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Time Domain Physical Optics Integral Evaluation on Third Order Triangular Patches

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Abstract

A scheme to evaluate the physical optics (PO) integral on third order triangles in time domain is presented. Using the Radon transform interpretation of the radiation integrals, the PO integral on the surface modeled with third order triangles is reduced to a line integral and evaluated exactly using Gauss-Legendre quadrature rule (GLQR). A numerical example that demonstrates the accuracy and validity of the proposed scheme is presented.

1 Introduction

The physical optics (PO) approximation is one of the most preferred high-frequency techniques to analyze scattering and radiation from electrically large objects [1]. Analysis using the PO approximation requires evaluation of a highly-oscillatory radiation integral, which is called PO integral, on the illuminated surface of the scatterer. The PO integral can be evaluated analytically or approximately on different surface models that are used to discretize the scatterer, such as linear triangles [2], quadratic triangles [3]-[6], NURBS surfaces [7], [8]. It is possible to evaluate the PO integral in closed-form using the Radon transform interpretation, when the scatterer is modeled with linear triangles [2]. Using the closed-form expressions for linear triangles removes the numerical errors in the evaluation of the PO integral, however modeling errors remain, especially for curved scatterers. In order to reduce the modeling errors, high order surface models can be used [3]-[8]. Recently, a scheme to exactly evaluate the PO integral on quadratic (second order) triangles is proposed in [5]. In [5], the PO integral on the quadratic triangle is reduced to a line integral on a quadratic curve that is formed by the intersection of the quadratic triangle and a plane formed by the incidence and observation directions. The intersecting curve is determined in barycentric coordinates of the quadratic triangle. Using the appropriate coordinate transformations depending on the type of the quadratic curve, the intersecting curve is parametrized and Gauss-Legendre quadrature rule (GLQR) is applied on the parametrized curve to exactly evaluate the line integral. It is shown that using quadratic triangles models the curved surfaces with higher accuracy using less number of patches [5], [6].

In this work, the scheme presented in [5] is extended to the exact evaluation of the PO integral on third order triangles. Similar to the scheme presented in [5], the intersecting curve is determined in barycentric coordinates, however using the Dirac delta function's properties, GLQR is applied to the intersecting curve directly in barycentric coordinates without parametrizing the curve and requiring a coordinate transformation. As a result, the PO integral on third order triangles can be determined exactly. A preliminary example is presented to demonstrate the accuracy of the proposed scheme.

2 Formulation

Let S denote a perfect electrically conducting (PEC) scatterer's surface. S is illuminated by an impulsively excited plane wave with the electric field component $\mathbf{E}^{\text{inc}}(\mathbf{r},t) = \hat{\mathbf{p}}\delta(t - \hat{\mathbf{k}}_{\text{i}} \cdot \mathbf{r}/c)$, where $\hat{\mathbf{k}}_{\text{i}}$ and $\hat{\mathbf{p}}$ are the propagation direction and polarization of the incident plane wave, respectively, $\delta(\cdot)$ denotes the Dirac delta function, and c is the speed of light. Assuming that the illuminated part of the scatterer S, i.e. S_{ill}, is discretized with third order triangles as $S_{\text{ill}} \simeq \bigcup S_n$, where S_n denotes the nth third order triangle, the scattered electric field [7] can be determined as

$$\mathbf{E}^{\text{sca}}(\mathbf{r},t) = -\frac{1}{2\pi c}\frac{\partial_t \delta(t - r/c)}{r} * \mathbf{E}^{\text{sca}}_{\text{rc}}(\hat{\mathbf{k}}_{\text{s}},t), \tag{1}$$

where ∂_t and "$*$" denote time derivative and convolution, respectively, $\hat{\mathbf{k}}_{\text{s}}$ is the observation direction, and $\mathbf{E}^{\text{sca}}_{\text{rc}}(\hat{\mathbf{k}}_{\text{s}},t)$ is the range-corrected scattered electric field [9]

$$\mathbf{E}^{\text{sca}}_{\text{rc}}(\hat{\mathbf{k}}_{\text{s}},t) = \hat{\mathbf{k}}_{\text{s}} \times \hat{\mathbf{k}}_{\text{s}} \times (\hat{\mathbf{k}}_{\text{i}} \times \hat{\mathbf{p}}) \times \textstyle\sum_{n=1}^{N} \mathbf{h}_n(t). \tag{2}$$

In (2), $\mathbf{h}_n(t)$ denotes the PO integral and defined as

$$\mathbf{h}_n(t) = \int_{S_n} \hat{\mathbf{n}}(\mathbf{r}')\delta\left(t - \frac{\mathbf{k}_{\text{r}} \cdot \mathbf{r}'}{c}\right) d\mathbf{r}', \tag{3}$$

where $\hat{\mathbf{n}}(\mathbf{r})$ is the outward pointing unit normal vector of S_n and $\mathbf{k}_{\text{r}} = (\hat{\mathbf{k}}_{\text{i}} - \hat{\mathbf{k}}_{\text{s}})/2$ defines a plane, called $\mathbf{k}_{\text{r}}$-plane [5]. As a first step, it is assumed that coordinate system is rotated as $\mathbf{r}' \to \mathbf{r}'_p$, where $\mathbf{k}_{\text{r}} \parallel \hat{\mathbf{x}}'_p$. Using the Dirac delta function's properties and barycentric coordinates of the third order triangle (α,β), $\mathbf{h}_n(t)$ can be given as

$$\mathbf{h}_n(t) = \frac{c}{2\,|\mathbf{k}_{\text{r}}|}\int_{S_n} \hat{\mathbf{n}}(\mathbf{r}'_p)\delta\left(\frac{ct}{2\,|\mathbf{k}_{\text{r}}|} - x'_p\right) d\alpha d\beta. \tag{4}$$

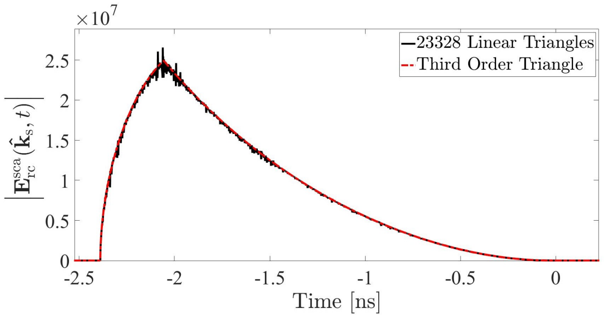

Figure 1. Norm of the range corrected scattered electric field.

Note that $\mathbf{r}'_p$ and x'_p depend on (α,β), since any point on the third order triangle is represented in barycentric coordinates by $\mathbf{r}(\alpha,\beta)=\sum_{i=1}^{10} N_i(\alpha,\beta)\mathbf{r}_i$, where $\mathbf{r}_i$ and $N_i(\alpha,\beta)$, $i=1,\ldots,10$, are the nodes and shape functions of the third order triangle, respectively. The intersecting curve of the third order triangle and the $\mathbf{k}_{\mathrm{r}}$-plane can be determined using

$$\begin{aligned} 0 &= \frac{ct}{2\left|\mathbf{k}_{\mathrm{r}}\right|} - x_p(\alpha,\beta) \\ &= \frac{ct}{2\left|\mathbf{k}_{\mathrm{r}}\right|} - \sum_{i=1}^{10} N_i(\alpha,\beta)x_{p,i}. \end{aligned} \tag{5}$$

Once the intersecting curve is determined, using the Dirac delta function's properties, the integral in (5) can be reduced to a line integral on the intersecting curve and the $\mathbf{h}_n(t)$ integral can be determined exactly using a suitable order GLQR.

3 Numerical Example

In this section, backscattering from a third order triangle with the nodes $\mathbf{r}_1=(-0.901,0.434,0)$, $\mathbf{r}_2=(-0.874,0.327,0.358)$, $\mathbf{r}_3=(-0.586,0.749,0.308)$, $\mathbf{r}_4=(-0.906,0.405,0.121)$, $\mathbf{r}_5=(-0.897,0.368,0.242)$, $\mathbf{r}_6=(-0.802,0.482,0.352,)$, $\mathbf{r}_7=(-0.703,0.627,0.335)$, $\mathbf{r}_8=(-0.715,0.666,0.213,)$, $\mathbf{r}_9=(-0.824,0.557,0.106)$, $\mathbf{r}_{10}=(-0.818,0.526,0.232)$ is analyzed for $\hat{\mathbf{k}}_{\mathrm{i}}=-\hat{\mathbf{z}}$, $\hat{\mathbf{k}}_{\mathrm{s}}=\hat{\mathbf{z}}$, as a preliminary example. 3-point GLQR is used to evaluate the line integral and time step size $\Delta t=1.11$ ps. Fig. 1 plots the norm of the range corrected scattered electric field $\mathbf{E}_{\mathrm{rc}}^{\mathrm{sca}}(\hat{\mathbf{k}}_{\mathrm{s}},t)$ for the third order triangle and compares them with the results obtained for 23328 linear triangles model produced by repeatedly dividing the third order triangle into third order and linear sub-triangles and using the closed-form expressions given in [2]. Fig. 2 plots the backscattered radar cross section (RCS) obtained by discrete Fourier transforming the time domain results in $0-2$ GHz frequency band with $\Delta f=10$ MHz as explained in [5]. It can be seen from Figs. 1 and 2, the results obtained for the third order triangle and 23328 linear triangles coincide very well.

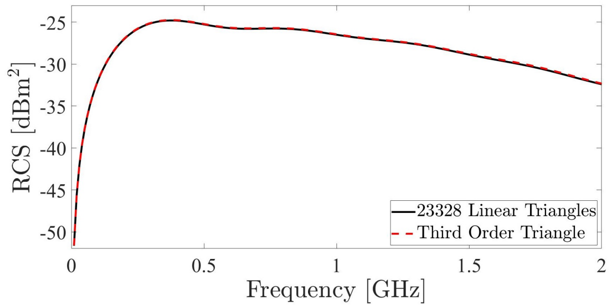

Figure 2. Backscattered RCS results.

4 Conclusion

In this work, a scheme for exact evaluation of the time domain PO integral on third order triangle patches using Radon transform interpretation is proposed. A preliminary example that demonstrates the accuracy of the proposed method is presented.

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