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Global Point-to-Point Ionospheric Ray Tracing Using the Direct Variational Method

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Abstract

Global optimization of phase distance functional is a promising approach to the multipath problem of the point-to-point ionospheric ray tracing. The approach is involves systematic algorithm where all relevant rays between the fixed points are found one after another in a systematic manner, without the need to provide an accurate initial estimation for each solution. In particular for each local ray searching the direct variational method is applied. The global optimization is applied to a point-to-point ionospheric ray tracing, where the modeled ionosphere is three-dimensional inhomogeneous medium.

1 Introduction

The geometric optics approximation, which has become widespread for a propagation of wave in various fields of science, is widely used in numerical modeling of the propagation of high frequency (HF) radio waves. At present, many numerical models have been developed on the numerical solution of the eikonal equation [1, 2, 3, 4]. This approach allows to effectively solve the Cauchy problem with given angular parameters. In the case of a boundary-value problem, when the positions of the transmitter and receiver points are fixed, and the initial angles are unknown, the shooting method is additionally applied.

Coleman [5] proposed an alternative approach to the point-to-point ionospheric ray tracing problem. The approach directly uses Fermat's variational principle and bases on transforming some initial trajectory to an optimal configuration. There is, however, a complication arising from different characters of extrema that need to be identified. While high rays correspond to minima of the phase distance functional and can therefore be found by a straightforward minimization of the objective function, low rays correspond to saddle points [6, 7], which are difficult to locate. Coleman [5] solves the problem of low rays by applying the Newton-Raphson method that requires a good initial guess for the solution. However, the selection of the initial estimate is a difficult task, especially for a strongly perturbed ionosphere. At the same time, multipath propagation creates additional difficulties, since it is necessary to sample the initial approximations so that the iterative procedure converges to various extrema.

In this paper we consider a computational scheme for global point-to-point ray tracing, where all relevant rays between the fixed points are found one after another in a systematic manner, without the need to provide an accurate initial estimate for each solution.

2 Direct Variational Method

The phase distance of the ray in isotropic medium is given by the following equation:

$$S(\mathbf{r}) = \int_A^B n(\vec{r})dl. \qquad (1)$$

Here, the integration is performed along the ray path, which connects transmitter A and receiver B; $n(\vec{r})$ – refractive index at point $\vec{r}$; dl – the length element along the ray. According to Fermat's principle, ionospheric point-to-point ray tracing then reduces to an identification of stationary points of function $S(\mathbf{r})$. To simplified problem, stationary points of only two types are relevant, i.e. minima and first order saddle points. The former correspond to high ionospheric rays while the latter correspond to low rays [7].

This can be visualized as a three-dimensional surface, for which a reduced description of the model can be used. This is accomplished by choosing a three-point representation of the radio ray, where two points are fixed according to the boundary conditions and the third one defines the apex position (hypothetical reflection point) [7]. With this representation, the radio ray is completely defined by two essential variables – horizontal and vertical coordinates of the apex point – and a surface contour of the phase distance can be constructed.

Resulting surface of the phase distance is presented in Fig. 1 (a detailed description of the ionospheric model and the ray tracing for this example are presented in [8]), which demonstrates that high rays correspond to minima of the phase distance, while the low rays correspond to saddle points. This

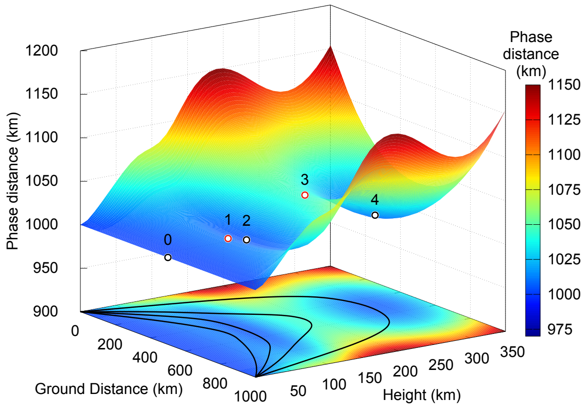

Figure 1. Surface of the phase distance for frequency 12 MHz in the two-layer ionospheric model [8]. The projection of the phase distance surface by contour map is presented on the bottom plane. Black solid lines show high and low ionospheric rays. The numbers indicate the positions of the minima and saddle points corresponding to the high and low rays.

explains why high rays can be reliably identified by direct minimization. Saddle points are, however, difficult to locate. The difficulty arises from the need to minimize the phase distance with respect to all but one degree of freedom for which a maximization should be carried out and it is not known a priori which degree of freedom should be treated differently. The approach for searching both the high and low rays is presented below.

The high rays can be obtained by simply minimizing the phase distance by Eq. (1). The minimization is guided by an antigradient or generalized force, **F**, components of which are given by the following equation:

$$\mathbf{F}^h = -\nabla S(\mathbf{r})|_\perp + \mathbf{F}^s, \tag{2}$$

where $\mathbf{F}^h$ – generalized force for the high rays, $\mathbf{F}^s$ – artificial spring force. A detailed description of the direct minimization is presented in [7].

The low rays correspond to saddle points of the phase distance functional, which are difficult to locate. The main idea of the method of finding saddle points is moved away from the convex region near the minimum and brought to the basin of attraction of a first order saddle point. For this, the Hessian matrix of the functional $S(\mathbf{r})$ is estimated. Taking into account force projections and spring forces, the generalized force guiding the optimization of low rays is therefore defined as:

$$\mathbf{F}^l = \begin{cases} \nabla S(\mathbf{r})|_\perp + \mathbf{F}^s \quad \text{or} \quad (\nabla S(\mathbf{r})|_\perp \cdot \mathbf{Q}_\lambda)\,\mathbf{Q}_\lambda + \mathbf{F}^s, \\ \text{if } \lambda \geqslant 0, \\ -\nabla S(\mathbf{r})|_\perp + 2\,(\nabla S(\mathbf{r})|_\perp \cdot \mathbf{Q}_\lambda)\,\mathbf{Q}_\lambda + \mathbf{F}^s, \\ \text{if } \lambda < 0. \end{cases} \tag{3}$$

Here, λ is the minimal eigenvalue of the Hessian and $\mathbf{Q}_\lambda$ is the corresponding normalized eigenvector, the minimum mode. A detailed description of the saddle point search is presented in [8].

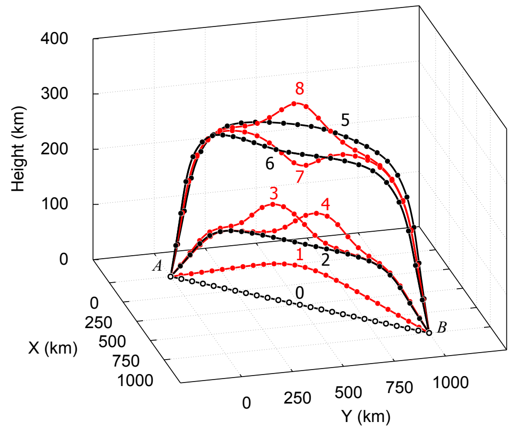

Figure 2. Results of point-to-point ray tracing using the global optimization. The medium is represented by the disturbed two-layer ionospheric model [8]. Operating frequency is 10 MHz. High and low rays are shown with black and red solid lines, respectively. Circles indicate the positions of the ray vertices included in the calculations.

Thus, the formulas (2) and (3) form the basis for the definition of the generalized force. Observe that the difference in the identification of high and low rays is only in the definition of the generalized force.

3 Global Optimization

Based on the relationship of minima and saddle points of the functional $S(\mathbf{r})$, a procedure for searching for multiple rays using the global optimization method has been created [8]. The desired ray path can can be systematically explored by traversing from one high ray to another via low rays. The algorithm for the global ionospheric ray tracing is presented below:

1. Create an initial path (It can be a straight line connecting the fixed points).
2. Find the first high ray by the direct minimization of the phase distance functional.
3. Generate a starting configuration for a low ray optimization in the vicinity of the high ray.
4. Perform a low ray search.
5. Repeat step 3 and step 4 until a predetermined number of unsuccessful attempts to locate new low rays has been exceeded.
6. For each low ray found, generate a starting point for a high ray optimization by moving along the minimum mode in the direction away from the known high ray. The high ray search starts in the vicinity of the low ray.

7. Find an adjacent high ray.
8. For each new high ray, repeat steps 3 through 7 until all relevant rays have been found.

4 Discussion

The global optimization is applied here to the point-to-point ray tracing in the two-layer ionospheric model [8]. To demonstrate the three-dimensional propagation, a local inhomogeneity in the $F2$ layer was added to the background model. The ray tracing was performed between the positions of the endpoints A and B for the operating frequency of 10 MHz. In each local ray searching, the generalized force converges iteratively to zero using the velocity projection optimization method [8].

Application of the global ray tracing method reveals eight rays found by traversing from one minimum of the phase distance $S(\mathbf{r})$ to another via saddle points, including three high rays (rays 2, 5 and 6 in Fig. 2) and five low rays (rays 1, 3, 4, 7 and 8 in Fig. 2). The ionospheric inhomogeneity strongly affects the radio wave propagation. The pair of high rays (rays 5 and 6) skirt the ionospheric inhomogeneity along the shortest optical path and, as a consequence, deviate from the vertical plane exhibiting three-dimensional structure. For comparison, the ray paths have also been calculated using a traditional ray tracing method based on the homing-in approach [9]. Both methods give almost identical results.

Finally, the calculations presented here illustrate the ability of the direct vatiational method and global ray tracing technique to resolve complex ray configurations such as three-dimensional propagation. The certainty about the character of the rays has made it possible to establish an efficient global ray tracing technique where all relevant rays are identified one after another and every local ray search starts at a previously found solution so that there is no need to provide an accurate initial estimate for each ray.

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