWM and RMT approximation, the expression of S-DGF is given by [8]:

$$\overline{\overline{\mathbf{G}}}_{\mathrm{S}}\left(\mathbf{r},\mathbf{r}'\right)=\mathrm{Re}\left[\overline{\overline{\mathbf{G}}}_{0}\left(\mathbf{r},\mathbf{r}'\right)\right]+\sum_{m}\frac{\overline{\overline{\mathbf{D}}}(\mathbf{r},\mathbf{r}';k_m)}{\tilde{\lambda}_m-j\alpha}\frac{kV}{2\pi^2} \tag{3}$$

where $\overline{\overline{\mathbf{G}}}_{0}\left(\mathbf{r},\mathbf{r}'\right)$ is the free-space dyadic Green's function, $\overline{\overline{\mathbf{D}}}(\mathbf{r},\mathbf{r}';k_m)$ is a dyadic product of correlated Gaussian variables, the eigenvalues $\tilde{\lambda}$ are calculated from the Gaussian Orthogonal Ensemble (GOE) of random matrices, and the α is a macroscopic dimensionless loss-parameter defined by $\alpha=k^3V/\left(2\pi^2Q\right)$.

2.3 Calculation of Statistical Cavity Quality Factor: The S-DGF can be directly utilized to analyze dipole radiation problems inside large enclosures using the electric field integral equation (EFIE). For the aperture leakage problem, one can obtain an equivalent exterior problem by filling the interior sub-region with a perfect electric conductor based on the equivalence principle [12]. The exterior sub-region is then formulated by a surface integral equation (SIE), whose unknowns involve the magnetic current at the aperture. Another equivalent problem can be obtained for the interior cavity sub-region with the aperture covered by an electric conductor. The aperture SIE of the interior sub-region is formulated with the electric stochastic dyadic Green's function of the second kind.

With the Galerkin method and appropriate basis and testing functions, the problem in Fig. 1 can be modeled by an IE matrix equation of the following compact form:

$$\begin{bmatrix}\mathbf{Y}_0^{\mathrm{a}}+\mathbf{Y}_{\mathrm{S}}^{\mathrm{a}} & \mathbf{C}_{\mathrm{S}}^{\mathrm{a,x}} & \mathbf{C}_{\mathrm{S}}^{\mathrm{a,y}} & \mathbf{C}_{\mathrm{S}}^{\mathrm{a,z}}\\ \mathbf{C}_{\mathrm{S}}^{\mathrm{x,a}} & \mathbf{Z}_{\mathrm{S}}^{\mathrm{x,x}} & & \\ \mathbf{C}_{\mathrm{S}}^{\mathrm{y,a}} & & \mathbf{Z}_{\mathrm{S}}^{\mathrm{y,y}} & \\ \mathbf{C}_{\mathrm{S}}^{\mathrm{z,a}} & & & \mathbf{Z}_{\mathrm{S}}^{\mathrm{z,z}}\end{bmatrix}\begin{bmatrix}\mathbf{M}_a\\ \mathbf{J}_{\mathrm{r}}^{\mathrm{x}}\\ \mathbf{J}_{\mathrm{r}}^{\mathrm{y}}\\ \mathbf{J}_{\mathrm{r}}^{\mathrm{z}}\end{bmatrix}=\begin{bmatrix}\mathbf{C}_{\mathrm{S}}^{\mathrm{a,t}}\\ \mathbf{C}_{\mathrm{S}}^{\mathrm{x,t}}\\ \mathbf{C}_{\mathrm{S}}^{\mathrm{y,t}}\\ \mathbf{C}_{\mathrm{S}}^{\mathrm{z,t}}\end{bmatrix}\cdot\mathrm{J}_{\mathrm{t}} \tag{4}$$

where the $\mathbf{M}_a$ is the solution vector for the magnetic current at the aperture. The $\mathbf{Y}_0^{\mathrm{a}}$ is the aperture admittance matrix for the exterior region in terms of free-space dyadic Green's function, and the $\mathbf{Y}_{\mathrm{S}}^{\mathrm{a}}$ is the aperture admittance matrix for the interior cavity region. The J_{t} is the electric current at the Tx dipole, and $\mathbf{J}_{\mathrm{r}}^{\mathrm{x/y/z}}$ are the solution vectors of electric current for a finite number N sets of $\hat{\mathbf{x}}$-, $\hat{\mathbf{y}}$-, and $\hat{\mathbf{z}}$- directed Rx dipoles. In addition to the dipole impedance matrices $\mathbf{Z}_S$, we have introduced IE matrices $\mathbf{C}_{\mathrm{S}}^{\mathrm{a,x/y/z}}$ accounting for the coupling between the aperture current and Rx dipole sets. It is noted that there are no coupling terms between Rx dipoles as they are used as probing devices.

We remark that the matrix equation (4) only involves the wire currents on the Tx/Rx dipoles and the surface currents on the aperture opening. The coupling between them as well as the interaction with the cavity interior are statistically characterized via the S-DGF model. Therefore, this is no need to discretize the cavity enclosure.

After the solution of (4), we can calculate the power leakage through the aperture as,

$$P_d=\frac{1}{2}\mathrm{Re}[\mathbf{M}_a^T\cdot(\mathbf{Y}_0^{\mathrm{a}})^*\cdot\mathbf{M}_a^*] \tag{5}$$

Since linear triangular basis functions are used for the Rx dipole currents, the vectorial electric field can be easily retrieved from the current solution. For example, the $\hat{\mathbf{x}}$-

directed E-fields at Rx probe locations can be obtained by:

$$\mathbf{E}^{x} = \frac{2}{l}\mathbf{Z}_{S}^{x,x}\cdot\mathbf{J}_{r}^{x} \tag{6}$$

The cavity energy density can then be approximated by:

$$W_c = \frac{\varepsilon}{N}\left[(\mathbf{E}^{x})^{T}\cdot(\mathbf{E}^{x})^{*}+(\mathbf{E}^{y})^{T}\cdot(\mathbf{E}^{y})^{*}+(\mathbf{E}^{z})^{T}\cdot(\mathbf{E}^{z})^{*}\right] \tag{7}$$

The energy stored in the cavity is calculated as: $U_s = W_c \cdot V$ and the cavity Q-factor is obtained by using (1). By repeating the above procedure with different S-DGF IE matrices, we can predict the probability distribution of cavity Q-factor due to the aperture leakage.

2.4 Further Technical Discussion: It seems counterintuitive at first that the construction of the S-DGF in (3) also requires an input parameter known as the cavity loss-parameter α. Nevertheless, the attribute of α will affect both the stored energy (numerator) and dissipated power (denominator) in (1). As a result, the choice of α does not affect the PDF of the calculated cavity Q-factor due to localized losses.

The research result is applicable to electrically large cavities with high modal density and modal overlap, where a macroscopic cavity Q-factor is a more suitable measure than the modal Q-factor. Since the S-DGF is a statistical model, the result does not give precise Q-factor values for a specific, well-defined geometry. Rather, the outcome predicts the PDF of cavity Q-factor for an ensemble of statistically similar cavity environments.

3 Experimental Validation

The experimental setup is illustrated in Fig. 2. A SMART 800 mode-stirred reverberation chamber (MSRC) is placed inside an anechoic chamber. The dimension of the MSRC is 0.784 m by 1.494 m by 1 m. Two square apertures of dimension 0.3048 m by 0.3048 m are located on the opposite walls of MSRC. We can open one aperture at a time to examine the effects of power loss, as depicted in Fig. 3(a). Two monocone antennas are used as transmitter and receiver shown in Fig. 3(b). The configuration ensemble is obtained by rotating a paddle wheel mode stirrer.

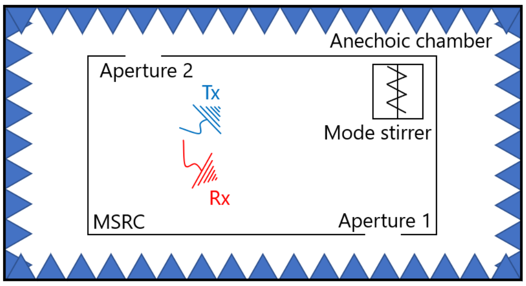
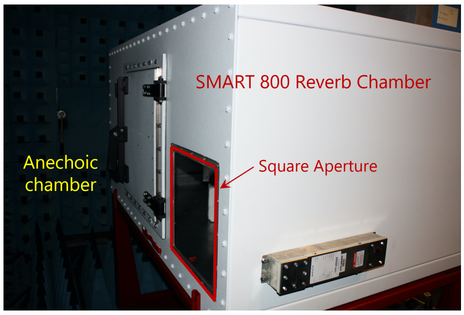
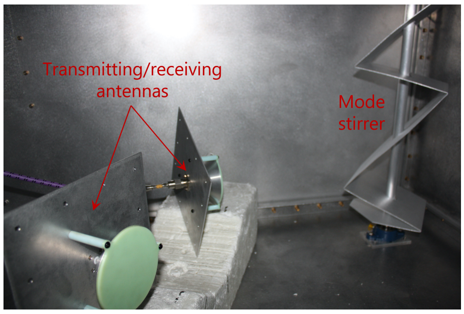


Figure 2. Illustration of the experiment setup.

We first perform the S-parameter measurement with both apertures closed. The stirrer is rotated 100 positions over 360 degrees. At each stir state, S-parameters of Tx and Rx antennas, S_{11} and S_{21}, are measured and recorded from 1 to 1.25 GHz with 0.025 MHz frequency stepping. To avoid measurements at multiple locations to calculate cavity energy density, we utilize the ensemble average over the stirrer states. The frequency-dependent Q-factor is obtained by [13, 14]:



(a) Exterior view



(b) Interior view

Figure 3. Configuration of the MSRC testing environment.

$$Q_{\mathrm{o}}(f) = \frac{16\pi^2 V\left\langle |S_{21}(f)|^2\right\rangle_{\mathrm{stirr}}}{\eta_{TX}\eta_{RX}\lambda^3\left\langle 1-|S_{11}(f)|^2\right\rangle_{\mathrm{stirr}}} \tag{8}$$

where V is the volume of MSRC, η_{TX} and η_{RX} are the Tx and Rx antenna efficiencies. A total number of 10,001 Q-factor samples are obtained from 1 GHz to 1.25 GHz. We then perform the measurement with the same experimental setting while opening the aperture at the front panel of the MSRC. The resulting cavity Q-factor is denoted by Q_{t}. Therefore, the quality factor due to the aperture leakage Q_{lkg} can be recovered by,

$$Q_{\mathrm{lkg}} = 1/\left(1/Q_{\mathrm{t}} - 1/Q_{\mathrm{o}}\right) \tag{9}$$

Finally, we repeat the experiment by opening the aperture on the back panel of the MSRC. Thereby, two sets of measurement data (denoted by front panel and back panel) for Q_{lkg} are obtained. As depicted in Fig. 4, quasi-random fluctuations between frequencies are observed due to the stochastic nature of cavity field.

The objective of this study is to predict the probability distribution of Q_{lkg} for the frequency range 1 GHz – 1.25 GHz. To do so, we first subdivide it into five frequency bands of bandwidth 50 MHz. The proposed procedure in Sec. II.E is

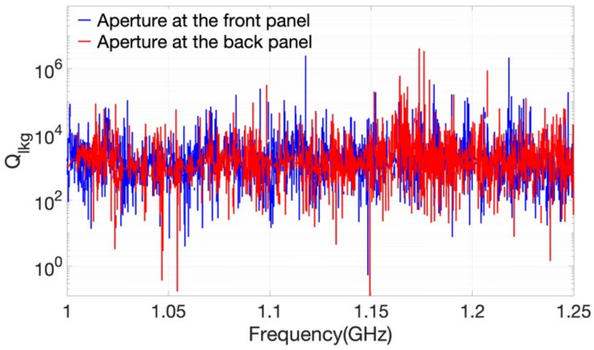


Figure 4. The measurement result of Q-factor, Q_{lkg}.

applied at frequencies of 1 GHz, 1.05 GHz, ..., to 1.25 GHz. At each frequency, 10k GOE random matrices are used to generate an ensemble of S-DGF IE matrices in (4). Therefore, 60k quality factor samples are computed with the proposed work. The resulting PDF is compared to the measurement results shown in Fig. 5, where good agreements are observed.

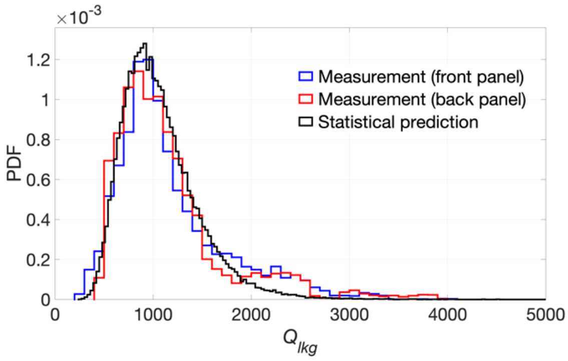


Figure 5. PDF of the cavity Q-factor for aperture leakage.

4 Conclusion

This paper aims to address an important question about the statistical characterization of cavity Q-factor caused by localized losses such as aperture leakage and absorptive loading. A novel hybrid deterministic-stochastic computing model is proposed that integrates component-specific (loading) and site-specific (aperture) features with the statistical representation of complex cavity environments. Experimental results are supplied to validate the proposed work.

References

[1] D. A. Hill, *Electromagnetic Fields in Cavities: Deterministic and Statistical Theories*. New Jersey: John Wiley & Sons, 2009.

[2] P. Corona, G. Latmiral, and E. Paolini, "Performance and analysis of a reverberating enclosure with variable geometry," *IEEE Trans. Electromagnetic Compatibility*, vol. EMC-22, no. 1, pp. 2–5, 1980.

[3] D. A. Hill, M. T. Ma, A. R. Ondrejka, B. F. Riddle, M. L. Crawford, , and R. T. Johnk, "Aperture excitation of electrically large, lossy cavities," *IEEE Trans. Electromagnetic Compatibility*, vol. 36, no. 3, pp. 169–178, Aug 1994.

[4] L. R. Arnaut and G. Gradoni, "Probability distribution of the quality factor of a mode-stirred reverberation chamber," *IEEE Trans. Electromagnetic Compatibility*, vol. 55, no. 1, pp. 35–44, 2013.

[5] L. R. Arnaut, P. Besnier, J. Sol, and M. I. Andries, "On the uncertainty quantification of the quality factor of reverberation chambers," *IEEE Trans. Electromagnetic Compatibility*, vol. 61, no. 3, pp. 823–832, 2019.

[6] A. Gifuni, G. Ferrara, M. Migliaccio, and A. Sorrentino, "Estimate of the probability density function of the quality factor of mode tuned, source stirred and mode stirred reverberation chambers," *IEEE Trans. Electromagnetic Compatibility*, vol. 57, no. 5, pp. 926–936, 2015.

[7] S. Lin, Z. Peng, and T. M. Antonsen, "A stochastic Green's function for solution of wave propagation in wave-chaotic environments," *IEEE Transactions on Antennas and Propagation*, vol. 68, no. 5, pp. 3919–3933, 2020.

[8] S. Lin, Y. Shao, and Z. Peng, "On the vectorial property of stochastic dyadic green's function in complex electronic enclosures," in *2022 IEEE International Symposium on Electromagnetic Compatibility & Signal/Power Integrity (EMCSI)*, 2022, pp. 350–355.

[9] E. Ott and W. M. Manheimer, "Cavity Q for ergodic eigenmodes," *Phys. Rev. A*, vol. 25, pp. 1808–1810, Mar 1982. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.25.1808

[10] M. V. Berry, "Regular and irregular semiclassical wavefunctions," *Journal of Physics A: Mathematical and General*, vol. 10, no. 12, p. 2083, 1977.

[11] E. Wigner, "Random matrices in physics," *SIAM Review*, vol. 9, no. 1, pp. 1–23, 1967.

[12] R. F. Harrington and J. R. Mautz, "Electromagnetic coupling through apertures by the generalized admittance approach," *Computer Physics Communications*, vol. 68, no. 1, pp. 19–42, 1991.

[13] M. Magdowski, B. R. Banjade, and R. Vick, "Measurement of the coupling to shielded cables above a ground plane in a reverberation chamber," in *2016 International Symposium on Electromagnetic Compatibility - EMC EUROPE*, 2016, pp. 234–239.

[14] N. Wellander, M. Elfsberg, and T. Hurtig, "Finding frequencies of enhanced electromagnetic coupling to electronic devices by the use of mode stirred reverberation chambers," in *2017 International Symposium on Electromagnetic Compatibility - EMC EUROPE*, 2017, pp. 1–6.