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Investigation of Electromagnetic Field Coupling to a Transmission Line Terminated with Non-Vertical Risers

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Abstract

Recently, a simple and efficient method was proposed to take into account non-vertical risers at the end of transmission lines through an equivalent partial inductance. This paper evaluates the frequency above which the inductance needs to be taken into account. In addition, the proposed method is validated in the time domain.

1 Introduction

Transient electromagnetic fields may couple to transmission lines inducing currents and voltages which may cause interferences in the connected equipment. Therefore, it is important to perform numerical simulations to assess the susceptibility of the equipment and the relevant protection systems.

Several models have been used in the literature to estimate the voltages and currents induced on transmission lines due to an impinging transient electromagnetic field [1-5]. These studies are mainly based on the classical transmission line theory and they only consider the case of a transmission line terminated by vertical risers at both ends. However, in some cases, the risers at the end of the transmission lines are not vertical and may have an arbitrary shape. Such a problem can be handled by full-wave methods, which entail high computational cost.

A simple and efficient method taking into account non-vertical risers at the ends of the transmission lines has been proposed recently to solve this problem [6]. In the method, the classical transmission line theory is adopted and the non-vertical risers are taken into account through an equivalent partial inductance.

The aim of this paper is to study the effect of the equivalent inductance and, in particular, the frequencies above which the inductance needs to be taken into account. Moreover, the performance of the proposed method in the time domain will be assessed.

The remainder of this paper is organized as follows. Section 2 describes the basic concept of the proposed approach. Section 3 presents the results associated with several case studies to investigate the performance and the characteristics of the modeling method. Finally, Section 4 presents the conclusions of the work.

2 Basic Concept of the Proposed Approach

In the case where the transmission line is terminated with vertical risers, the field-induced response of the line can be evaluated using the classical transmission line theory [1-3]. In this study, the Agrawal *et al.* [1] model in the frequency domain is adopted:

$$\begin{aligned} &\frac{dV^s(x,\omega)}{dx} + (R' + j\omega L')I(x,\omega) = E_x^e(x,0,h,\omega) \\ &\frac{dI(x,\omega)}{dx} + j\omega C' V^s(x,\omega) = 0 \end{aligned} \tag{1}$$

in which R', L' and C' are, respectively, the per-unit-length resistance, inductance and capacitance of the line, $I(x,\omega)$ is the induced current, $V^s(x,\omega)$ is the scattered voltage, defined as

$$v^s(x,t) = -\int_0^h E_z^s(x,y,z,t)dz \tag{2}$$

and h is the height of the conductor above the ground. The boundary conditions at the two line ends terminated on impedances Z_A and Z_B are given by:

$$\begin{aligned} &V^s(0,\omega) = -Z_A I(0,\omega) + \int_0^h E_z^e(0,0,z,\omega)dz \\ &V^s(L,t) = Z_B I(L,\omega) + \int_0^h E_z^e(L,0,z,\omega)dz \end{aligned} \tag{3}$$

Consider a transmission line terminated by a non-vertical riser that has an arbitrary shape at the left end, as shown in Figure 1.

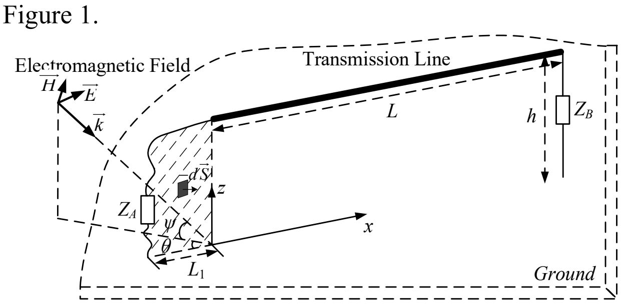


Figure 1. Transmission line with a non-vertical riser excited by an incident plane wave.

Since the riser is not vertical, the source at the termination is the line integral of the exciting electric field along the riser's non-vertical geometry. Moreover, the termination impedance, represented by Z_A in Figure 2, is the series combination of the actual termination and an additional inductive impedance stemming from the more general geometry for the risers. For the sake of completeness, we present the derivation of the boundary conditions [6].

If we focus on the left end of the transmission line, we can write the integration of the electric field along the non-vertical riser as

$$-\int_{Riser} \vec{E}\cdot dl = -I(0)Z_A \tag{4}$$

Separating the total field into the exiting and scattered components, (4) can be written as follows

$$-\int_{Riser} \vec{E}^s\cdot dl - \int_{Riser} \vec{E}^e\cdot dl = -I(0)Z_A \tag{5}$$

Now, applying Ampere-Maxwell's Equation to the scattered field around the loop hashed in Figure 1, we can write

$$\int_{Riser} \vec{E}^s\cdot dl - \int_0 E_z^s dz = -j\omega\iint \vec{B}^s\cdot dS \tag{6}$$

Solving for the first term on the left hand side of (6) and substituting the second term by the scattered voltage given in (2), we obtain

$$\int_{Riser} \vec{E}^s\cdot dl = -V^s - j\omega\iint \vec{B}^s\cdot dS \tag{7}$$

Replacing (7) into (5) and solving for the scattered voltage yields

$$V^s = \int_{Riser} \vec{E}^e\cdot dl - I(0)Z_A - j\omega\iint \vec{B}^s\cdot dS \tag{8}$$

The surface integral in the third term on the right hand side of (8) is a scattered magnetic flux. Considering the assumption of an electrically small line cross section, this flux is proportional to the termination current, through an inductive term. As a result, the boundary condition at the left end can be rewritten as

$$V^s(0,\omega) = -(Z_A + j\omega L_A)I(0,\omega) + \int_{Riser} \vec{E}^e\cdot dl \tag{9}$$

in which [6]

$$L_A = \frac{\iint \vec{B}^s\cdot dS}{I(0)} \tag{10}$$

The surface over which the scattered magnetic field is integrated in shown in Figure 1.

The value of the inductance L_A depends only on the geometry of the riser. Figure 2 illustrates the equivalent circuit for the left hand side of the line based on (9).

Thus, the integrals of the exciting electric field at the two line ends are evaluated along a path defined by the geometry of the risers.

3 Case Studies to Investigate the Effect of the Equivalent Inductance

In this section, three validation examples are considered to further investigate the capability and the applicability of the proposed method. The configuration of the transmission line is shown in Figure 1. The terminal riser at the right end is assumed to be vertical, while two different shapes, namely rectangular and triangular, will be considered for the riser at the left-end, as illustrated in Figures 3a and 3b). The equivalent (partial) inductance L_A will be evaluated using the Biot-Savart law according to (10). The total magnetic flux is computed by numerical integration, taking into account the riser image, assuming the ground as a perfectly conducting plane, as illustrated in Figure 4.

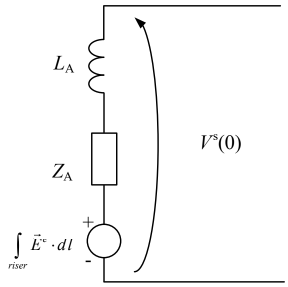
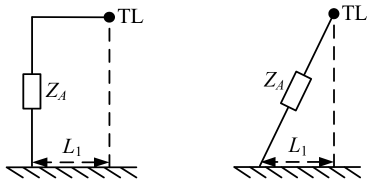
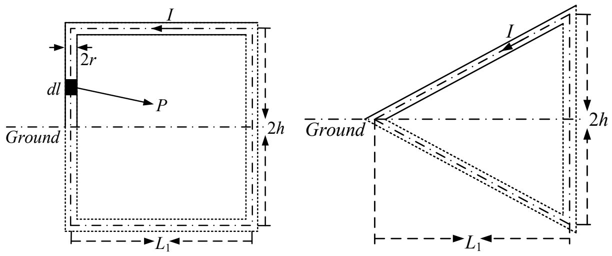


Figure 2. Equivalent circuit for the left termination when the riser is not vertical (adapted from [6]).



(a) (b)

Figure 3. Cross-section of the two considered geometries for the left-end riser. (a) rectangular, and (b) triangular.



(a) (b)

Figure 4. Cross-section of the considered non-vertical geometries for the left-end riser. (a) Rectangular, and (b) triangular.

The field-to-transmission-line coupling equations including the treatment of non-vertical risers are solved using the BLT equations [7]. In order to validate the calculation results, the Numerical Electromagnetics Code NEC-4, a full-wave solver based on the Method of Moments [8], is adopted. In what follows, we will consider a 20-m long wire located at a height of 0.1 m above a perfectly conducting ground. The conductor radius is 1 mm. The azimuth, elevation, and polarization angles of the exciting plane wave are, respectively, 0°, 45° and 0°.

3.1 Validation in the Frequency Domain

In the first example, we present in Figure 5 the comparison between the BLT equations and NEC-4 with the reference case, in which the terminal risers at the both ends are vertical. As expected, it can be seen that the results based on the transmission line theory (BLT equations) are in excellent agreement with full-wave results obtained using NEC-4.

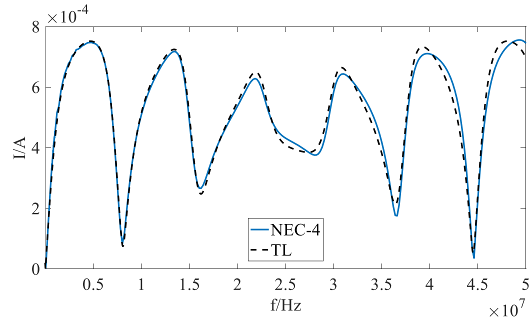
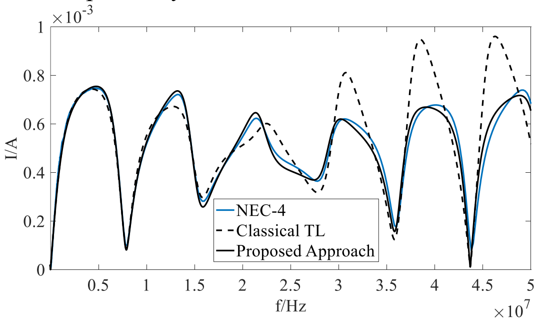
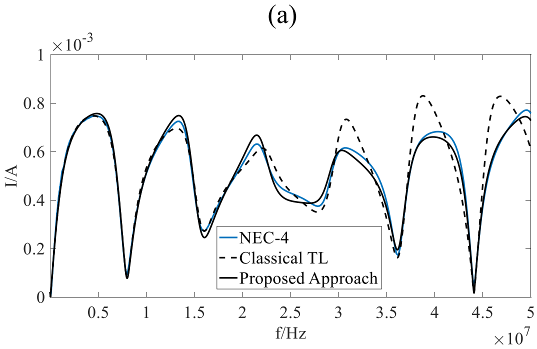


Figure 5. Left-end induced current as a function of frequency. The line is terminated in vertical risers at both ends.

We now consider the two cases for the geometry of the left-end riser shown in Figure 3.
In both cases, the value of L_1 (see Figure 3) was set to 0.5 m. The calculated equivalent inductances L_A for the rectangular (Figure 3a) and for the triangular (Figure 3b) terminations are, respectively, 0.66 μH and 0.49 μH. The line is terminated at both ends in 100 Ω resistive loads. The frequency range of the wave is 10 kHz-50 MHz, and the amplitude of the *E*-field is 1 V/m across the complete frequency spectrum. The calculated results for the induced currents at the left end for both considered cases (rectangular and triangular risers) are shown in Figures 6a and 6b, respectively.



(a)



(b)

Figure 6. Left-end induced currents as a function of frequency when the left end geometry is considered to be: (a) rectangular, and (b) triangular.

It can be seen that the results calculated using the classical transmission line theory deviate from the full-wave results obtained using NEC-4. Taking into account the non-vertical riser using the equivalent inductance leads to significantly more accurate results.

3.2 Investigation of the Frequency Beyond which the Inductance Needs to be Considered

To evaluate the frequency above which the equivalent inductance L_A needs to be taken into account, a parameter $A(f)$ is defined as the logarithm of the ratio of the results obtained using the classical transmission line method and those obtained using NEC-4:

$$A(f) = 20\left|\log_{10}\left(\frac{|I_{\mathrm{CTL}}(f)|}{|I_{\mathrm{NEC}}(f)|}\right)\right| \tag{11}$$

where $A(f)$ is a measure of the accuracy at a given frequency f, I_{CTL} is the current obtained from the classical TL method, and I_{NEC} is the current obtained using NEC-4.

The threshold value of the calculation accuracy to indicate that the obtained solution is acceptable is defined to be 3 dB. If $A(f)$ is larger than 3 dB, the result obtained from the classical TL method is no longer considered as acceptable, and the equivalent inductance L_A needs to be taken into account. The threshold frequency, defined as the smallest frequency at which $A(f)$ is larger than 3 dB, is denoted by f_a. For the case in which the geometry of the left riser is considered to be rectangular, the value of f_a in Figure 6(a) is 37.6 MHz.
It is reasonable to consider that the value of f_a may be related to the ratio of $j\omega L_A$ to the total impedance. To test this hypothesis, 7 cases with the rectangular-shape left riser but different values of Z_A and L_1 are considered; the parameters are listed in Table 1. All other parameters are the same as in the first example. The classical transmission line method and NEC-4 were used to calculate the current response at the left-end in these 7 cases. Then, the values for f_a were calculated and listed in the second to the last column in Table 1.

Table 1. Parameters for the Validation Examples

	Z_A (Ω)	Z_B (Ω)	L_1 (m)	L_A (μH)	f_a (MHz)	$f_{a\text{-}e}$ (MHz)
Case1	100	100	0.5	0.66	37.6	40.8
Case2	100	100	1	1.25	27.6	21.6
Case3	100	100	1.5	1.83	16.2	14.7
Case4	100	100	2	2.43	9.3	11.0
Case5	20	20	0.5	0.66	7.4	8.1
Case6	50	50	0.5	0.66	22.8	20.4
Case7	80	80	0.5	0.66	35.1	32.6

Let us now define a parameter K as the ratio, at the cutoff frequency f_a in Table 1, of the inductive to the total impedance:

$$K = \frac{2\pi f_a L_A}{|2\pi f_a j L_A + Z_A|} \tag{12}$$

The parameter K was calculated for the 7 cases and it is presented in Figure 7. It can be seen that the value of K remains relatively constant for the seven cases regardless of the differences in the transmission line parameters. This suggests that the ratio K of ωL_A to the magnitude of the total impedance can be used to roughly estimate the value of the frequency f_a beyond which the equivalent inductance L_A needs to be taken into account. Indeed, by setting K in (12) to its average value of 0.86 (obtained from Figure 7) and solving for f_a, we obtained estimates of the threshold frequency for each of the 7 cases. The estimates, which we call $f_{a\text{-}e}$, are listed in the last column of Table 1. It can be seen that f_a can indeed be estimated roughly by the calculated $f_{a\text{-}e}$.

It is to be noted that the inferred average of the parameter K cannot be generalized to any arbitrary riser. There are still many other factors (*e.g.*, the height or the radius of the line and the shape of the riser) that may influence the results.

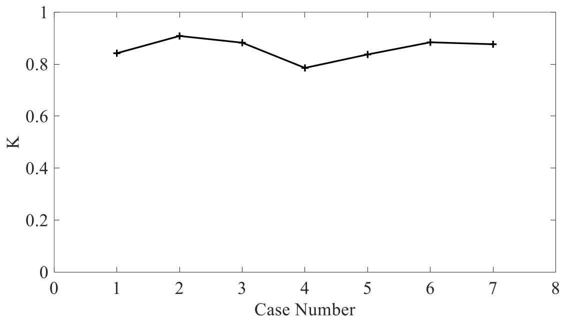


Figure 7. The value of parameter K for the considered 7 cases.

3.3 Validation in the Time Domain

In order to investigate the capability of the proposed method to predict induced signals over a wide frequency band, time domain simulations were also carried out. We considered a value for length L_1 equal to 1 m. The terminal loads at both ends were assumed to be 100 Ω. The EMP standard waveform defined in IEC 61000-2-5 was adopted for the electric field. The calculated results for the induced current at the left-end terminal are shown in Figure 8.

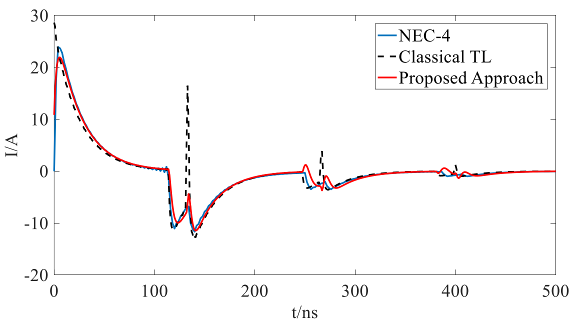


Figure 8. Time-domain waveform for the left-end current induced by a standard EMP.

It can be seen that the time domain results calculated using the classical transmission line theory deviate from the results obtained using NEC-4, whereas the results obtained from the proposed approach agree well with those obtained using NEC-4.

4 Conclusions

In this paper, a simple and efficient method to take into account non-vertical risers through an equivalent partial inductance was further discussed and validated.

In particular, the frequency f_a beyond which the equivalent inductance L_A needs to be taken into account was evaluated. It was shown that the value of f_a is related to the magnitude of the ratio of $j\omega L_A$ to the total impedance of the riser.

Finally, the efficiency of the proposed method was also illustrated when applied to wideband signals.

4 References

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