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A Novel Array Response Control Algorithm via Oblique Projection

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Abstract

This paper presents a novel array response control algorithm and its application to array pattern synthesis. The proposed algorithm considers how to flexibly and precisely adjust the array responses at multiple points, on the basis of one given weight vector. With the principle of adaptive beamforming, it is shown that the optimal weight vector for array response control can be equivalently obtained with a different manner, in which a linear transformation is conducted on the quiescent weight. This new strategy is utilized to realize multi-point precise array response control from one given weight vector, and it obtains a closed-form solution. A careful analysis shows that the response levels at given points can be independently, flexibly and accurately adjusted by simply varying the parameter vector, and that the uncontrolled region remains almost unchanged. By applying the proposed algorithm, an effective pattern synthesis approach is devised. Simulation results are provided to demonstrate the performance of the proposed algorithm.

1 Introduction

Array antenna has found extensive applications in the fields like radar, wireless communication, remote sensing, navigation and so forth [1]. In general, how to flexibly control the array response is of great significance for practical application. In radar systems, for instance, it is an effective manner to suppress undesirable interference by shaping a beampattern with fixed nulls. In some communication systems, it facilitates user reception by synthesizing multiple-beam patterns.

Quite a number of approaches to array response control have been devised during the past several decades. For example, the so-called linearly constrained minimum variance (LCMV) method [2] is able to control array responses of specific points by imposing linear constraints on the output of beamformer. A variant of LCMV is reported in [3] to adjust mainlobe response and control sidelobe level. A simple iterative algorithm is proposed in [4] to precisely adjust the responses of certain points to their desirable levels. It should be noticed that in general the aforementioned methods are lack of flexibilities, because they cannot control array response under the condition of any given weight vectors.

To tackle the above drawback, an accurate array response control (A^2RC) algorithm is reported in [5]. It has been shown that this method can accurately adjust the response at a given direction to an arbitrary level by making some simple modification to the initial weight vector. However, it is worth pointing out that the A^2RC approach can only deal with the response control of one single point. More recently, in [6], a multi-point accurate array response control (MA^2RC) method has been developed to overcome this shortcoming of A^2RC. Unfortunately, the array responses are interrelated in MA^2RC, so that the procedure of MA^2RC has to be completely re-conducted even if only a slight change of the desired level is needed.

The imperfections of the existing approaches motivate us to develop a new array response control algorithm. More precisely, an innovative flexible and precise array response control scheme is developed. First, it is shown that the classical optimal beamformer can be equivalently realized by making a linear transformation on the quiescent weight vector, and then the array responses at the directions of interferences can be alternatively adjusted by changing the parameter vector in the transformation matrix. On this basis, an extension to the generalized array response control is devised by carrying out a linear transformation on any given weight vector. In such a manner, a closed-form expression of weight is obtained and the responses of multiple points can be independently and accurately controlled, with other uncontrolled sectors staying almost unaltered. The application of the devised algorithm on pattern synthesis [7, 8, 9, 10] can be easily carried out and its superiority is clearly observed as we will show later.

2 Preliminaries

2.1 Adaptive Beamforming

According to the principle of adaptive beamforming, the beampattern can be adaptively synthesized with the data received. In this case, the weight vector $\mathbf{w}$ is called adaptive beamformer. It is known that the optimal weight vector $\mathbf{w}_{opt}$, which maximizes the signal-to-interference-plus-noise ratio (SINR), is given by $\mathbf{w}_{opt} = \alpha \mathbf{R}_{n+i}^{-1}\mathbf{a}(\theta_0)$, where α is the normalization factor that does not affect the output SINR, $\mathbf{a}(\theta_0)$ is the N-dimensional signal steering vector, $\mathbf{R}_{n+i}$ denotes the $N \times N$ noise-plus-interference covariance matrix. By defining the normalized noise-plus-interference covariance matrix [11] as $\mathbf{B}=\mathbf{R}_{n+i}/\sigma_n^2=\mathbf{I}+\sum_{q=1}^{Q}\beta_q\mathbf{a}(\theta_q)\mathbf{a}^{\mathrm{H}}(\theta_q)$, with β_q and $\mathbf{a}(\theta_q)$ representing the interference-to-noise ratio (INR) and steering vector of the qth interference, respectively, the optimal weight vector can be expressed as

$$\mathbf{w}_{\star}=\mathbf{B}^{-1}\mathbf{a}(\theta_0). \tag{1}$$

Obviously, it is seen that $\mathbf{w}_{\star}$ is equal to $\mathbf{w}_{opt}$ up to a scalar.

2.2 Oblique Projection

Assume that $\mathbf{Z}$ is a complex matrix of size $m\times(p+l)$ having full column rank, and $\mathbf{Z}$ can be partitioned as $\mathbf{Z}=\begin{bmatrix}\mathbf{G} & \mathbf{S}\end{bmatrix}$, where $\mathbf{G}\in C^{m\times p}$ and $\mathbf{S}\in C^{m\times l}$. Let us use $\mathscr{R}(\cdot)$ to denote the range space of the input matrix. Then, the well-known formula to build an orthogonal projection whose range is $\mathscr{R}(\mathbf{Z})$ is $\mathbf{P_Z}=\mathbf{Z}(\mathbf{Z}^{\mathrm{H}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathrm{H}}$. The $\mathbf{P_Z}$ is termed as the orthogonal projector onto $\mathscr{R}(\mathbf{Z})$. Accordingly, the orthogonal projector whose range is $\mathscr{R}^{\perp}(\mathbf{Z})$ (representing the orthogonal complementary space of $\mathscr{R}(\mathbf{Z})$) is given by $\mathbf{P}_{\mathbf{Z}}^{\perp}=\mathbf{I}-\mathbf{P_Z}$.

Projection matrices that are not orthogonal are referred to as oblique projections [12]. To see this, we can decomposes $\mathbf{P_Z}$ as $\mathbf{P_Z}=\mathbf{P}_{[\mathbf{G}\ \mathbf{S}]}=\mathbf{E_{G|S}}+\mathbf{E_{S|G}}$, where $\mathbf{E_{G|S}}$ and $\mathbf{E_{S|G}}$ are given by [12]

$$\mathbf{E_{G|S}}=\begin{bmatrix}\mathbf{G} & \mathbf{0}\end{bmatrix}(\mathbf{Z}^{\mathrm{H}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathrm{H}}=\mathbf{G}(\mathbf{G}^{\mathrm{H}}\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{H}}\mathbf{P}_{\mathbf{S}}^{\perp}$$
$$\mathbf{E_{S|G}}=\begin{bmatrix}\mathbf{0} & \mathbf{S}\end{bmatrix}(\mathbf{Z}^{\mathrm{H}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathrm{H}}=\mathbf{S}(\mathbf{S}^{\mathrm{H}}\mathbf{P}_{\mathbf{G}}^{\perp}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{H}}\mathbf{P}_{\mathbf{G}}^{\perp}.$$

It can be readily verified that $\mathbf{E_{G|S}G}=\mathbf{G}$, $\mathbf{E_{G|S}S}=\mathbf{0}$, $\mathbf{E_{S|G}G}=\mathbf{0}$ and $\mathbf{E_{S|G}S}=\mathbf{S}$. In [12], $\mathbf{E_{G|S}}$ and $\mathbf{E_{S|G}}$ are named as oblique projectors. More precisely, the oblique projector $\mathbf{E_{G|S}}$ projects vectors onto $\mathscr{R}(\mathbf{G})$ along the direction that is parallel to $\mathscr{R}(\mathbf{S})$, and $\mathbf{E_{S|G}}$ projects onto $\mathscr{R}(\mathbf{S})$ along the direction that is parallel to $\mathscr{R}(\mathbf{G})$.

3 Flexible Array Response Control Algorithm

3.1 An Equivalent Realization of Optimal Beamformer

As is known to all, the optimal weight vector (1) is capable of rejecting the undesirable interferences by shaping specific notches accordingly. In general, different sets of INRs contribute to different array responses at the directions of interferences. Owing to this fact, array responses can thus be adjusted by imposing virtual interferences [8] and carefully selecting appropriate INRs. Unfortunately, the array responses are mutually affected by INRs. Moreover, the scheme of response control by imposing virtual interferences cannot be extended directly to a more generalized case, in which the array responses of specific points are needed to be adjusted from a given weight vector.

To circumvent these shortcomings, an equivalent realization of the optimal beamformer is devised. For notation convenience, we first define

$$v(i,j)=\mathbf{a}^{\mathrm{H}}(\theta_i)\mathbf{a}(\theta_j). \tag{2}$$

Then, taking advantage of the principle of oblique projection, the following proposition can be developed.

Proposition 1: Suppose that $\mathbf{a}(\theta_0),\mathbf{a}(\theta_1),\cdots,\mathbf{a}(\theta_Q)$ are linearly independent ($Q+1\leq N$) and $v(q,0)\neq 0$ ($q=1,\cdots,Q$). Define $\mathbf{A}=[\mathbf{a}(\theta_0),\cdots,\mathbf{a}(\theta_Q)]$, then for $\forall\beta_1,\cdots,\beta_Q$, there exist $\eta=[\eta_1,\eta_2,\cdots,\eta_Q]^{\mathrm{T}}$ such that

$$\mathbf{w}_{ob}=\mathbf{T}_{\eta}\mathbf{w}_0=c_{ob}\mathbf{w}_{\star} \tag{3}$$

where $\mathbf{w}_0=\mathbf{a}(\theta_0)$, c_{ob} is a constant, and $\mathbf{T}_{\eta}$ is a transformation matrix defined as

$$\mathbf{T}_{\eta}=\left(\mathbf{I}-\mathbf{E}^{\mathrm{H}}_{\mathbf{A}_{0-}|\mathbf{a}(\theta_0)}\right)+\sum_{q=1}^{Q}\eta_q\mathbf{E}^{\mathrm{H}}_{\mathbf{a}(\theta_q)|\mathbf{A}_{q-}} \tag{4}$$

where $\mathbf{E}_{\mathbf{A}_{0-}|\mathbf{a}(\theta_0)}$ and $\mathbf{E}_{\mathbf{a}(\theta_q)|\mathbf{A}_{q-}}$ are oblique projectors, $\mathbf{A}_{q-}$ ($q=0,\cdots,Q$) stands for the resultant matrix by removing the column $\mathbf{a}(\theta_q)$ from $\mathbf{A}$, i.e., $\mathbf{A}_{q-}=[\mathbf{a}(\theta_0),\cdots,\mathbf{a}(\theta_{q-1}),\mathbf{a}(\theta_{q+1}),\cdots,\mathbf{a}(\theta_Q)]$, particularly, we have $\mathbf{A}_{0-}=[\mathbf{a}(\theta_1),\cdots,\mathbf{a}(\theta_Q)]$ and $\mathbf{A}_{Q-}=[\mathbf{a}(\theta_0),\cdots,\mathbf{a}(\theta_{Q-1})]$.

Proof: *First, according to the definition of oblique projector, we have*

$$\begin{aligned}\mathbf{E}_{\mathbf{a}(\theta_q)|\mathbf{A}_{q-}}&=\mathbf{a}(\theta_q)(\mathbf{a}^{\mathrm{H}}(\theta_q)\mathbf{P}^{\perp}_{\mathbf{A}_{q-}}\mathbf{a}(\theta_q))^{-1}\mathbf{a}^{\mathrm{H}}(\theta_q)\mathbf{P}^{\perp}_{\mathbf{A}_{q-}}\\&=\xi_q\mathbf{a}(\theta_q)\mathbf{a}^{\mathrm{H}}(\theta_q)(\mathbf{I}-\mathbf{P}_{\mathbf{A}_{q-}}),\ q=0,\cdots,Q\end{aligned} \tag{5}$$

where ξ_q and $\mathbf{P}_{\mathbf{A}_{q-}}$ are respectively denoted as

$$\xi_q=(\mathbf{a}^{\mathrm{H}}(\theta_q)\mathbf{P}^{\perp}_{\mathbf{A}_{q-}}\mathbf{a}(\theta_q))^{-1}\in R \tag{6}$$
$$\mathbf{P}_{\mathbf{A}_{q-}}=\mathbf{A}_{q-}(\mathbf{A}^{\mathrm{H}}_{q-}\mathbf{A}_{q-})^{-1}\mathbf{A}^{\mathrm{H}}_{q-}. \tag{7}$$

Recalling the definition of $\mathbf{T}_{\eta}$ in (4) and $\mathbf{w}_0=\mathbf{a}(\theta_0)$, and utilizing the properties of the oblique projection, it can be derived that

$$\begin{aligned}\mathbf{w}_{ob}&=\left(\mathbf{I}-\mathbf{E}^{\mathrm{H}}_{\mathbf{A}_{0-}|\mathbf{a}(\theta_0)}\right)\mathbf{w}_0+\sum_{q=1}^{Q}\eta_q\mathbf{E}^{\mathrm{H}}_{\mathbf{a}(\theta_q)|\mathbf{A}_{q-}}\mathbf{w}_0\\&=\mathbf{E}^{\mathrm{H}}_{\mathbf{a}(\theta_0)|\mathbf{A}_{0-}}\mathbf{a}(\theta_0)+\sum_{q=1}^{Q}\eta_q\cdot[\mathbf{a}(\theta_q),\ \mathbf{A}_{q-}]\mathbf{d}_q\\&=\mathbf{A}\mathbf{d}_0+\sum_{q=1}^{Q}\eta_q\mathbf{A}\mathbf{y}_q\\&=\mathbf{A}\mathbf{Y}_0\bar{\eta}\end{aligned} \tag{8}$$

where $\mathbf{d}_q$ is given by

$$\mathbf{d}_q=\xi_q\cdot v(q,0)\begin{bmatrix}1\\-(\mathbf{A}^{\mathrm{H}}_{q-}\mathbf{A}_{q-})^{-1}\mathbf{A}^{\mathrm{H}}_{q-}\mathbf{a}(\theta_q)\end{bmatrix} \tag{9}$$

$\bar{\eta}=[1,\ \eta]^{\mathrm{T}}$ *and* $\mathbf{Y}_0=[\mathbf{d}_0,\ \mathbf{y}_1,\cdots,\mathbf{y}_Q]$. *Here,* $\mathbf{y}_q$ *is a* $(Q+1)$*-dimensional vector that permutates the entries of*

$\mathbf{d}_q$ *($q = 1, \cdots, Q$). It can be validated (the details are omitted in this paper due to the space limitation) that* $\mathbf{Y}_0$ *is an invertible matrix provided that* $\mathbf{a}(\theta_0), \cdots, \mathbf{a}(\theta_Q)$ *are linearly independent and* $v(q,0) \neq 0$ *for* $q = 1, \cdots, Q$. *This implies that* $\mathbf{AY}_0$ *has full column rank, and therefore, spans the same column space as* $\mathbf{A}$*, i.e.,* $R(\mathbf{AY}_0) = \mathscr{R}(\mathbf{A})$. *On the other hand, by applying the Woodbury Lemma to the matrix inversion, it can be obtained that* $\mathbf{w}_\star \in \mathscr{R}(\mathbf{A})$. *As a consequence, it can be concluded that there must exist a certain* η *such that* $\mathbf{w}_{ob} = c_{ob}\mathbf{w}_\star$.

Proposition 1 indicates that the multi-point array responses control can be alternatively achieved by selecting the parameters η_q ($q = 1, \cdots, Q$) and then constructing the weight vector $\mathbf{w}_{ob}$ by (3). In fact, the response adjustments of multiple points (at most $N-1$) are mutually unaffected by $\mathbf{w}_{ob}$ in (3), as shall be explained later.

3.2 Generalization to Array Response Control

Formulation (3) in Proposition 1 suggests that the response levels of specific points can be adjusted by carrying out a linear transformation on the previous weight vector. In light of the basic model of (3), and for a given weight $\mathbf{w}_{k-1}$, we propose to find out a desirable weight vector $\mathbf{w}_k$ that is able to adjust the responses at $\theta_1, \theta_2, \cdots, \theta_Q$ to certain levels, by selecting the parameter vector η_k and conducting a linear transformation on $\mathbf{w}_{k-1}$ as

$$\mathbf{w}_k = \mathbf{T}_{\eta_k}\mathbf{w}_{k-1} \tag{10}$$

where the transformation matrix $\mathbf{T}_{\eta_k}$ is given in (4) by setting the subscript η_k as $\eta_k = [\eta_{k,1}, \eta_{k,2}, \cdots, \eta_{k,Q}]^{\mathrm{T}}$.

To see the merits of (10), we recall the property of oblique projection and obtain the following facts that $\mathbf{E}_{\mathbf{a}(\theta_q)|\mathbf{A}_{q-}}\mathbf{a}(\theta_0) = \mathbf{0}$ ($q = 1, \cdots, Q$), and $\mathbf{E}_{\mathbf{A}_{0-}|\mathbf{a}(\theta_0)}\mathbf{a}(\theta_0) = \mathbf{0}$. As a result, the weight $\mathbf{w}_k$ in (10) satisfies

$$\begin{aligned}\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_0) &= \mathbf{w}_{k-1}^{\mathrm{H}}\mathbf{T}_{\eta_k}^{\mathrm{H}}\mathbf{a}(\theta_0)\\ &= \mathbf{w}_{k-1}^{\mathrm{H}}\Big[(\mathbf{I}-\mathbf{E}_{\mathbf{A}_{0-}|\mathbf{a}(\theta_0)}) + \sum_{q=1}^{Q}\eta_{k,q}^{*}\mathbf{E}_{\mathbf{a}(\theta_q)|\mathbf{A}_{q-}}\Big]\mathbf{a}(\theta_0)\\ &= \mathbf{w}_{k-1}^{\mathrm{H}}\mathbf{a}(\theta_0).\end{aligned} \tag{11}$$

In other words, the response output at $\mathbf{a}(\theta_0)$ remains unaltered after carrying out this specific linear transformation (i.e. (10)) on the previous weight $\mathbf{w}_{k-1}$. This property facilitates the task of normalized response control, since the response level at beam axis remains unchanged.

More importantly, for the given i and q satisfying $i, q = 1, 2\cdots, Q$, one learns that

$$\mathbf{E}_{\mathbf{a}(\theta_i)|\mathbf{A}_{i-}}\mathbf{a}(\theta_q) = \begin{cases}\mathbf{a}(\theta_q), & \text{if } i = q\\ \mathbf{0}, & \text{if } i \neq q\end{cases} \tag{12}$$

and that $(\mathbf{I}-\mathbf{E}_{\mathbf{A}_{0-}|\mathbf{a}(\theta_0)})\mathbf{a}(\theta_q) = \mathbf{0}$ for $\forall q = 1, \cdots, Q$. Therefore, the weight vector $\mathbf{w}_k$ in (10) satisfies

$$\begin{aligned}\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_q) &= \mathbf{w}_{k-1}^{\mathrm{H}}\mathbf{T}_{\eta_k}^{\mathrm{H}}\mathbf{a}(\theta_q)\\ &= \mathbf{w}_{k-1}^{\mathrm{H}}\Big[(\mathbf{I}-\mathbf{E}_{\mathbf{A}_{0-}|\mathbf{a}(\theta_0)}) + \sum_{i=1}^{Q}\eta_{k,i}^{*}\mathbf{E}_{\mathbf{a}(\theta_i)|\mathbf{A}_{i-}}\Big]\mathbf{a}(\theta_q)\\ &= \eta_{k,q}^{*}\mathbf{w}_{k-1}^{\mathrm{H}}\mathbf{a}(\theta_q).\end{aligned} \tag{13}$$

From this important result in (13), we know that for the given $q = 1, \cdots, Q$, the term $\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_q)$ is a simple scaling of the previous $\mathbf{w}_{k-1}^{\mathrm{H}}\mathbf{a}(\theta_q)$ with scaling factor being $\eta_{k,q}^{*}$.

Combine the results of (11) and (13), one gets $L_k(\theta_q, \theta_0) = |\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_q)|^2/|\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_0)|^2 = |\eta_{k,q}|^2 L_{k-1}(\theta_q, \theta_0)$ with $L_k(\theta, \theta_0)$ denoting the normalized response of $\mathbf{w}_k$. It is shown that the resultant $L_k(\theta_q, \theta_0)$ is amplified by $|\eta_{k,q}|^2$, when compared to the previous $L_{k-1}(\theta_q, \theta_0)$. In addition, for a fixed index q, the resultant response level $L_k(\theta_q, \theta_0)$ has no relation with the other $\eta_{k,i}$'s, where $i = 1, \cdots, Q$ and $i \neq q$. As a consequence, the response level at θ_q can be independently adjusted by selecting $\eta_{k,q}$ in $\mathbf{T}_{\eta_k}$, with other responses at θ_i ($i = 1, \cdots, Q$ and $i \neq q$) staying unchanged.

Consequently, for a given weight $\mathbf{w}_{k-1}$ and the prescribed array responses control task, e.g., $L_k(\theta_q, \theta_0) = \rho_{k,q}$ ($q = 1, \cdots, Q$), we can simply set the ultimate η_k as

$$\eta_k = \left[\sqrt{\frac{\rho_{k,1}}{L_{k-1}(\theta_1, \theta_0)}}, \cdots, \sqrt{\frac{\rho_{k,Q}}{L_{k-1}(\theta_Q, \theta_0)}}\right]. \tag{14}$$

In such a manner, a closed-form expression of $\mathbf{w}_k$ can be obtained for the given response control requirement. Interestingly, if η_k is taken as the all-one vector, it yields $\mathbf{T}_{\eta_k} = \mathbf{I}$ and thus $\mathbf{w}_k = \mathbf{w}_{k-1}$.

To have a better understanding of the weight vector $\mathbf{w}_k$ in (10), we can derive that $\mathbf{w}_k = \mathbf{T}_{\eta_k}\mathbf{w}_{k-1} = \mathbf{P}_{\mathbf{A}}^{\perp}\mathbf{w}_{k-1} + \mathbf{AY}_{k-1}\bar{\eta}_k$, where $\mathbf{Y}_{k-1}$ is an invertible matrix and $\bar{\eta}_k = [1,\ \eta_k]^{\mathrm{T}}$. A careful observation shows that the component on $\mathscr{R}^{\perp}(\mathbf{A})$ stays unchanged when carrying out the operator (10). In such a manner, the response variation at the uncontrolled point θ_p (where $p \neq 0, 1, \cdots, Q$) would be small.

4 Application to Pattern Synthesis

The proposed approach can be directly applied to pattern synthesis. Specifically, for a given weight vector (can be arbitrarily initialized), multiple directions are first determined according to the current pattern and the desired pattern (denoted by $L_d(\theta)$). These angles can be in either sidelobe region or mainlobe region. For sidelobe synthesis, we select several peak angles where the response deviations (from the desired level) are relatively large. For mainlobe synthesis, few discrete angles (not required to be peak angles) where the responses deviate large from the desired ones are chosen. Once those angles have been picked out, the proposed array response control algorithm is applied (i.e., updating the weight by (10)) to adjust the corresponding responses to their desired values. This step is iteratively carried out until the beampattern is satisfactorily synthesized.

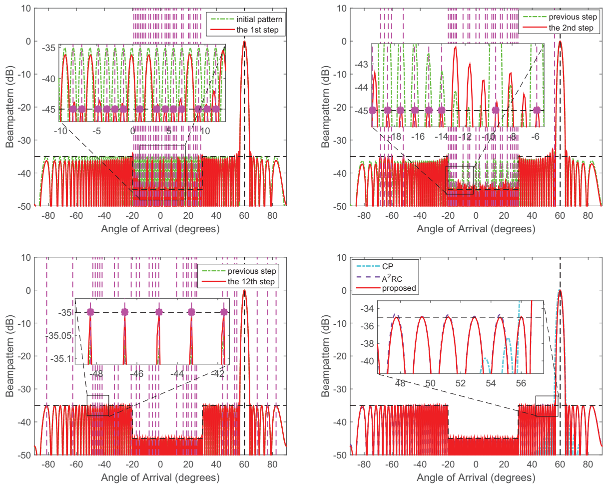

Figure 1. Resultant patterns. (a) Result at the 1st step. (b) Result at the 2nd step. (c) Result at the 12th step. (d) Result comparison.

5 Numerical Results

In this section, pattern synthesis for a large linearly half-wavelength-spaced array with $N = 100$ isotropic elements is considered. The desired pattern steers at $\theta_0 = 60^\circ$ with a nonuniform sidelobe level. More specifically, the upper level is -45dB in the sidelobe region $[-20^\circ, 30^\circ]$ and -35dB in the rest of the sidelobe region. Apparently, the desired pattern is similar to Chebyshev pattern. For this reason, we take the initial weight of the proposed approach as the Chebyshev weight with a -35dB of sidelobe attenuation, in the hope that the synthesis procedure can be simplified.

In this scenario, we select $Q = 30$ sidelobe peak angles in each step and then adjust their responses to the desired levels by employing the proposed approach. Several intermediate results of the proposed method are demonstrated in Fig. 1, where we can clearly see that it only requires 12 steps to synthesize a satisfactory beampattern. At each step of response control, the response levels of selected peak angles are precisely adjusted to their desired values, with other uncontrolled region keeping almost unaltered. The comparison with other existing approaches is displayed in Fig. 1(d). It is observed from the resultant sidelobe that the pattern envelope of the convex programming method in [7] is not aligned with the desired one. A careful observation shows that the A^{2}RC method in [5] is outperformed by the the proposed approach.

6 Conclusions

In this paper, a novel array response control scheme has been developed to precisely control the array responses of multiple points. The devised algorithm stems from the adaptive array theory. It has been shown that the optimal weight vector can be equivalently obtained by taking a linear transformation on the quiescent weight. We have generalized this concept to realize responses control from a given weight vector. Analysis shows that the proposed scheme is able to adjust array responses of multiple points independently, with other uncontrolled points staying almost unchanged. A closed-form expression of the proposed method is obtained and its application on pattern synthesis has been demonstrated. As a future work, we shall consider the extension of the proposed approach to response control under non-ideal circumstances.

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