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Scattering of Sub-Relativistic Electrons by Oblique EMIC Waves through Nonlinear Fractional Resonances

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Abstract

Electromagnetic ion cyclotron (EMIC) waves are well recognized as an important driver of scattering and loss of relativistic electrons in the Earth's outer radiation belt. Observations of sub-relativistic (hundreds of keV) atmospheric precipitation associated with EMIC waves have long lacked theoretical support, but the nonresonant scattering by short wave packets has been recently shown as the most likely explanation. In this paper, we numerically demonstrate the existence of a resonant interaction between oblique EMIC waves and electrons at fractions of the minimum fundamental resonance energy. This fractional resonance is revealed to be a purely nonlinear phenomenon arising from finite Larmor radius effects along the strongly perturbed particle orbit. It is further shown that the pitch angle diffusion due to $n = -1/2$ fractional resonance scales with amplitude faster than the fundamental resonance $n = -1$. Comparison of nonresonant and fractional resonant scattering based on test-particle simulations shows that for high-amplitude, moderately oblique waves, the contribution to diffusion from fractional resonances becomes comparable to the nonresonant effect. Under such conditions, the fractional resonances are expected to significantly enhance the low-energy part of the precipitating electron spectrum.

1 Introduction

Electromagnetic ion cyclotron (EMIC) waves are naturally occurring electromagnetic emissions in Earth's magnetosphere generated by unstable anisotropic hot ion populations. In the outer radiation belt, the wave frequencies in the near-equatorial source fall mainly into the Pc1 range of 0.2–5 Hz [1]. Initially generated in the left-handed mode, the waves may convert to the linear or right-handed mode at higher latitudes [2]. These elliptically polarized waves can scatter relativistic electrons (kinetic energies E_k around 1 MeV and larger) in pitch angle α through cyclotron resonant interactions, which leads to significant losses of radiation belt electrons to the atmosphere [3].

During geomagnetically active times, EMIC waves at lower L-shells ($L < 6$) can reach peak magnetic field amplitudes B_w above one percent of the background magnetic field strength B_0 [4]. Trajectories of particles resonating with strong waves experience large perturbations, and a variety of associated nonlinear effects appear [5]. Phase-trapping of ions in the wave potential leads to nonlocal transport to higher pitch angles and the formation of phase space density (PSD) holes in the gyrophase space, while phase-trapped electrons experience a decrease in pitch angle [6]. Below the fundamental cyclotron resonance energy, nonresonant scattering by amplitude-modulated waves takes place and may extend the energy range of precipitating electrons down to hundreds of keV [7, 8].

In this summary paper, we present results from test-particle simulations of nonlinear electron interactions with oblique monochromatic EMIC waves, demonstrating the existence of nonlinear resonant scattering at fractions of the fundamental resonance energy. This new type of resonance arises due to finite Larmor radius effects along the strongly perturbed orbit of energetic electrons. It is shown that the diffusive scattering induced by the fractional resonance of order $n = -1/2$ scales with amplitude faster than the fundamental resonance $n = -1$ and the associated nonresonant scattering. Consequently, when the wave is at least moderately oblique ($\theta_\mathrm{k} > 45^\circ$) and reaches amplitudes close to 1% of the background magnetic field B_0, the fractional resonant scattering may become the dominant driver of sub-relativistic electron precipitation. An abbreviated theoretical explanation is provided to support the numerical results.

2 Methods

We run test-particle simulations of electrons streaming along an EMIC wave packet, starting at the equator and stopping at the mirror point (one quarter-bounce). The integrator utilizes an adaptive relativistic Boris push with phase correction [9] with 128 steps per local electron gyroperiod. When choosing the wave and plasma parameters, we must consider under what conditions can the wave-particle interaction influence electrons at energies below 1 MeV. Figure 1 shows the minimum resonance energy E_Rmin at different frequencies and magnetic latitudes (λ_m) for a hydrogen band wave. A high plasma-to-cyclotron frequency ratio $\omega_\mathrm{pe0}/\Omega_\mathrm{e0}$ is chosen, which is less typical for the hydrogen band [10], but it dramatically decreases the resonance energy. For our simulation, we choose wave frequency $\omega/\Omega_{p0} = 0.7$, which gives us an E_{Rmin} value slightly above 1 MeV, comparable to but still larger than the exceptionally low value $E_{Rmin} = 0.83$ MeV derived from spacecraft observations presented in [8]. The electron density latitude dependence $n_e = n_{e0}/\cos\lambda_m$ and the ion concentrations are taken from [11].

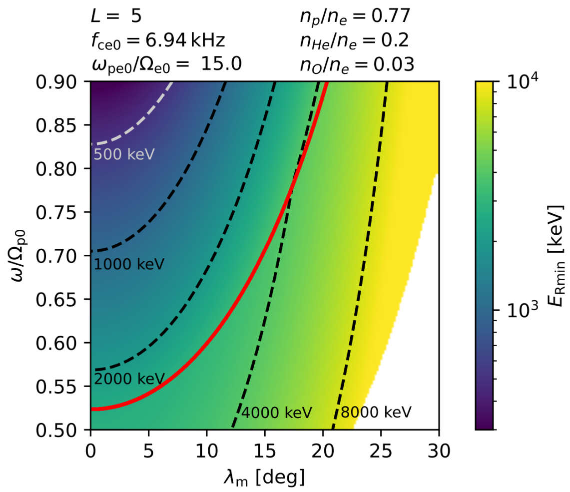

Figure 1. Minimum resonant energies E_{Rmin} of electrons interacting with a parallel-propagating hydrogen-band EMIC wave at different frequencies and latitudes. The solid red line represents polarization reversal. Energy values for frequencies $\omega/\Omega_{He} \leq 1$ are excluded (white color).

The distribution of amplitudes in λ_m is represented by two models: a constant amplitude packet, going from the equator to the point where wave frequency meets the local helium gyrofrequency; or a packet with smoothened edges ($\cos^2$ half-period over field-aligned distance $\Delta h = 310c\Omega_{e0}^{-1}$ at each end of the packet), as shown in Figure 2. Smooth edges heavily suppress nonresonant effects.

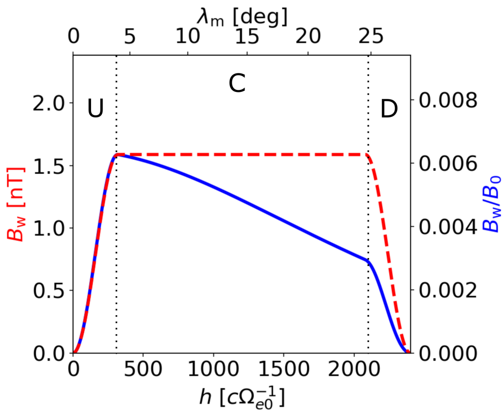

Figure 2. Smoothened distribution of wave amplitudes along the field line. The wave grows in region U, stays constant in region C (1.6 nT in this example), and decreases smoothly back to zero in region D, as shown by the dashed red line. The solid blue line shows the relative wave amplitude with respect to the background field B_0.

3 Results

We start by inspecting individual electron trajectories obtained from propagation through the smooth wave packet model with wave normal angle $\theta_k = 45°$ and amplitude $B_w/B_{0eq} = 0.0064$. Figure 3a shows particles released from the equator with $\alpha_{ini} = 25.5°$ and $E_k = 1.36$ MeV, slightly above the minimum energy for the $n = -1$ (fundamental) resonance with left-handed waves near the equator. We observe strong scattering, with an asymmetry towards lower pitch angles that arises from partial trapping of the electrons in the wave potential. At energies of 0.61 MeV, where the nonresonant scattering should be utterly negligible due to the small amplitude gradients [7], we still observe minor changes in the equatorial pitch angle of electrons escaping the wave packet. This weak scattering is not associated with any resonance known from the quasilinear theory [12].

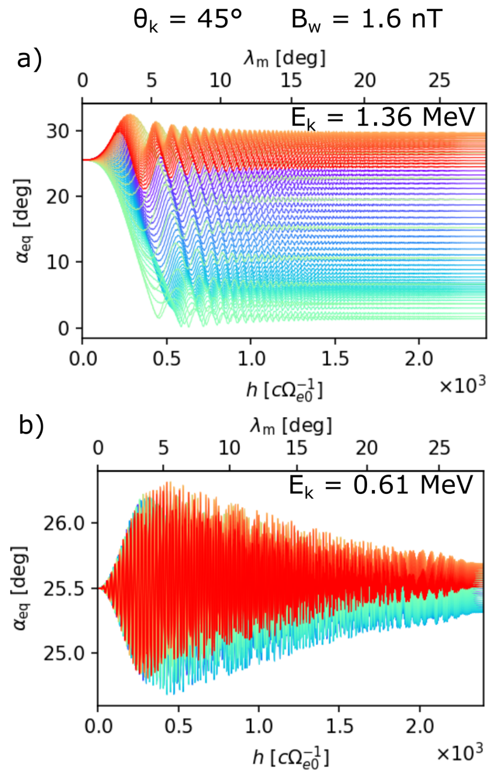

Figure 3. Trajectory examples showing the change in equatorial pitch angle over latitude due to interaction with a high-amplitude, moderately oblique wave ($B_w/B_{0eq} = 0.0064$, $\theta_k = 45°$) with smooth edges. Line colors represent the initial gyrophase (uniform distribution). a) Electrons propagating from the equator with initial pitch angle $\alpha_{ini} = 45°$ and energy $E_k = 1.36$ MeV. The particles experience strong pitch angle scattering due to the fundamental cyclotron resonance. b) At energies of 0.61 MeV, the nonresonant scattering associated with the $n = -1$ resonance is not measurable, and the only persisting deviations in α_{eq} come from fractional resonances.

To explain the unexpected behavior at low energies, we calculate the standard deviation in equatorial pitch angle

$\sigma_\varphi(\alpha_{\rm eq})$ (related to the diffusion coefficient from quasilinear theory) for all initial energies and pitch angles and plot it in Figure 4. Above 1 MeV, we can see the well-known diffusive effects of fundamental and harmonic resonances. However, we also observe a new, weaker resonance starting near 500 keV. We identify it as the $n=-1/2$ fractional resonance. These fractional resonances arise from finite Larmor radius effects acting on the perturbed electron trajectory.

To provide a simplified theoretical explanation, we start by splitting the gyrophase evolution into a zero-order motion and a wave-induced perturbation, $\varphi = \varphi_0 + \varphi_1$. Based on the equations of electron motion in an elliptically polarized wave field, it is reasonable to model the perturbation with a harmonic function. The wave-particle power transfer equation [13] (and the related pitch angle evolution equation) contains terms of the type $\sin(\varphi \pm \psi)$, where ψ is the wave phase. Under some additional simplifying assumptions (omitted here for reasons of space), these terms can be expanded into sums of products of Bessel functions, e.g.,

$$\sin(\varphi+\psi) \approx \sum_{n,r,l=-1}^{1} J_n(\beta) J_r((n-1)R_1) J_l((n-1)L_1) \sin\left((r-n+l+1)\varphi_0 + (l-r+1)\psi_B\right), \tag{1}$$

where J are Bessel functions of the first kind, ψ_B is the gyroaveraged wave phase, and R_1 and L_1 are quantities dependent on amplitude, particle velocity, and resonance mismatch. Permutations of the integers n, r, and l provide various fractional resonances.

It can be proven that the scattering due to the fractional resonance $n=-1/2$ depends on a higher power of amplitude than the integer resonance. We omit the analytical derivation here, but we provide a demonstration in Figure 4, where the ratio of $\sigma_\varphi(\alpha_{\rm eq})$ values related to $n=-1/2$ and $n=-1$ resonances increases with amplitude ($B_{\rm w} = 400$ pT in Fig. 4a vs. $B_{\rm w} = 1.6$ nT in Fig. 4b). Note that the fractional resonances are a purely nonlinear phenomenon and do not appear in the quasilinear theory.

The scattering due to the fractional resonance gets stronger at lower pitch angles when we increase the wave normal angle to $\theta_{\rm k} = 70^\circ$ (Fig. 5a). However, this increase is not very significant, suggesting that moderate obliquity is sufficient for this type of resonance. Switching to the rectangular amplitude profile (zero-width U and D regions in Figure 2), we can compare the maximum effects of nonresonant scattering to fractional resonances. In Figure 5b, we observe that at lower pitch angles near 500 keV, the contribution from the $n=-1/2$ resonance is comparable to the nonresonant scattering. As we increase the kinetic energy and move towards the fundamental resonance, the nonresonant effects quickly become dominant.

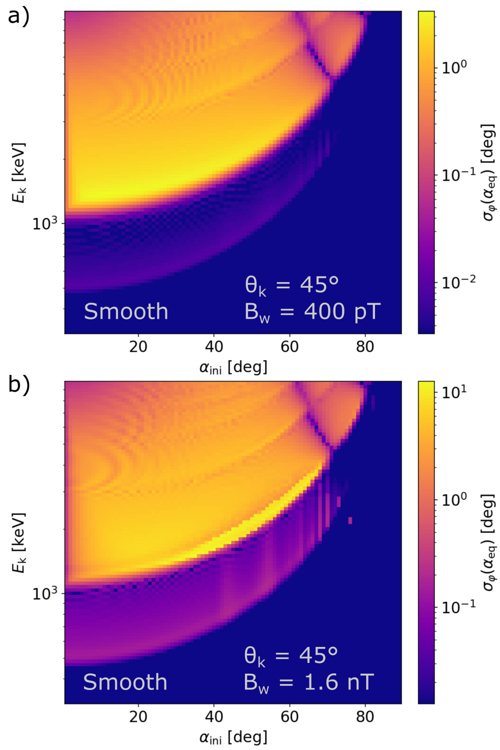

Figure 4. Standard deviation $\sigma_\varphi(\alpha_{\rm eq})$ in equatorial pitch angle for propagation along the smooth EMIC wave packet (quarter-bounce). The color bar has a fixed range starting at $10^{-3}\max(\sigma_\varphi(\alpha_{\rm eq}))$. a) The low-amplitude case ($B_{\rm w}/B_{\rm 0eq} = 0.0016$) shows that the diffusive scattering due to fractional resonances is very weak compared to the fundamental resonance. b) For higher amplitudes, the relative strength of the fractional resonance increases.

4 Conclusion

We performed test-particle simulations of electrons propagation through oblique EMIC waves and observed a new type of interaction identified as nonlinear fractional resonance. A similar type of resonance has been noticed before in relation to Langmuir waves [14], but to the best of our knowledge, it has never been discussed before in the context of cyclotron resonance and electron pitch angle diffusion. We have shown that under favorable plasma conditions, the fractional resonance of order $n=-1/2$ can contribute to pitch angle scattering (and subsequent precipitation) of subrelativistic electrons. Under a simple constant-amplitude wave model, the contribution to scattering from the fractional resonance can be comparable to the nonresonant scattering as long as the EMIC wave is strong and at least moderately oblique ($\theta_{\rm k} \approx 45^\circ$). While only hydrogen band waves were shown here, additional analysis confirms the same behavior for the helium band. We conclude that this new type of resonance may contribute to subrelativistic electron precipitation in a limited energy range; however, further investigation into the occurrence and amplitudes of oblique hydrogen-band and helium-band EMIC waves is needed to assess the importance of this type of scattering.

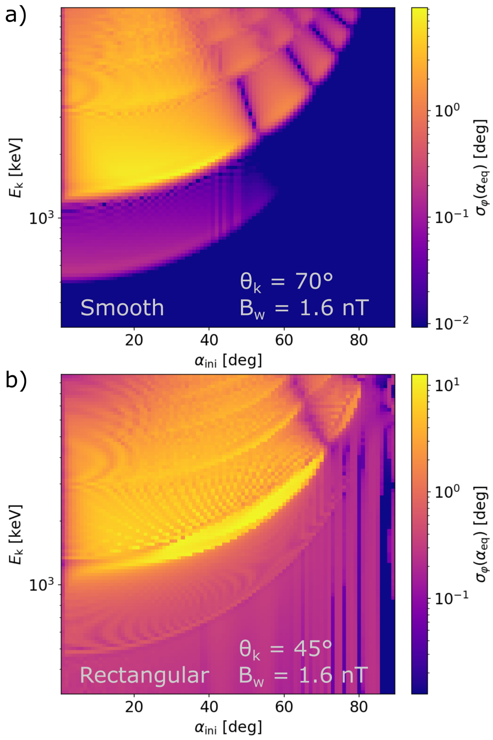

Figure 5. Similar to Figure 4b, but the wave normal angle in panel a) is increased to $\theta_k = 70°$, and the amplitude profile in panel b) has been changed to rectangular, introducing strong nonresonant scattering. The peculiar vertical stripes, which are very apparent at higher pitch angles, are related to particle oscillations at mirror points located within the wave field and are not important for our analysis.

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