Multi-Frequency Inversion of Scattered-Field Data in Lebesgue Spaces with Nonconstant Exponents

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Abstract

The quantitative retrieval of the dielectric properties of structures from scattered-field measurements at microwave frequencies is relevant for a wide range of applications. To accomplish this task, a key point is represented by the effective exploitation of the available information, including frequency diversity. In this contribution, a quantitative imaging method that works in Lebesgue spaces with nonconstant exponents is extended in order to perform a multi-frequency inversion. The proposed technique is validated against experimental data.

1 Introduction

The development of microwave imaging techniques based on full-wave inversion approaches has attracted significant efforts in the last years, with a notable increase in the number of potential applications [1]. Nevertheless, the complexity of the underlying inverse scattering problem motivates a continuous research of novel solution strategies, which follows different directions. On the one hand, the definition of the inversion method has a key importance. Among the various possible approaches, Newton-based methods exhibit good potentialities in dealing with the nonlinearity of the relationship between the dielectric properties of the targets under test and the corresponding scattered field [2]–[5]. Recently, alternative formulations developed outside the standard framework of Hilbert spaces, which seem capable of enhancing the reconstruction quality, have also been proposed [6]. On the other hand, the complexity of the problem stimulates the proposal of strategies to extract as much information as possible from the measured data. In this respect, the use of data acquired at multiple frequencies – when available – appears as one of the most viable ways to improve inversion results [7] and has been exploited in different contexts and applications [8]–[11].
In this work, the electromagnetic problem is solved by an inexact-Newton procedure defined in Lebesgue spaces with nonconstant exponents $L^{p(\cdot)}$, which has several benefits with respect to the fixed-exponent counterparts [6]. This kind of technique has been initially proposed to process single-frequency data in [12]. Moreover, it has been applied to brain stroke detection with a frequency-hopping scheme [13]. However, despite the simplicity of employing frequency-hopping approaches, they require a series of successive single-frequency inversions, which usually lead to a long computational time. In order to overcome such a limitation, an approach aimed at simultaneously exploiting multi-frequency data inside an $L^{p(\cdot)}$ space inversion method is proposed and preliminarily validated in this contribution.
The paper is structured as follows. The multi-frequency inversion method is outlined in Section 2. Section 3 presents some initial experimental results obtained with the Fresnel dataset. Conclusions follow.

2 Multi-frequency inversion method

Let us consider a target under test characterized by space-dependent relative dielectric permittivity $\epsilon_r(\mathbf{r})$ and electric conductivity $\sigma(\mathbf{r})$, which is located inside an investigation domain $\mathbb{D}$ (i.e., with $\mathbf{r} \in \mathbb{D}$). For the sake of simplicity, a two-dimensional transverse-magnetic (TM) configuration is considered, where electric fields are polarized along the z axis and $\mathbb{D}$ lies on the xy plane. In order to characterize the dielectric properties of the target, we assume to have at our disposal a set of measurements of the electric field it scatters at the frequency f, denoted as $E_f^{scat}(\mathbf{r})$. This time, $\mathbf{r} \in \mathbb{O}$, where $\mathbb{O}$ is the so-called observation domain. Under such hypotheses, the single-frequency electromagnetic problem for one view can be formulated as

$$\mathcal{F}_f\left(T_f x\right) = E_f^{scat}, \tag{1}$$

where $\mathcal{F}_f$ is the nonlinear operator that describes the scattering phenomena at the frequency f [1], $T_f = [1 \quad -j\, f_0/f]$, and the unknown function (which describes the dielectric properties inside $\mathbb{D}$) is given by $x = [(\epsilon_r - 1) \quad \sigma/(2\pi f_0 \epsilon_0)]^T$ (f_0 being the lowest available frequency).
If measurements at different frequencies are available (that is, a set of F frequencies $f_1, \dots, f_F$), all the information can be combined together by defining

$$\mathcal{F}_{MF}(x) = \begin{bmatrix} \Re\{\mathcal{F}_{f_1}(T_{f_1}x)\} \\ \Im\{\mathcal{F}_{f_1}(T_{f_1}x)\} \\ \vdots \\ \Re\{\mathcal{F}_{f_F}(T_{f_F}x)\} \\ \Im\{\mathcal{F}_{f_F}(T_{f_F}x)\} \end{bmatrix}; \quad E_{MF}^{scat} = \begin{bmatrix} \Re\{E_{f_1}^{scat}\} \\ \Im\{E_{f_1}^{scat}\} \\ \vdots \\ \Re\{E_{f_F}^{scat}\} \\ \Im\{E_{f_F}^{scat}\} \end{bmatrix} \tag{2}$$

where the problem is recast as a real-valued one splitting into real and imaginary parts both data and operators, since the unknown function x is real. Based on (1) and (2), the resulting multi-frequency problem becomes

$$\mathcal{F}_{MF}(x) = E_{MF}^{scat}. \quad (3)$$

The unknown x is found by a full-wave inversion method formulated in the framework of Lebesgue spaces with nonconstant exponents $L^{p(\cdot)}$ [14]. In more details, the algorithm is structured with an external iterative loop which linearizes (3) around the currently reconstructed unknown x at the nth step (indicated with x_n). The linear equation obtained in this way is then handled by an internal loop, which implements the Landweber method in $L^{p(\cdot)}$ spaces and is outlined in [12]. In the present implementation, the exponent $p(\cdot)$ is free of assuming different values for each point of the investigation domain $\mathbb{D}$, and is adaptively updated at each external iteration. The resulting sequence of functions $p_n(\mathbf{r})$, $\mathbf{r} \in \mathbb{D}$ is connected to the magnitude of the unknown in the investigation domain at the previous Newton step x_{n-1}, according to the relationship

$$p_n(\mathbf{r}) = \bar{p} + \Delta p \frac{\|x_{n-1}(\mathbf{r})\|}{\max_{\mathbf{r}\in\mathbb{D}} \|x_{n-1}(\mathbf{r})\|}, \qquad \mathbf{r} \in \mathbb{D}. \quad (4)$$

where $\bar{p}$ and Δp are the minimum and the extent of the range of admissible values of the function p_n. In the first iteration, when no a-priori information about the target under test is available, a uniform distribution $p_n(\mathbf{r}) = \bar{p}, \mathbf{r} \in \mathbb{D}$ is chosen. It is worth noting that (4) causes the assignment of higher values of p in the regions where targets are detected, and lower values in the other parts of the investigation domain.

3 Results

Preliminary experimental results are presented here in order to validate the proposed multi-frequency inversion approach. The dataset of experimental measurements provided by the Institut Fresnel (Marseille, France) has been used [15]. A multi-view measurement setup is adopted, where the investigation region $\mathbb{D}$ is sequentially illuminated from $V = 8$ different locations (positioned on a circumference of radius $r_{\mathbb{O}} = 1.67$ m with equal angular spacing). For each view, scattered-field data are measured in $N = 241$ points located on the same circumference. In [15] more details about the adopted configuration can be found. The data related to the *FoamDielExt* target have been considered, which is formed by two circular cylinders. The first one (foam, relative dielectric permittivity $\epsilon_{r,1} = 1.45$) is centered at the origin and has a radius $r_1 = 0.04$ m. The second cylinder (plastic, $\epsilon_{r,2} = 3$) is centered at $\mathbf{r}_2 = (-0.0555; 0)$ m and has a radius $r_2 = 0.0155$ m.
A square investigation domain $\mathbb{D}$ with side length $l_{\mathbb{D}} = 0.2$ m centered at the origin has been considered. Inside the inversion procedure, $\mathbb{D}$ has been partitioned into 40×40 square cells with 5-mm sides. Measurements obtained in the band $B = [2, 8]$ GHz have been used for the reconstruction. Moreover, the other parameters of the inversion procedure have been fixed as follows: $\bar{p} = 1.5$, $\Delta p = 0.5$, maximum numbers of inexact-Newton and Landweber iterations $I_{MAX}^{IN} = 50, I_{MAX}^{LW} = 10$, respectively; threshold on the relative variation of the residual in both loops $\Delta r = 0.05$.
The reconstructed distribution of the relative dielectric permittivity of the *FoamDielExt* target obtained with the single-frequency inversion at 2 GHz is reported in Figure 1, whereas Figure 2 shows the result of applying the multi-frequency approach with seven frequencies (equally spaced in the band B). A significant improvement obtained by adopting the proposed multi-frequency method can be noticed. This is also confirmed by the relative reconstruction errors computed in the investigation domain, which are equal to 0.085 in the single-frequency approach and 0.051 when multiple frequencies are exploited.

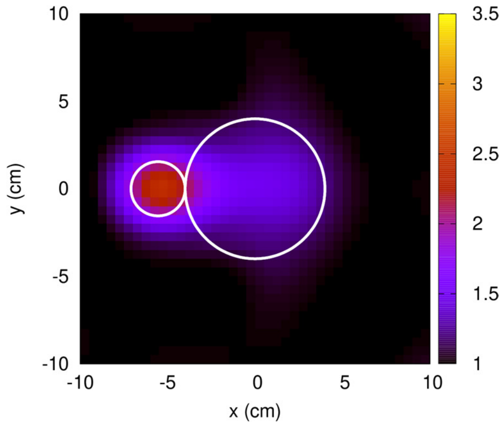

Figure 1. Reconstructed distribution of the relative dielectric permittivity ϵ_r inside $\mathbb{D}$ obtained by the single-frequency approach [12]. Experimental reconstruction of the *FoamDielExt* target.

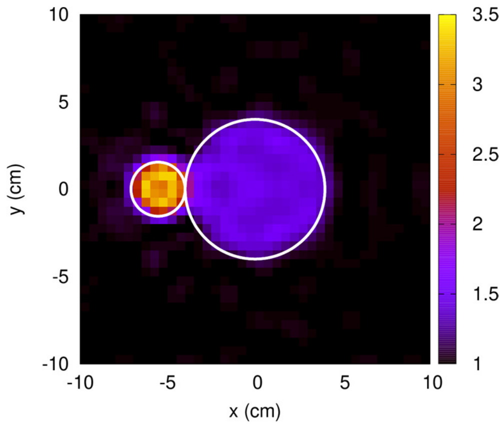

Figure 2. Reconstructed distribution of the relative dielectric permittivity ϵ_r inside $\mathbb{D}$ obtained by the proposed approach by considering seven frequencies. Experimental reconstruction of the *FoamDielExt* target.

4 Conclusions

One of the possible ways to enhance the quantitative inversion of scattered-field data comes from exploiting the information contained in frequency diversity. This paper proposed an approach to simultaneously use the data acquired at multiple frequencies inside an inexact-Newton scheme formulated in the unconventional mathematical framework of Lebesgue spaces with nonconstant exponents. Experimental data from the Fresnel dataset have been used to initially assess the capabilities of the proposed inversion approach with promising results.

6 References

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