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# Monte-Carlo simulations of ion velocity distributions and resulting incoherent radar spectra under strong ion frictional heating conditions

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## Abstract

During periods where strong electric fields penetrate weakly ionized plasma at high latitudes the ion velocity distribution can differ enough from a Maxwellian shape to substantially change Incoherent Scatter Radar (ISR) spectra, and thus the analysis of those spectra. In this work an advanced description of the ion velocity distribution and spectra is found through improvements made to previous studies that employed Monte-Carlo simulations. These improvements include: 1) a higher resolution one-dimensional ion velocity distribution, 2) a new velocity distribution fitting technique, 3) the use of Nyquist diagrams to check the plasma stability, 4) a study of more recent published $O^+$-O resonant charge exchange collision cross-sections, 5) the option to incorporate ion-ion and ion-electron collisions in the ion velocity distribution, and 6) an improved filtering technique to reduce statistical noise. Through these improvements, it has been found that: 1) ion-ion and ion-electron collisions have a minimal impact on the $NO^+$ line-of-sight temperature, but a strong impact on the $O^+$ temperature parallel to the magnetic field, 2) the choice of $O^+$-O resonant charge exchange collision cross-section can change the ion temperature along a line-of-sight by several 1000 K under strong electric fields, as well as possibly drive a plasma to be stable, 3) $NO^+$ spectra can generally be modeled using Maxwellian velocity distributions, and 4) $O^+$ spectra parallel to the magnetic field show an apparent increase in electron temperature. This work is discussed in greater detail in this report, and will aid future studies into ion temperature anisotropy and ISR spectra.

## 1 Introduction

It is now well-known that in the presence of strong electric fields the ion velocity distribution of the weakly ionized plasma at high latitudes can differ enough from a Maxwellian shape in such a way as to substantially affect Incoherent Scatter Radar (ISR) spectra. This in turn is known to seriously influence the analysis of those spectra and the retrieval of parameters like the ion and electron temperatures. Until now, studies of this topic have focused on a first order description of the ion velocity distribution based on a semi-empirical toroidal shape, which has proven to produce ISR spectra that were qualitatively similar to the observed ones. However, a precise description of the velocity distribution used in such calculations has been found lacking, particularly for line-of-sight directions parallel or near-parallel to the magnetic field. To remedy these shortcomings and provide the best possible tools to analyze spectra, a state-of-the-art Monte-Carlo (MC) simulation has been used to retrieve high-resolution one-dimensional ion velocity distributions for any electric field, ion-neutral particle interaction, and direction relative to the magnetic field.

Here, Section 2 will discuss the new procedures being done in this research in the context of previous work. Then Section 3 will highlight the results of using an improved simulation procedure in exploring ion temperature anisotropy and spectra. Then Section 4 will discuss these findings in more detail. Lastly, Section 5 will provide a summary.

## 2 Background and New Procedures

Through the use of MC simulations, it is possible to reproduce the interaction of an ion particle with a background of neutral particles under a given electric field and predetermined neutral particle parameters. As the simulation runs, an ion velocity distribution is inferred, which can then be used to simulate hypothetical ISR spectra and better understand temperature anisotropy ([1] and references therein). By both improving older techniques and introducing better methods, it is possible to return to previous MC simulation studies and explore them in greater detail with less limitations.

One way in which this work is improved over previous studies is that this work is capable of a creating higher resolution ion velocity distributions through an increased number of simulated ion-neutral particle collisions. Previous studies were not capable of doing this (or doing this as easily) because the technology needed to simulate a high volume of ion-neutral particle collisions was limited at the time. Since then it is now possible to simulate several tens of millions of collisions within minutes, rather than taking hours and days, making multiple simulation runs more feasible.

In previous studies, to fully describe a non-Maxwellian one-dimensional ion velocity distribution, a Maxwellian distribution was distorted to fit the results to leading order [2]. Although this was needed at the time, this technique would only work efficiently for electric fields less than 100 mV/m. Instead, this work uses a new fitting technique in which a least squares fit is performed to the logarithm of the one-dimensional ion velocity distribution. This entirely describes the distribution, and prevents a less-than zero-fit.

In previous simulation studies similar to this work, the question of whether or not a plasma was stable was generally ignored. However, the stability of a plasma is important to consider because the standard spectrum formulations are only correct when the plasma is stable. Through the use of Nyquist diagrams it is possible to check the stability of a given plasma. Quite simply, if the imaginary component of a given function is graphed against it's real component and the origin is circled, the plasma is unstable. An example of this can be seen in Figure 1, where the blue curve reflects the unstable plasma produced by the “Knof" cross-section, and the green curve reflects the stable plasma produced by the “Pesnell" cross-section. For this reason, and given the contention over the $O^+$-O resonant charge exchange (RCE) cross-section, this work explores a variety of published $O^+$-O RCE cross-sections [3, 4].

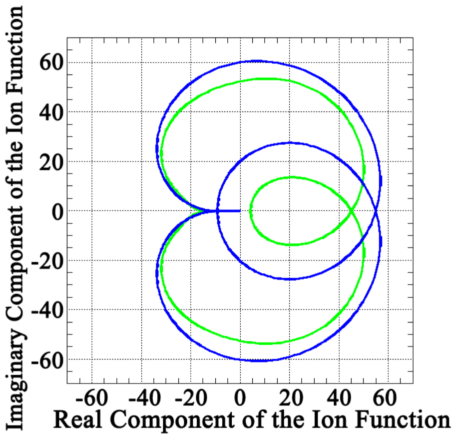


**Figure 1.** The imaginary component of the spectrum ion function against the real component at a 100 mV/m electric field. The green line uses the “Pesnell" cross-section, and the blue uses “Knof".

An additional advantage of this work is that it uses a very simple procedure to incorporate ion-ion and ion-electron collisions into the total ion velocity distribution. This is done using:

$$\nu_T f_T = \nu_{in} f_{in} + \nu_{ie} f_e + \nu_{ii} f_i \tag{1}$$

where $\nu_T = \nu_{in} + \nu_{ie} + \nu_{ii}$, $\nu_{in}$ is the ion-neutral momentum transfer collision frequency, $\nu_{ie}$ is the ion-electron momentum transfer collision frequency, $\nu_{ii}$ is the ion-ion momentum transfer collision frequency, $f_T$ is the total ion velocity distribution, $f_{in}$ is the ion velocity distribution as determined by just ion-neutral particles collisions, $f_e$ is the isotropic electron velocity distribution, and $f_i$ is the isotropic ion velocity distribution.

A final technique that is used in this work is a smoothing filter that is able to reduce the influence of statistical noise from the MC calculations on the spectra.

## 3 Results

Using the techniques described earlier to resolve the line-of-sight ion temperature for a number of scenarios, the influence of ion-neutral frictional heating and temperature anisotropy is explored. Figure 2 examines the relative temperature difference between ion and neutral particles as a function of their relative drift for $O^+$-O collisions, the dominate collision type at an altitude of roughly 300 km to 400 km. This figure shows the influence of two different RCE cross-sections, the “Pesnell" RCE cross-section shown in green and the “Knof" RCE cross-section shown in blue [3, 4]. The solid lines only consider ion-neutral particle collisions, while the dashed lines incorporate ion-neutral, ion-ion, and ion-electron collisions. Here it can be seen that as the relative drift increases, the relative temperature difference increases parabolically. However, depending on the $O^+$-O RCE cross-section and the aspect angle, this parabolic increase can vary. It can also be seen that the influence of ion-ion and ion-electron collisions is much larger parallel to the magnetic field.

Figure 3 examines temperature anisotropy in a similar way as Figure 2, but with respect to $NO^+$ and with a 50% O and 50% $N_2$ neutral particle background, simulating an altitude of roughly 180 km. As seen in Figure 2, as the relative drift increases, the relative temperature difference increases parabolically. However, in Figure 3 the temperature difference parallel to the magnetic field is higher and the temperature difference perpendicular to the magnetic field is lower. In addition, ion-ion and ion-electron collisions do not have nearly as much of an impact in Figure 3 as compared to Figure 2. The temperature anisotropy seen in Figures 2 and 3 parallel those that have been observed in ISR data [5].

From the velocity distributions created through MC simulations, spectra can be explored using improved fitting techniques. Figure 4 shows the simulated MC spectra for a variety of cases, along with spectra created using equivalent (same line-of-sight ion temperature) Maxwellian velocity distributions, and spectra where ion-ion and ion-electron collisions have been incorporated. In all of these cases the electron temperature is 2000 K. Figures 4a through 4c show spectra created from simulated $O^+$-O collisions. For these figures the “Pesnell" RCE cross-section was used because it was found to produce stable plasma more consistently than the “Knof" RCE cross-section. Figure 4a shows the spectra at an electric field of 0 mV/m parallel to the magnetic field, which are identical to the spectra perpendicular to the mag-

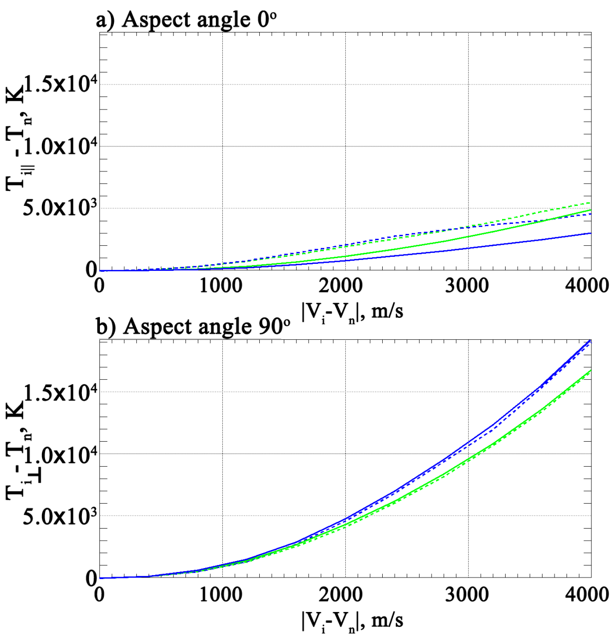


**Figure 2.** The relative difference in ion and neutral particle temperature as a function of the relative drift for $O^+$-O collisions. The solid lines reflect the influence of just ion-neutral particle collisions, while the dashed lines have also incorporated the influence of ion-ion and ion-electron collisions. The green lines represent the “Pesnell" RCE cross-section, while the blue lines represent the “Knof" RCE cross-section [3, 4]. a) Results parallel to the magnetic field. b) Results perpendicular to the magnetic field.

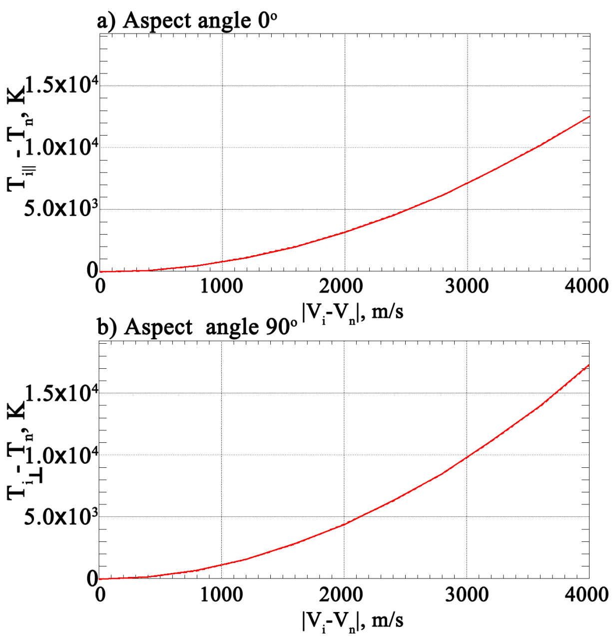


**Figure 3.** The relative difference in ion and neutral particle temperature as a function of the relative drift for $NO^+$ collisions with 50% O and 50% $N_2$. The solid lines reflect the influence of just ion-neutral particle collisions, while the dashed lines (which are directly underneath the solid lines) have also incorporated the influence of ion-ion and ion-electron collisions. a) Results parallel to the magnetic field. b) Results perpendicular to the magnetic field.

netic field at 0 mV/m. Figures 4b and 4c show spectra for an electric field of 200 mV/m, parallel and perpendicular to the magnetic field, respectively. In these two figures, a distinct difference in MC simulated spectra shapes can be seen, and the peak to trough ratio of the MC simulated spectra parallel to the magnetic field is much larger than the MC simulated spectra perpendicular to the magnetic field.

Figures 4d and 4e show spectra for $NO^+$ collisions with a mixture of 50% O and 50% $N_2$. The spectra in Figure 4d are parallel to the magnetic field for an electric field of 0 mV/m and the spectra in Figure 4e are perpendicular to the magnetic field for an electric field of 200 mV/m. For a given electric field, the spectra parallel and perpendicular to the magnetic field are identical. Both Figures 4d and 4e are minimally influenced by ion-ion and ion-electron collisions, and the MC simulated spectra are similar to the Maxwellian distribution based spectra.

## 4 Discussion

From Figures 2 and 3, it can be seen that the influence of ion-ion and ion-electron collisions only has a significant influence on the ion temperature and spectra in $O^+$ dominant environments near-parallel or parallel to the magnetic field. These findings could be used to explain unexpectedly high ion temperatures found in other works [6]. It was also seen in Figure 2 that the use of different $O^+$-O RCE cross-sections is capable of changing the ion temperature along a line-of-sight by several 1000 K under strong electric fields.

As for the spectra examined, Figures 4b and 4c show that the $O^+$ spectra parallel to the magnetic field has a larger “peak-to-trough" ratio than the spectra perpendicular to the magnetic field. Through standard ISR fitting routines this would show an apparent increase in electron temperature parallel to the magnetic field, even though all the spectra were simulated with an electron temperature of 2000 K. This high “peak-to-trough" ratio parallel to the magnetic field is the result of a particularly wide velocity distribution, or rather, a “hot tail". Lastly, in Figures 4d and 4e it can be seen that for $NO^+$ the MC simulated spectra can be modeled using Maxwellian velocity distributions of an equivalent line-of-sight ion temperature.

## 5 Conclusions

In this work, specific improvements and new techniques were applied to previous MC simulations in order to further study temperature anisotropy, ion-neutral frictional heating, and ISR spectra. The improvements and new techniques used included: 1) a higher resolution one-dimensional velocity distribution through an increased number of simulated ion-neutral particle collisions, 2) a new fitting technique to fully characterize the distribution over a wide range of velocities, 3) the use of Nyquist diagrams to check the plasma stability (which is important given that an unstable plasma would alter the velocity distribution, and therefore

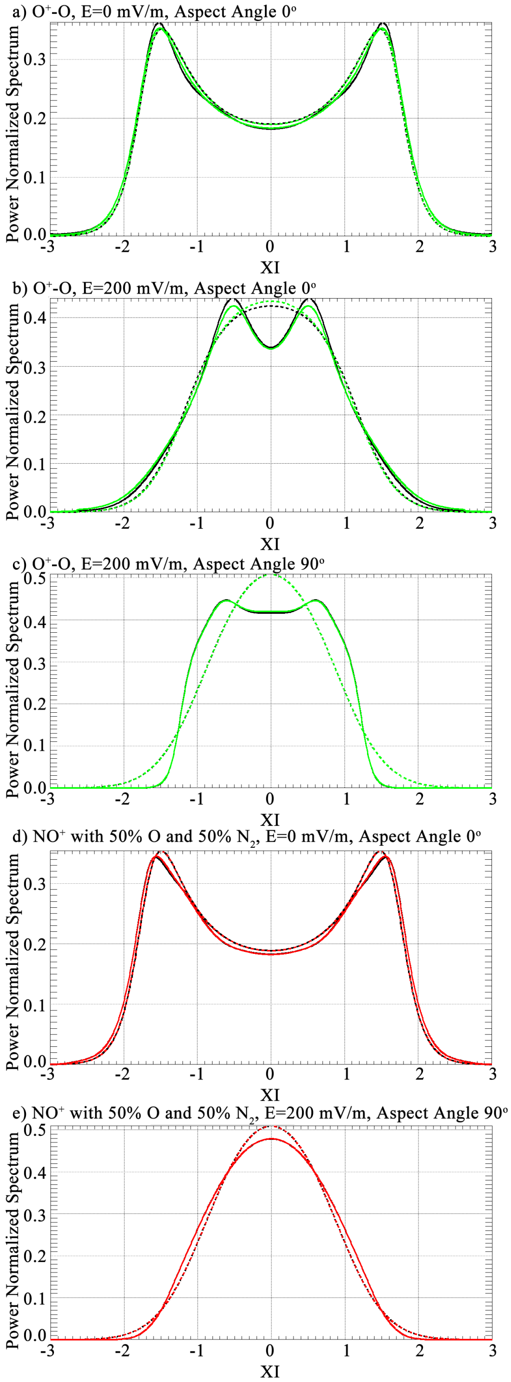


**Figure 4.** Spectra as a function of XI $= \omega/bk$ ($\omega$ is angular frequency, $b$ is the ion thermal speed, and $k$ is the wavenumber), where the electron temperature is 2000 K. Black lines reflect ion-neutral particle collisions, red/green lines include ion-ion and ion-electron collisions, solid lines are from MC simulated velocity distributions, and dashed lines are from Maxwellian velocity distributions of the same line-of-sight ion temperature. a) $O^+$-O collisions parallel to the magnetic field with no electric field. b) $O^+$-O collisions parallel to the magnetic field with a 200 mV/m electric field. c) $O^+$-O collisions perpendicular to the magnetic field with a 200 mV/m electric field. d) $NO^+$ with O and $N_2$ collisions parallel to the magnetic field and no electric field. e) $NO^+$ with O and $N_2$ collisions perpendicular to the magnetic field and a 200 mV/m electric field.

the spectra), 4) A study of the impact of more recent published RCE cross-sections for RCE collisions between $O^+$ and O [4], 5) a relatively simple way to incorporate ion-ion and ion-electron collisions into ion velocity distribution calculations, and 6) an improved filtering technique to reduce the influence of statistical noise from the MC calculations on the spectra. Through these improvements, it has been found that: 1) although ion-ion and ion-electron collisions have a minimal impact on the $NO^+$ ion velocity distribution, things are very different for $O^+$ ions higher up, where collisions with other charged particles greatly increase the ion temperature parallel to the magnetic field (which could explain some of the findings uncovered in [6]), 2) the use of a more recent determination by [4] for the RCE cross-section between $O^+$ and O not only can change the ion temperature along a line-of-sight by several 1000 K under strong electric fields, but can also stabilize the plasma against electrostatic fluctuations in spite of a strong toroidal distribution function, 3) $O^+$ spectra parallel to the magnetic field show an apparent increase in electron temperature due to an $O^+$ "hot tail", and 4) for the most part $NO^+$ spectra can be modeled using Maxwellian velocity distributions having the same line-of-sight ion temperature as the simulated non-Maxwellian distribution.

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