32nd URSI GASS, Montreal, 19–26 August 2017

Quantifying the samplng in uv-plane: implications for imaging extended radio sources

Ruta Kale[(1)]
(1) National Centre for Radio Astrophysics, Tata Institute of Fundamental Research, Post Bag 3, Pune 411007, India

Abstract

Radio interferometers measure the Fourier transform of the sky at the spatial frequencies sampled by the interferometer baselines. Inherently, the shortest projected baseline in an observation towards a source limits the largest sampled angular scale. The sampling in the 'uv-plane'- the plane in which the projected baseline vectors are measured - is critical for obtaining high sensitive images and to quantify the angular structures which may be missed in an observation. In this work we define a filling fraction F_{fill} of the uv-plane in a given data and the overlap fraction (F_{ov}) between the uv-coverages of two data. These fractions have applications in calculating the errors introduced by interpolation in the process of gridding the visibilities for imaging. The methods are demonstrated using data from the Giant Metrewave Radio Telescope. With wide-band receivers in the upcoming upgrade of existing interferometers and the future Square Kilometer array, the uv-overlap fraction will be much higher leading to much better spectral index mapping and impact the related science.

1 Introduction

Radio interferometers measure the Fourier transform of the sky at the spatial frequencies sampled by the interferometer baselines [1]. Inherently the shortest projected baseline (b_{min}) in an observation towards a source limits the largest sampled angular scale (θ_l) to λ/b_{min} at the observing wavelength of λ. Imaging radio sources of extents that are as large as that scale is a challenge. It was shown by [2] using one dimensional simulations that for a Gaussian source of the size of θ_l, the recovery of central brightness is limited to 3%.

In radio astronomy, extended radio sources are common - the Galaxy, supernova remnants, radio galaxies and radio halos in galaxy clusters being some of the examples. The continuum spectra of these sources are typically of synchrotron origin and are often approximated as power-laws ($S \propto \nu^{-\alpha}$, where S is the flux density at the frequency ν), the power-law index referred to as the 'spectral index' (α). By taking the ratio of radio images at two well separated frequencies, ν_1, ν_2 a spatial distribution of the spectral indices- called the spectral index map defined by,

$$\alpha_{\nu_1}^{\nu_2} = \frac{log(S_1/S_2)}{log(\nu_1/\nu_2)}, \tag{1}$$

can be constructed, where S_1 and S_2 are the flux densities in the given pixels in the images at the two frequencies. The spectral index is among the most important quantities for understanding the physical processes in the source as it is affected by the local magnetic field and the age of the source. Therefore it is necessary to make accurate maps and quantify the errors involved.

The primary errors in a spectral index map are introduced by the incomplete and unequal sampling of angular scales at the two frequencies involved. An artificial steepening (or flattening) of the spectral index can result if large angular scales are not sampled equally at the two frequencies. Matched sampling is key to making spectral index maps with meaningful values for the spectral indices. Commonly the matching is made at the shortest and longest scales but the inequalities in the intermediate scales are completely neglected and left to the interpolation in the process of gridding the visibilities. The inequalities in the intermediate scales are important to consider because in general astrophysical sources can have emission from large to small scales depending on the physical process involved.

In this work we propose new measures of the sampling in the 'uv-plane' called the filing fraction and the uv-overlap fraction – the 'uv' refers to the spatial coordinates in the imaginery plane on which the radio interferometer samples the sky. This is useful in interpretation of the recovery of extended emission and in designing matched uv-sampling for making meaningful spectral index maps. The paper is organized as follows: Sec. 2 defines the overlap fraction, Sec. 3 describes the method to calculate it using an example. Applications of this method are described in Sec. 4. Summary is presented in Sec. 5.

2 Definition

A radio interferometer measures the visibilities which are recorded as a function of the standard coordinates u, v and w. The distances along the three coordinates are measured in kilo wavelengths (kλ). Here we will assume w to be negligible and work with only the two dimensional plane of $u-v$. The intensity distribution in the sky plane is measured as its Fourier transform components at the locations where the baselines sample the uv-plane. For a radio interferometer with elements fixed in a given configuration, the uv-coverage will be a function of observing frequency, location of the source, duration of the observation and the bandwidth. Consider a dataset that contains observations in a single frequency channel at the wavelength, λ for a duration that is equal to or less than the up-time of the source in the sky. Due to earth rotation, the baselines will trace ellipses in the 'uv-plane' [1] and that is the sampling. If the maximum uv-distance is given by the maximum baseline b_{max}, then the sampling is confined to a region of $u = b_{max}, v = b_{max}$ and $u = -b_{max}, v = -b_{max}$. This region is binned into bins of the size according to the bandwidth of the channel given in wavelengths (BWL), for example bin size= $2 * b_{max}/BWL$. A 2-dimensional histogram of the visibilities in these bins is made and the bins with non-zero bins are replaced by 1 and those with zero visibilities with 0. The filling fraction is then defined as,

$$F_{fill} = \frac{N_{1s}}{N_{bins}}, \tag{2}$$

where N_{1s} are number of 1s and N_{bins} are the total number of bins. This fraction gives a measure of how well the uv-plane is filled within the region that is sampled.

The F_{fill} measures the filling fraction for a single dataset. For comparison of two datasets as is relevant for spectral index maps, the overlap fraction, F_{ov} is defined. First the region of uv-plane common for the two datasets is found. The maximum uv-distance in wavelengths at the lower frequency and the minimum uv-distance at the higher frequency will define a uv-range common to both the frequencies. Following the same procedure as for a single dataset, the matrix of 1s and zeros in the common region in the uv-plane is defined. In this case the bin size selected is common for both the frequencies, equal to the smaller of the two bin sizes. The two matrices with 1s and zeros are then added to get a matrix of zeros, 1s and 2s. The 2s denote the overlap, 1s denote data at either one of the datasets and zeros are where none of the two datasets contain data. The overlap fraction is,

$$F_{ov} = N_{2s}/N_{bins}, \tag{3}$$

where N_{2s} is the number of 2s. The fraction where there are points at both the dataset but do not overlap but are separate is defined as,

$$F_{sep} = N_{1s}/N_{bins}, \tag{4}$$

where N_{1s} is the number of 1s.

3 Method: illustration

The steps to obtain the F_{fill} and F_{ov} fractions using real datasets are described here. Two datasets observed with the Giant Metrewave Radio Telescope (GMRT) are taken for the purpose of illustration. The two data are from simultaneous dual frequency measurements using the GMRT at 610 and 240 MHz. We consider single channel data for illustration. The maximum uv-distances sampled at 610 and 240 MHz are 55.4 and 21.7 kλ and the mini uv-distances are 189.8 and 60.1 kλ, respectively. Therefore the overlapping uv-range is 189.8 to 21.7 kλ. We divided this range into 500 bins which implies that the bin width is 0.55 kλ. The filling fraction, $F_{fill} = 0.11$ and the overlap fraction, $F_{ov} = 0.71$. The fraction of points where bins in only one frequency were non-zero is 0.28.

4 Applications

The above method allows the quantification of the actual overlap in the sampling in the uv-plane for two different datasets. The datasets can be from the same or different telescopes and at different frequencies. An important application of this method is to the data used to make spectral index maps of extended sources. Astronomical sources have emission on small to large scales depending on the involved physical mechanism. Matched uv-coverage is key to making the most accurate spectral index maps. The method above gives a simple way to compare the absolute filling fraction in the uv-plane and the overlapping fraction. The overlap fraction represents a number that quatifies how close are the angular scales probed by the sampling at the two frequencies. In particular the ratio,

$$R_{grid} = F_{ov}/F_{sep}, \tag{5}$$

gives an estimate of the error introduced due to interpolation in gridding while making spectral index maps from two datasets.

A further extension of this method is to obtain the overlap fraction as a function of frequency. This can be done by carrying out the overlap fraction calculation as a function of uv-distance.

The upcoming telescopes such as the upgraded GMRT, the Square Kilometer Array are designed to have broadband receivers. This implies that the tracks in the uv-plane will get wider along the radial axis and result in a much more filled uv-plane than is possible with the current relatively narrow band systems.

5 Discussion and summary

Quantifying the filling fraction in the uv-plane for a given dataset and the overlap in the uv-coverage in two datasets is important while imaging with radio interferometers. In this work the filling fraction of uv-plane and the overlap fraction in uv-plane were defined. The method was illustrated using GMRT datasets. The methods defined here are useful in the context of making matched uv-coverage for spectral index mapping. The method also has wider applications in comparing data from different telescopes.

The codes for these calculations can be obtained from the author. In future a task in Common Astronomy Software Applications will be made for these calculations.

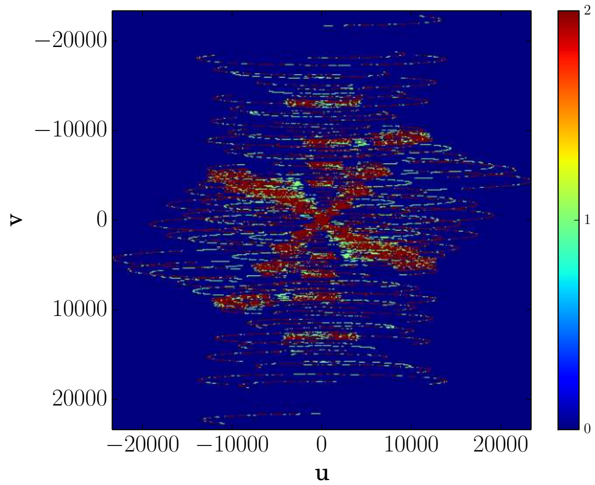

Figure 1. The image shows the zeros, 1s and 2s are shown in blue, green and red colours. The two datasets contain overlapping points only in the red regions.

6 Acknowledgements

I thank Kaushal Buch for discussions on this topic. We thank the staff of the GMRT that made these observations possible. GMRT is run by the National Centre for Radio Astrophysics of the Tata Institute of Fundamental Research.

References

[1] A. R. Thompson, J. M. Moran, and G. W. Swenson, *Interferometry and synthesis in radio astronomy*, 1986.

[2] D. J. Wilner and W. J. Welch, "The S140 core: Aperture synthesis HCO(+) and SO observations," *ApJ*, vol. 427, pp. 898–913, Jun. 1994.