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Preamble Design for Data-Aided Synchronization of Single Side Band Continuous Phase Modulation

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Abstract

In this paper, we calculate the Cramer-Rao bound (CRB) for single side band continuous phase modulation signals (SSB-FSK). The CRB is useful to design an optimum preamble sequence to jointly estimate the frequency offset, symbol timing, and carrier phase. The goal is to find preamble sequences that minimize the CRBs of all estimated parameters. The results provide some useful preambles, which can be added to data packets for synchronization in burst-mode SSB-FSK transmissions.

1 Introduction

Synchronization is necessary in demodulation block for a complete information detection process. The synchronization objective is to estimate certain parameters such as frequency offset, symbol timing, and carrier phase. Therefore, it is interesting to find a bound on synchronization performance to perform a comparison benchmark between different synchronizers. One tool to obtain such bound, is the Cramer-Rao bounds (CRBs) [1], which presents a lower limit of the variance of the estimated parameters. The authors in [2, 3, 4] used the CRB to design an optimum data-aided (DA) preamble sequence for joint estimation of all these three synchronization parameters (frequency offset, symbol timing, and carrier phase) for different continuous phase modulation (CPM) schemes [5].

In this paper, we used the same approach to obtain the optimum DA preambles, based on deriving the CRBs of joint estimate of all three synchronization parameters, for single side band continuous phase modulation (SSB-FSK) signals [6].

Using the CRBs, we present the following contributions: we derive the CRBs closed form expression for frequency offset, symbol timing, and carrier phase for joint DA estimation of SSB-FSK signals over additive white Gaussian noise (AWGN). We present the optimum preamble sequences for the specific SSB-FSK presented in [6]. We explain why for SSB-FSK signals it is not possible to obtain a single optimum DA preamble for all synchronization parameters. We present a sub-optimum DA preamble that can be used for jointly estimate of all three parameters achieving a certain reasonable trade-off.

The rest of this paper is structured as follows. Section2 presents the SSB-FSK signal model. In Section 3, we derived the CRBs for SSB-FSK signals for a known preamble sequence. In Section 4, we obtain the optimum preamble sequences via a computer exhaustive search. In Section 5, we show simulation results, that illustrate a comparison between the optimum CRBs preambles sequences, and the sub-optimal one. Finally, conclusions are drawn in Section 6.

2 SSB-FSK Signal Model

The complex envelope of an SSB-FSK signal has the form

$$s(t) = \sqrt{\frac{E_s}{T_s}} e^{j\varphi(t;\alpha)} \tag{1}$$

where E_s is the signal energy per symbol, T_s is the symbol interval. The phase of the signal $\varphi(t;\alpha)$ is represented as

$$\varphi(t;\alpha) = h\sum_i \alpha_i \varphi_0(t-iT_s) \tag{2}$$

where α_i is the information symbol assumed to be independent and identically distributed and takes values in the alphabet $\{0,1\}$ (i.e., no antipodal coding is made in order to maintain the SSB property), h is an integer modulation index ensuring a 2π phase increment. $\varphi_0(t)$ is a Leviton phase-shift function given by

$$\varphi_0(t) = \int_{-\infty}^{t} g(\tau)d\tau \tag{3}$$

where $g(t)$, is the derivative of the phase-shift function (truncated Lorentzian pulse), defined as

$$g(t) = \frac{d\varphi_0(t)}{dt} = \mu\frac{2w}{t^2+w^2}, t \in [-LT_s/2, LT_s/2]$$
$$\int_{-t}^{t} g(\tau)d\tau = \varphi_0(LT_s/2) - \varphi_0(-LT_s/2) = 2\pi, t \geq LT_s/2. \tag{4}$$

Parameter μ is a correcting factor to ensure a 2π phase increment when the frequency pulse $g(t)$ is truncated to a finite $L > 1$ symbol length (this correcting factor is necessary because the 2π phase increment is only achieved for an extremely long/infinite Lorentzian pulse). This factor is defined as the ratio between the total phase increment without any truncation and the one attained after Lorentzian truncation

$$\mu(L) = \frac{2\pi}{\int_{-LT_s/2}^{LT_s/2} \frac{2w}{t^2+w^2} dt} = \frac{\pi}{arctan(\frac{LT_s}{2w})}. \quad (5)$$

3 CRBs for SSB-FSK signals

The complex envelope of the received signal is given by

$$r(t) = \sqrt{\frac{E_s}{T_s}} e^{j(2\pi f_d t+\theta)} e^{j\varphi(t-\tau;\alpha)} + w(t) \quad (6)$$

where f_d is the frequency offset, θ is the unknown carrier phase, τ is the time offset, and $w(t)$ is complex baseband AWGN with zero mean and power spectral density N_0. We take $s(t,\mathbf{u},\alpha)$ as the first term of the signal component of $r(t)$ where $\mathbf{u} = [f_d, \theta, \tau]^T$ is the vector of the unknown but deterministic parameters which are to be jointly estimated at the receiver. We refer to the unbiased estimate of the unknown vector by $\hat{\mathbf{u}}$. In general for estimation, we are interested in finding a lower bound on the variance of the elements to be estimated as a performance metric. Therefore, we use the CRB as a lower bound on the error co-variance matrix $C_{\hat{\mathbf{u}}}$ for the joint estimation of each parameter in $\mathbf{u}$

$$var(\hat{\mathbf{u}}_i) = [C_{\hat{\mathbf{u}}}]_{i,i} \geq I(\mathbf{u})^{-1}_{i.i} \quad (7)$$

where $I(\mathbf{u})$ is the function used to obtain the Fisher information matrix (FIM), given by

$$I(\mathbf{u})_{i,j} = -E[\frac{\partial^2}{\partial \mathbf{u}_i \partial \mathbf{u}_j} ln(p(\mathbf{r};\mathbf{u}))]. \quad (8)$$

In this part, we use the same FIM used in [3] in function of the log-likelihood function (LLF) $\Lambda[r(t);\mathbf{u}]$

$$\begin{aligned} I(\mathbf{u})_{i,j} &= -E\left[\frac{\partial^2 \Lambda[r(t);\mathbf{u}]}{\partial u_i \partial u_j}\right] = \\ & \frac{2}{N_0}\int_0^{T_0} Re\left[\frac{\partial s(t,\mathbf{u},\alpha)}{\partial u_i}\frac{\partial s^*(t,\mathbf{u},\alpha)}{\partial u_j}\right]dt \end{aligned} \quad (9)$$

where $T_0 = L_0 T_s$ is the duration of the transmitted symbols in seconds. Using Eq. 9 we calculated all the FIM elements

$$I_{13} = I_{31} = \frac{-4\pi h(\frac{E_s}{N_0})}{T_s} \sum_{i=0}^{L_0-1} \alpha_i \int_0^{T_0} tg(t-iT_s-\tau)dt \quad (10)$$

$$I_{23} = I_{32} = \frac{-2h(\frac{E_s}{N_0})}{T_s} \sum_{i=0}^{L_0-1} \alpha_i \int_0^{T_0} g(t-iT_s-\tau)dt \quad (11)$$

$$I_{33} = \frac{2h^2(\frac{E_s}{N_0})}{T_s} \sum_{i=0}^{L_0-1}\sum_{j=0}^{L_0-1} \alpha_i\alpha_j \int_0^{T_0} g(t-iT_s-\tau)g(t-jT_s-\tau)dt \quad (12)$$

After obtaining all the elements, the FIM is given by:

$$I = \frac{1}{T_s}\left(\frac{E_s}{N_0}\right)\begin{bmatrix} \frac{8\pi^2 T_0^3}{3} & 2\pi T_0^2 & -4\pi hA \\ 2\pi T_0^2 & 2T_0 & -2hB \\ -4\pi hA & -2hB & 2h^2C \end{bmatrix} \quad (13)$$

where the variables A, B, and C in the matrix are defined as

$$A = \sum_{i=0}^{L_0-1} \alpha_i \int_0^{T_0} tg(t-iT_s-\tau)dt \quad (14)$$

$$B = \sum_{i=0}^{L_0-1} \alpha_i \int_0^{T_0} g(t-iT_s-\tau)dt \quad (15)$$

$$C = \sum_{i=0}^{L_0-1}\sum_{j=0}^{L_0-1} \alpha_i\alpha_j \int_0^{T_0} g(t-iT_s-\tau)g(t-jT_s-\tau)dt. \quad (16)$$

For the three latter variables, we can compute them numerically, but we are going to approximate them using the properties of the CPM phase. At first, we begin with the variable A. By taking $k = t - iT_s - \tau$, A becomes:

$$\begin{aligned} A &= \sum_{i=0}^{L_0-1} \alpha_i \int_{-iT_s-\tau}^{T_0-iT_s-\tau} (k+iT_s+\tau)g(k)dk \\ &= \sum_{i=0}^{L_0-1} \alpha_i \int_{-iT_s-\tau}^{T_0-iT_s-\tau} kg(k)dk + (iT_s+\tau)\int_{-iT_s-\tau}^{T_0-iT_s-\tau} g(k)dk \\ &= \sum_{i=0}^{L0-1} \alpha_i(\Gamma + 2\pi(iT_s+\tau)). \end{aligned} \quad (17)$$

We denoted in the second line the first integral as Γ. Moreover the 2π comes from the integral of the second part (the area defined by the frequency pulse of the SSB-FSK scheme). Based on [3], this approximation is valid only for a full response. However, for partial response, which is our case, we need to truncate the last $[L/2]$ symbols, which makes the effective length of the sequence approximated by $L_0 - [L/2]$. Likewise, for B, we are going to use the same approximation as A to get

$$B = \sum_{i=0}^{L0-1} \alpha_i 2\pi. \quad (18)$$

For C, we are going to use the same equations used in [3], where at first we introduce the correlation function of $g(t)$ as

$$R_g(\tau) = \int_{-\infty}^{+\infty} g(t)g(t+\tau)dt. \quad (19)$$

And the approximation of C in function of R_g

$$C \simeq \sum_{i=0}^{L_0-1}\sum_{j=0}^{L0-1} \alpha_i\alpha_j R_g((i-j)T_s). \quad (20)$$

Based on [3], the CRB is weakly dependent on τ, so we assume that $\tau = 0$ to facilitate the calculation of CRBs in the next part.

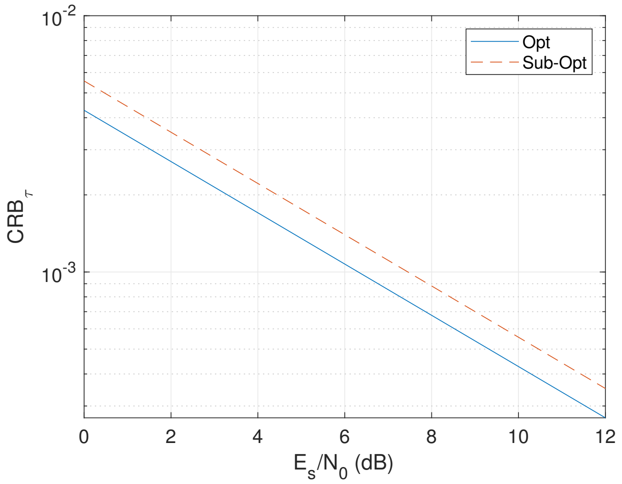

Figure 1. Optimum and sub-optimum symbol timing CRBs for 12SSB-FSK with $w = 0.37$ and $h = 1$, for a number of bits $L_0 = 16$.

4 Optimum and Sub-optimum Training Sequence

We can see that the FIM is a function of the data α, so the performance of the estimator will be based on the data-aided. Therefore, we are going to search for the optimum sequence. However, as mentioned in [4], it is impossible to obtain just one optimum sequence to jointly estimate of all parameters, because of the non-antipodal information symbol α_i (0's or 1's), which it is necessary to maintain the SSB form of the signal. Hence, optimum sequences are different for each of the synchronization parameters. Therefore, after obtaining the best sequence for the estimation of each parameter, we will try to find a sub-optimum sequence for the estimation of all the parameters together achieving a good trade-off.

From Eq.7, we can see that the CRBs functions are based on the inverse of the FIM(I^{-1}). However, since the inverse of the FIM matrix is a time-consuming task, detailed developments are not given, and we are going to show the final result equations of CRBs directly. The CRBs equation for symbol time, frequency offset and carrier phase are respectively given by

$$[I^{-1}]_{3,3} = \frac{T_s}{\frac{E_s}{N_0}} \frac{T_0^3}{2.h^2(CT_0^3 - (3A^2 + (3A - 2BT_0)^2))} \tag{21}$$

$$[I^{-1}]_{1,1} = \frac{T_s}{\frac{E_s}{N_0}} \frac{3}{2\pi^2 T_0} \frac{CT_0 - B^2}{CT_0 - B^2 - \frac{3}{4L_0^2}(2BL_0 - 4A)^2} \tag{22}$$

$$[I^{-1}]_{2,2} = \frac{T_s}{\frac{E_s}{N_0}} \frac{2}{T_0} \frac{CT_0^3 - 3A^2}{CT_0^3 - 3A^2 - (3A - 2BT_0)^2} \tag{23}$$

Now, after computing the CRBs, we can start our computer exhaustive search to find the best sequence. We apply exhaustive search only for $L_0 = 16$ bits, because the exhaustive search is not efficient for higher L_0, due to the vast search space. Our search results can be found in Table 1, for SSB-FSK with pulse length $w = 0.37$, modulation index $h = 1$, and pulse length $L = 12$, which are the same parameters used in [6], for $L_0 = 16$.

Partial response SSB-FSK binary case (L =12)			
Training sequence	CRB$_\tau$	CRB$_{f_d}$	CRB$_\theta$
1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0	0.0043	3.8837e-05	0.1378
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0	0.0314	3.7105e-05	0.1298
1 0 1 0 1 1 1 1 0 1 0 1 1 0 0 0	0.0107	6.9213e-05	0.1250
0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0	0.0056	4.74e-05	0.1252

Table 1. Training sequences for partial response binary SSB-FSK, with 16 bits data-added and $\frac{E_s}{N_0} = 1$. The first three rows present the optimum preamble sequences respectively for: symbol time, frequency offset, and phase carrier. The last row presents the sub-optimum preamble achieving a good trade-off for joint estimation of all three parameters.

Table.1 shows three different sequences in the first three rows that minimize symbol time, frequency offset and carrier phase CRBs, respectively. However, to save throughput we can not send three different preamble. Therefore, we need a unique sequence that can minimize all CRBs at the same time. Thus, we apply a new computer search, but this time we search for the sequence that minimizes an overall combination of CRBs and not each one separately. The sequence is presented in the last row in Table.1. We can see that this sequence slightly increases every single CRB, however it increases the overall performance of all CRBS together (it's best to achieve a unique trade-off). This sequence is presented as a sub-optimum preamble for all CRBs.

5 Simulation Results

We present the Optimum CRBs for symbol timing, frequency offset, and phase carrier respectively in Fig. 1, Fig. 2, and Fig. 3. Moreover, we showed the sub-optimum sequence alongside the optimum one for each parameter. Overall, we note that the sub-optimal sequence increases slightly the CRBs which reduces the synchronization performance, but the degradation is insignificant especially for the phase carrier in Fig. 3.

6 Conclusion

We presented an optimum and sub-optimum Data-Aided (DA) preamble, that can be used to improve the synchronization performance in burst-mode SSB-FSK transmission. We designed the preambles specially for SSB-FSK with pulse length $L = 12$, modulation index $h = 1$ and pulse

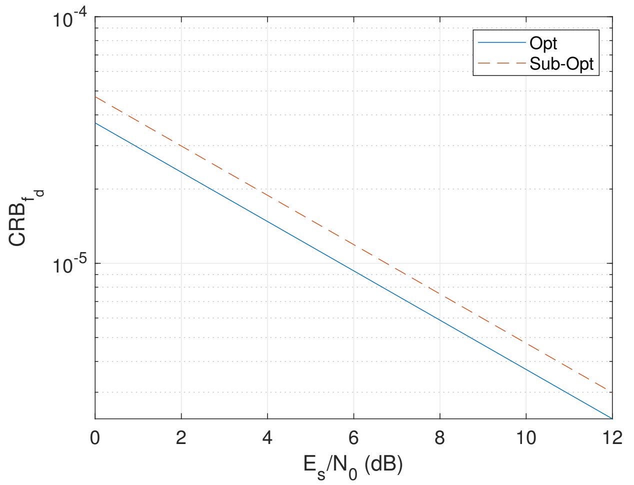

Figure 2. Optimum and sub-optimum frequency offset CRBs for 12SSB-FSK with $w = 0.37$ and $h = 1$, for a number of bits $L_0 = 16$.

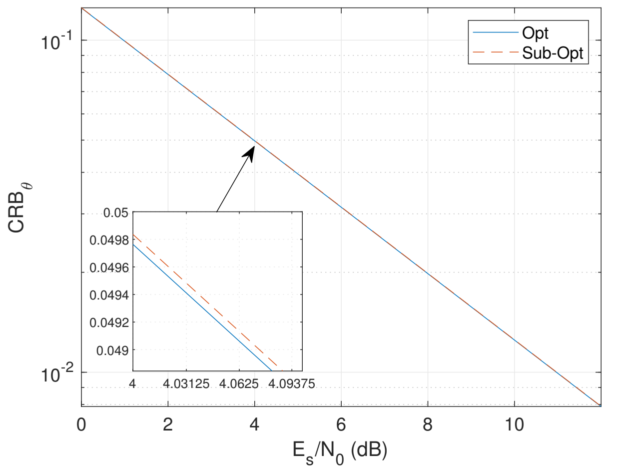

Figure 3. Optimum and sub-optimum carrier phase CRBs for 12SSB-FSK with $w = 0.37$ and $h = 1$, for a number of bits $L_0 = 16$.

with $w = 0.37$. The preamble design may be improved if we apply a search for higher space $L_0 > 16$ by using another computer search method (rather than exhaustive search). One of the methods that can be used for the computer's vast space search is the Genetic Algorithm (GA) [7].

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