



## A comparative study of analytic modelling, numerical simulations, and experimental measurements for rough soil surface scattering and emission

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### Abstract

This paper presents a comprehensive evaluation of the scattering, both backscattering and bistatic scattering, and emission of rough soil surface predicted by a well-established theoretical model, the advanced integral equation model (AIEM). Extensive numerical data simulated by approximate and numerically exact models as well as experimental datasets of well-characterized bare soil surfaces were used to examine the performance of AIEM in predicting the scattering coefficient and microwave emissivity over a wide range of geometric parameters and ground surface conditions. The results show that the AIEM predictions are generally in good consistency with both numerical simulations and experiment measurements in terms of angular, frequency and polarization dependences, except for some deviations in a few cases (e.g. at large incident angles and dry soil conditions). Possible explanations for the discrepancy between the model prediction and data are given, together with suggestions for model applications and refinements.

### 1. Introduction

In the field of both active and passive microwave remote sensing, a practical and reliable theoretical model is required to achieve a good understanding of the surface scattering and emission mechanism as well as to guide the development of retrieval algorithm for the parameters of interest (e.g. soil moisture). Accordingly, there is an increasing interest and importance in the study of the scattering and emission modeling of the rough surface. The theoretical models can be categorized into two principal classes, i.e., the numerical models and the analytical models. The former is based on numerical solution of Maxwell's equations for the electromagnetic field. Though the numerical models can yield very high accurate simulations, they are extremely computationally expensive which makes their direct application for analysis of the microwave satellite data rather difficult. Contrastly, the analytical models give a good balance between simulation accuracy and computational efficiency, which makes them easily and efficiently employed in numerous microwave remote sensing applications.

Among the analytical models, the integral equation method (IEM) developed by Fung et al. [1] is regarded as

a milestone to the development of surface scattering model, since it seamlessly bridges the gap between the Kirchhoff approximation (KA) and small perturbation method (SPM). Chen et al. [2] further improved the IEM, called the advanced IEM (AIEM), by removing some assumptions in IEM, resulting in more accurate simulations over the original IEM particularly for bistatic scattering and microwave emissivity simulations. Together with model development, the analysis and examination of the model validity offer insights into the potential and limitations of the model, which can be very beneficial for model refinements as well as model applications. Though some efforts have been devoted to evaluation of the IEM/AIEM through inter-comparison with numerical simulations or experimental measurements (e.g. [3] and [4]), most work focuses on the backscattering comparisons, and limited studies have looked into the capability of AIEM in reproducing the bistatic scattering and microwave emissivity. To the best of our knowledge, currently no work has yet been carried out for a comprehensive analysis and examination of the validity and limitations of rough soil surface scattering and emission predicted by AIEM. Apparently, quantitative and systematical understanding the AIEM performance should be implemented to explore potential model applications for microwave remote sensing. This forms the basis of our work presented here.

### 2. Advanced integral equation model

Detailed mathematical derivations of the AIEM have been explicitly reported in [2] and [3]. Here we only give the final expressions of the bistatic scattering coefficient and microwave emissivity in the AIEM.

In AIEM, considering only the single scattering, the bistatic scattering coefficient  $\sigma_{qp}^s$  consists of three parts, namely the Kirchhoff term  $\sigma_{qp}^k$ , the cross term  $\sigma_{qp}^{kc}$ , and the complementary term  $\sigma_{qp}^c$ , taking the following form

$$\begin{aligned} \sigma_{qp}^s = \sigma_{qp}^k + \sigma_{qp}^{kc} + \sigma_{qp}^c &= \frac{k^2}{2} \exp[-s^2(k_{iz}^2 + k_{sz}^2)] \\ &\times \sum_{n=1}^{\infty} \frac{s^{2n}}{n!} |\mathcal{J}_{qp}^n|^2 W^{(n)}(k_{sx} - k_{ix}, k_{sy} - k_{iy}) \end{aligned} \quad (1)$$

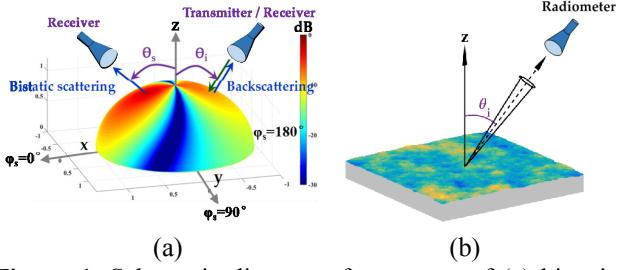
The surface effective reflectivity  $R_p^e$  consists of two components: the coherent component  $R_p^{coh}$  and the noncoherent component  $R_p^{non}$ , which is given by

$$R_p^e = R_p^{coh} + R_p^{non} = r_p \exp[-(2ks \cos \theta_i)^2] + \frac{1}{4\pi \cos \theta_i} \int_0^{2\pi} \int_0^{\pi/2} (\sigma_{pp}^s + \sigma_{pq}^s) \sin \theta_s d\theta_s d\phi_s \quad (2)$$

The corresponding coefficients in equation (1) and (2) can be found in [2] and [3]. Then the emissivity of a rough surface  $e_p$  can be computed by

$$e_p = 1 - R_p^e \quad (3)$$

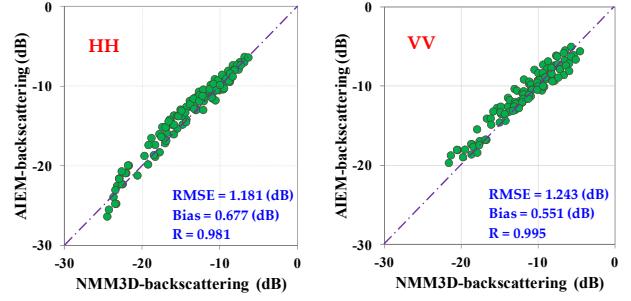
Figure 1 (a) and (b) illustrate a schematic diagram of geometry of bistatic scattering and emission respectively. In a bistatic radar system, the transmitter and receiver are separated and deployed in different locations, shown in Figure 1(a). The backscattering is a special case for bistatic scattering in which the transmitter and receiver are co-located. When an incident plane wave impinges onto a rough dielectric surface, it will be scattered as sum of coherent and incoherent signals. As given in equation (2) and (3), the emission from a rough surface can be calculated by one minus the total scattered energy.



**Figure 1.** Schematic diagram of geometry of (a) bistatic scattering, (b) emission.

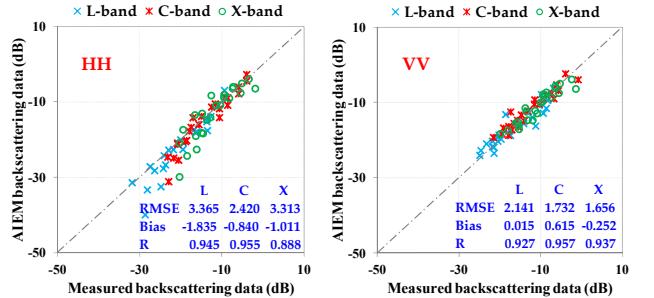
### 3. Comparison and validation of scattering predicted by AIEM

We firstly compare AIEM predictions with the numerical Maxwell model of three-dimensional simulations (NMM3D) for backscattering, shown in Figure 2. A look-up table includes a wide range of surface parameters with the real part of the soil dielectric constant from 3 to 30 and the RMS slope from 1/15 to 1/4 was pre-computed for both backscattering coefficients and microwave emissivity using the NMM3D [5]. Three commonly used error metrics including the root mean square error (RMSE), mean bias (Bias), and correlation coefficient (R) quantitatively demonstrate the high accuracy of AIEM compared with NMM3D. It can be clearly observed that the AIEM predictions are very close to the 1:1 line, suggesting that the AIEM is in excellent agreement with NMM3D simulations for both level and trend.



**Figure 2.** Comparison of backscattering coefficients between AIEM and NMM3D for an exponential correlated surface with an incident angle of 40°.

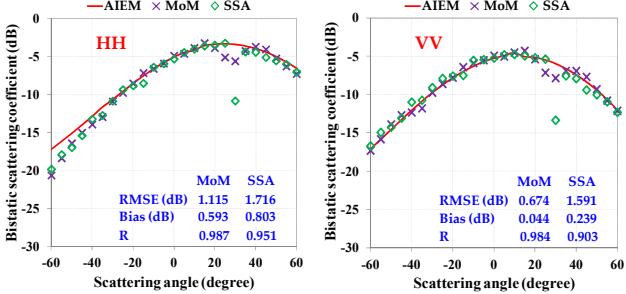
We also verify the AIEM backscattering predictions with experimental measurements, acquired by the University of Michigan's LCX POLARSCAT which provides backscattering measurements at three different frequencies (1.5 GHz, 4.75 GHz and 9.5 GHz) ranging from incident angles of 20° to 50° [6], shown in Figure 3. It is seen that the AIEM predictions at L/C/X-band are very close to the 1:1 line, indicating a quite favorable correlation between the model predictions and the measurement data. As a matter of interest, the AIEM performs better at VV polarization than at HH polarization with a lower RMSE and Bias value. This can be explained by the fact that we only use single scattering AIEM model. Multiple scattering generally exerts more impact on HH polarization than on VV polarization for backscattering. Also, the largest deviation of AIEM is found under the dry soil conditions (not shown), which may be due to the gradually increase of volume scattering in the dry soil surfaces that is somehow difficult to characterize.



**Figure 3.** Comparison of backscattering coefficient between AIEM and POLARSCAT data for an exponential correlated surface.

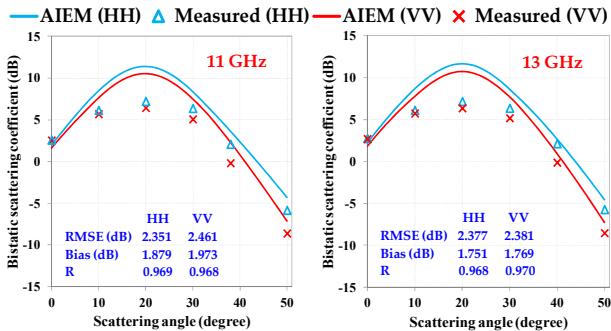
Figure 4 illustrates a comparison of the bistatic scattering coefficients in the incidence plane predicted from the AIEM and simulation results from the Small Slope Approximation (SSA) and method of moment (MoM). Results show that all three models are generally close to each other while slightly diverge after 40° in the backward directions especially for HH polarization. However, an obvious dip appears at the specular region for both the SSA and MoM, but not for the AIEM. This dip is physically unexpected despite the absence of coherent scattering like in this case. As a matter of fact, it

is commonly known that such kinds of dip feature generally occur at azimuthal direction. In addition, it can be observed that the AIEM predictions are closer to MoM than to SSA with lower RMSE and Bias values and higher correlation.



**Figure 4.** Comparison of bistatic scattering coefficient between AIEM and numerical results of MoM and SSA for a Gaussian correlated surface with  $\epsilon_r=4-j1$ ,  $ks=0.5$ ,  $kl=3.0$ , and incident angle of  $30^\circ$ .

Experimental measurements at 11 GHz and 13 GHz from the European Microwave Signature Laboratory (EMSL) were also used to assess the accuracy of bistatic scattering coefficients predicted by AIEM, shown in Figure 5. It can be observed that the bistatic scattering behavior of AIEM is generally in agreement with the EMSL data. The flatness of the angular trend between  $(10^\circ-30^\circ)$ , a forward scattering region, in measurements seems unexplainable, because in these scattering angles, stronger scattering signal is physically expected. Nevertheless, the AIEM well predicts the bistatic scattering for both HH and VV polarizations with a favorable RMSE value within 2.5 dB and a correlation value greater than 0.9.

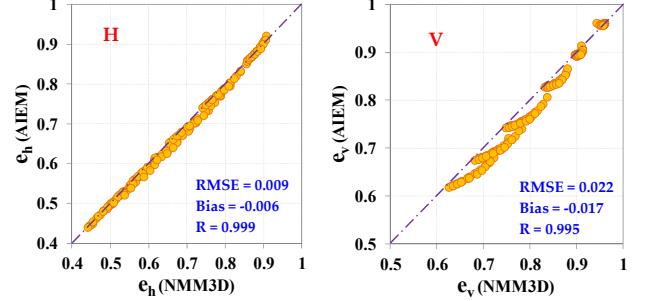


**Figure 5.** Comparison of bistatic scattering coefficient between AIEM and EMSL measurements for a Gaussian correlated surface with  $\epsilon_r=5.5-j2.1$ ,  $s=0.4$  cm,  $cl=6$  cm, and incident angle of  $20^\circ$ .

#### 4. Comparison and validation of emission predicted by AIEM

In this part, we adopt the emissivity look up table generated by NMM3D to assess the capability of AIEM in predicting the surface emissivity, shown in Figure 6. It is clearly observed that the AIEM predictions agree well with the NMM3D simulations in both level and trends. Though the AIEM emissivity slightly underestimates the

numerical results with a dry bias which is more obvious for the vertical polarization, the overall RMSE value of AIEM is only 0.009 and 0.022 for H and V polarizations respectively, indicating that both the AIEM and NMM3D predictions are quite close to each other. Since the emissivity look up table covers a broad range of soil permittivity and surface roughness, it is expected that AIEM is able to well reproduce the emissivity under various surface parameters.

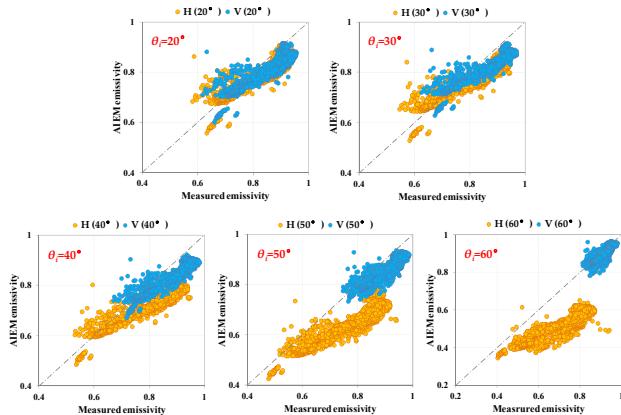


**Figure 6.** Comparison of emissivity between AIEM and NMM3D for an exponential correlated surface with a look angle of  $40^\circ$ .

An unique dataset, acquired at the SMOSREX06 site in the south of Toulouse [7], France, was used to examine the AIEM performance in predicting the emissivity of bare soil surfaces, shown in Figure 7. In SMOSREX06, a LEWIS radiometer was used to acquire both horizontal and vertical polarized brightness temperatures at five look angles (i.e.  $20^\circ$ ,  $30^\circ$ ,  $40^\circ$ ,  $50^\circ$ , and  $60^\circ$ ) from 1 January 2006 to 31 December 2006. Different performances of AIEM are observed in terms of angular and polarization dependences. It is seen visually that in general, the AIEM predictions are fairly close to the measured emissivity at small to intermediate look angles from  $20^\circ$  to  $40^\circ$  for both H and V polarizations. However, with the look angle increases to  $50^\circ$  and  $60^\circ$ , the divergence between these two data becomes apparent especially for H polarization. The error metrics concerning the AIEM emissivity are listed in Table I. Indeed, the RMSE is not quite satisfactory, particularly at large look angles (e.g.  $50^\circ$  and  $60^\circ$ ) for H polarization. The AIEM predictions correlate very well with the measured emissivity with the correlation coefficient close to 0.9 for both H and V polarizations. Since the observation period of the measurements is very long, collecting a total of 8125 samples and the comparison, as presented here, is made without adjusting the given input parameters to the AIEM model, it is convinced that the AIEM is able to correctly predict the temporal dynamic of the observed surface emissivity over a wide range of look angle and surface parameters. Furthermore, it has been recognized that large extent of uncertainties may fitfully exist in the field measurements of the soil moisture and surface roughness at the size of antenna footprint. Thus, it is highly suspected that the bias between the “ground truth” and the “model outputs” does not necessarily imply as large error. If we remove the bias by using the unbiased RMSE (ubRMSE, defined as  $\sqrt{RMSE^2 - Bias^2}$ ), we can find

that the absolute accuracy of AIEM reaches to 0.052 and 0.031 for H and V polarizations respectively.

It is also worth mentioning at this point that in this study only single scattering in AIEM was considered and the multiple scattering was ignored due to in part its higher computation load, and in part the surface slopes considered here were moderate. Nevertheless, in some cases especially when the surface root mean square (RMS) slope is large, multiple scattering needs to be considered to achieve a better accuracy in predicting bistatic scattering and in turn, the more accurate in predicting emissivity. Thus, the overlook of the impact of multiple scattering may be one of the reasons that cause the underestimation of the emissivity predicted by AIEM particularly for the H polarization. All these outcomes will be feedback to further appraise and improve the AIEM model in the future.



**Figure 7.** Comparison of H and V polarized emissivity between AIEM and SMOSREX06 measurements for an exponential correlated surface at 1.41 GHz with five look angles ( $20^\circ$ ,  $30^\circ$ ,  $40^\circ$ ,  $50^\circ$ , and  $60^\circ$ ).

**Table I.** Comparison of AIEM emissivity predictions with SMOSREX06 measurements

$\theta_i$ ( $^\circ$ )	RMSE		Bias		ubRMSE		R	
	H	V	H	V	H	V	H	V
20	0.061	0.059	-0.041	-0.044	0.046	0.039	0.863	0.873
30	0.064	0.054	-0.041	-0.038	0.049	0.039	0.906	0.908
40	0.090	0.056	-0.075	-0.045	0.050	0.032	0.916	0.905
50	0.134	0.053	-0.123	-0.046	0.053	0.026	0.902	0.868
60	0.196	0.026	-0.185	-0.015	0.063	0.021	0.866	0.833
All	0.109	0.050	-0.093	-0.038	0.052	0.031	0.891	0.877

## 5. Conclusions

In the study, we conducted a comprehensive analysis of the scattering and emission predicted by the AIEM over a wide range of geometric parameters and ground surface conditions by verifying with extensive numerical simulations (NMM3D, MoM, and SSA) and experimental measurements (POLARSCAT, EMSL and SMOSREX06 data) for the first time. The results show that the AIEM

predictions are in good accordance with both numerical simulations and experiment measurements with a low absolute error and a very favorable correlation. Larger divergence of AIEM appears at larger incident (look) angle or dry soil conditions, which may be caused by the increased contribution of multiple and volume scattering, especially for HH or H polarization. Overall, this study demonstrates the effectiveness and practicability of AIEM for both scattering and emission of rough soil surface, and inclusion of multiple scattering in the AIEM to account for a more complete scattering process is called to improve our understanding, predicting, and inferring of microwave scattering and emission from rough surfaces.

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