32nd URSI GASS, Montreal, 19-26 August 2017

A Semi-Analytical Simplified Approach to Compute Lightning Radiated Electric Fields at Long Distances Taking into Account Ionospheric Reflection

Mohammad Azadifar (1), Dongshuai Li (2), Marcos Rubinstein (3) and Farhad Rachidi (1)
(1) Electromagnetic Compatibility Laboratory, Swiss Federal Institute of Technology (EPFL), 1015 Lausanne, Switzerland
(2) Earth System Modeling Center and Climate Dynamics Research Center, Nanjing University of Information Science and Technology, Nanjing, China
(3) University of Applied Sciences of Western Switzerland, 1400 Yverdon-les-Bains, Switzerland

Abstract

In this paper, we present a semi-analytical simplified approach based on the ray tracing method to estimate radiated electric fields associated with lightning return strokes, taking into account ionospheric reflections.
The field transfer function at each frequency and each grazing angle is determined by applying the generalized Snell's law of refraction, and the antenna radiation pattern concept is used to obtain the source illumination efficiency, from which the radiated field is derived.
The proposed method is validated using as reference full-wave FDTD simulations.

1. Introduction

The electromagnetic fields radiated by a lightning flash can propagate very long distances along the earth-ionosphere waveguide [1]. Consequently, lightning electromagnetic fields can be used to probe ionospheric characteristics [2]. Various full-wave numerical methods both in the frequency domain and in the time domain have been developed to investigate electromagnetic field interaction with the ionosphere (see [3] for a recent review).
In this paper, we present a semi-analytical approach based on the ray tracing concept [4] to estimate lightning radiated electromagnetic fields taking into account the ionospheric reflection.
The paper is organized as follows. In Section 2, we present the methodology and the step-by-step procedure for the calculation of radiated fields. Section 3 presents the validation of the proposed approach by comparing its results with a full-wave FDTD approach [5]. Section 4 presents a discussion and general conclusions.

2. Proposed Approach

2.1 General considerations

Various numerical techniques have been adopted to compute lightning radiated electromagnetic fields (e.g., FDTD [6-9], MoM [10], FEM [11]). The application of these full-wave techniques at very far distances could be computationally burdensome.
In this paper, we present a semi-analytical approach based on the ray tracing concept which can be used for a fast computation of lightning electromagnetic fields at far distances, taking into account ionospheric reflections. The proposed approach can be also interesting in providing an insight into the physics of the wave propagation characteristics as a function of frequency and illumination angle.

2.2 General methodology and simplifying assumptions

Let us assume a vertical lightning channel of height H_o located along the z axis. As illustrated in Figure 1, the associated electromagnetic field radiates out from the channel in all directions and it depends both, on the frequency and on the direction. If the observation distance is sufficiently large compared to the channel length, the fields can be assumed to emanate from one point. The fields at any point in space can be obtained by the superposition of solutions at different frequencies and angles. The generalized Snell's law of refraction [4] along with the following simplifying assumptions were used to estimate the radiated electric field:

- An inhomogeneous, highly collisional and isotropic ionosphere environment is assumed (see Section 2.3 for more details).
- Multiple reflections in between layers of the absorbing media representing the ionosphere are neglected and only the first reflection at half distance is considered (see Section 2.5 for more details).
- The MTLE return stroke model [12-13] is used to compute the incident electric field, on which amplitude and phase corrections are applied to account for the presence of the ionosphere. Only the radiation field is considered in the analysis.
- The effect of the earth curvature is disregarded.
- The ground is assumed to be a perfectly-conducting half-space.

The applied methodology can be summarized as follows: First, the path of the field propagation is determined at each frequency and at each angle (frequency-angle space) using Snell's law of refraction for absorbing media, and the phase delay of the path and its associated attenuation loss are calculated analytically. Then, the lightning source illumination efficiency is estimated using the antenna radiation pattern concept [14], from which the radiated field is derived. Figure 2 shows a flowchart of the detailed step-by-step procedure which will be explained in detail in Sections 2.3 to 2.6.

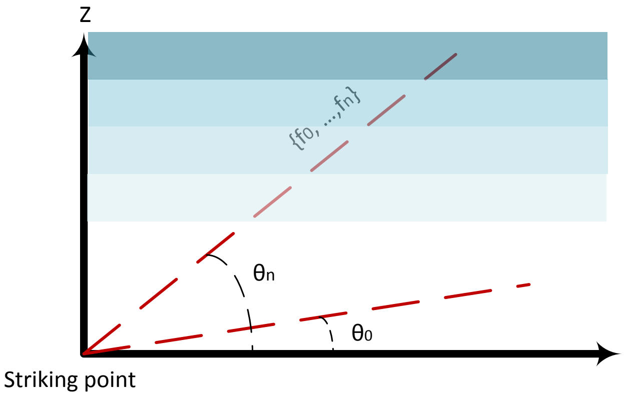

Figure 1. Schematic diagram of frequency-angle solution space. The shaded area represents the considered discretized stratified medium.

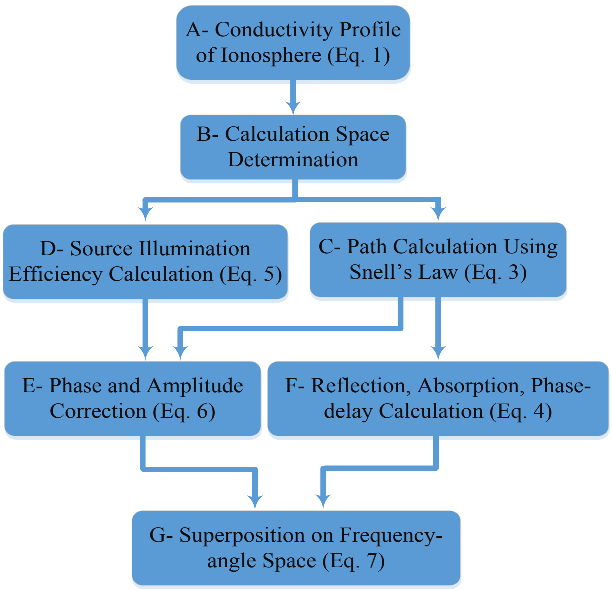

Figure 2. Schematic diagram of step by step procedure to estimate distant electric field.

2.3 Conductivity profile of the ionosphere (Step A)

An isotropic, inhomogeneous, highly collisional environment is considered for modeling the ionosphere [15]. The conductivity profile suggested in [15] is adopted in this work:

$$\sigma(z) = \varepsilon_0 (2.5\times10^5) e^{\beta(z-h')} \text{, z} > 0 \qquad (1)$$

where ε_0 is the permittivity of free space, β is the slope of the atmospheric conductivity profile, assumed to be $\beta = 0.7km^{-1}$ (nighttime conditions) and h' is the reference reflection height, which is assumed to be 90 km (nighttime conditions).

2.4 Calculation space (Step B)

According to the adopted conductivity profile, the calculation space is represented as discrete, stratified horizontal layers with variation of conductivity values determined by (1). The vertical discretization step is assumed to be $\lambda_{\min}/10$, where $\lambda_{\min}$ is the wavelength associated with the highest significant frequency of the source spectrum. For the sake of simplicity, we neglect the effect of the earth curvature and the finite ground conductivity in this preliminary study.

2.5 Path calculation using Snell's law (Step C)

To calculate the path of the propagating field between the origin (lightning strike point) and the observation point, we use the generalized Snell's law of refraction for absorbing media to calculate field properties at the intersection of medium adjacent layers.

A travelling plane wave with propagation direction $\vec{\mathbf{r}}$ can be written in the form $\vec{\mathbf{E}}(\omega) = \vec{\mathbf{E}}_0(\omega) e^{-\vec{\gamma}.\vec{r}}$ in the frequency domain, where:

$$\left|\vec{\gamma}\bullet\vec{\gamma}\right| = \left(j\omega\mu_0 \times (\sigma + j\omega\varepsilon_0)\right) = \alpha + j\beta = -\left|\vec{\mathbf{k}}\bullet\vec{\mathbf{k}}\right| \qquad (2).$$

where $\vec{\mathbf{k}}$ is the wave vector, ε_0 and μ_0 are, respectively, the permittivity and the permeability of free space.

The generalized Snell's law of refraction at the interface of two absorbing media can be obtained using the following equation:

$$\begin{aligned}
&\beta_i^2 - \alpha_i^2 = \omega^2 \mu_0 \varepsilon_0 \\
&\beta_i \times \sin\theta_i = \beta_{i-1} \times \sin\theta_{i-1} \\
&\alpha_i \times \sin\Psi_i = \alpha_{i-1} \times \sin\Psi_{i-1} \\
&\alpha_i \times \beta_i \times \cos(\theta_i - \Psi_i) = \alpha_i \times \beta_i \times (\vec{\mathrm{e}}_i . \vec{\mathrm{f}}_i) = \frac{\omega^2 \mu_0 \sigma}{2}
\end{aligned} \qquad (3).$$

where the incident field can be expressed in terms of the wave vector $\vec{\mathbf{k}}_{i-1} = \beta_{i-1}\vec{\mathbf{e}}_{\mathbf{i-1}} - \alpha_{i-1}\vec{\mathbf{f}}_{\mathbf{i-1}}$, where $\vec{\mathbf{e}}_{\mathbf{i-1}}$ and $\vec{\mathbf{f}}_{\mathbf{i-1}}$ are perpendicular to the planes of constant phase and constant amplitude, respectively, in the $(i-1)^{th}$ medium [4]. Assuming a propagation direction defined by the angle θ_0 as shown in Figure 3, when the two adjacent layers have very similar properties ($\sigma_{i-1} \approx \sigma_i$), we expect a very small value for the reflection coefficient ($\Gamma \ll 1$) at their interface. These higher order reflections are neglected in the analysis, since $\Gamma^n \ll \Gamma \ll 1$ $(n \geq 3)$. As a result, for each illumination angle θ_0, only one ray will reach the observation point (this ray is represented in bold in Figure 3 and corresponds to a reflection point at half distance).

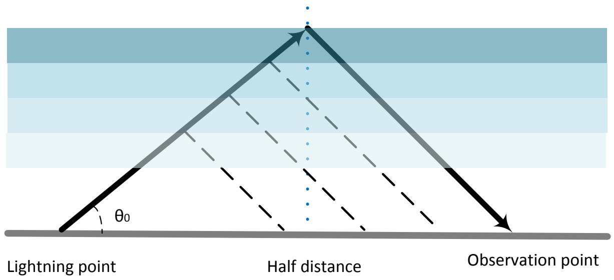

Figure 3. Schematic diagram of the first-hand reflections, the dashed lines show the first-hand reflected rays which

will not reach the observation point. The shaded area represents the considered discretized stratified medium.

2.5 Reflection, absorption and phase delay (Step F)

The field that passes through the dispersive ionosphere will be affected by reflection from the stratified layers (R_p), as well as absorption losses (AL) and phase delay (PD), which are dependent on the incidence angle and frequency. The following set of equations allows to determine these parameters [4,16]:

$$
\begin{aligned}
&\mathrm{R_p}(\theta,\omega) = \frac{\tilde{n}_i^2 k_{i-1} - \tilde{n}_{i-1}^2 k_i}{\tilde{n}_i^2 k_{i-1} + \tilde{n}_{i-1}^2 k_i} \\
&\mathrm{AL}(\theta,\omega) = \\
&\exp(-2\times \int_0^{L\_reflection} \alpha(\mathrm{l})\times \mathrm{l}\times\cos(\theta(\mathrm{l})-\Psi(\mathrm{l}))\times \mathrm{dl}) \\
&\mathrm{PD}(\theta,\omega) = \\
&\exp(-2\times \int_0^{L\_reflection} \beta(\mathrm{l})\times \mathrm{l}\times \mathrm{dl})
\end{aligned} \qquad (4).
$$

in which $\tilde{n}_i$ is the refractive index of the i^{th} layer, $k_i = \vec{\mathbf{n}}\bullet\vec{\mathbf{k}}_{\mathbf{i}}$ ($\vec{\mathbf{n}}$ is the normal unit vector to the incident plane), l is the path of the wave determined by $l_i = dh/\cos(\theta_i)$ and $L\_reflection$ is the distance corresponding to $x_i = \frac{d}{2}$. The factor 2 in the exponent accounts for the fact that the path of the down-going wave is just the flipped version of the up-going one (at each frequency and at each illumination angle).

2.5 Source illumination efficiency calculation at frequency-angle space (Steps D and E)

The current distribution along the return stroke channel was specified according to the modified transmission line model with exponential decay (MTLE) [12, 13], assuming a current decay constant of λ = 2 km and a return stroke speed of 1.5×10^8 m/s. The height of the lightning channel was set to 8 km. The incident radiation field of the lightning return stroke is calculated in the frequency domain using [17]:

$$
\begin{aligned}
E_z(\mathrm{r},\mathrm{z},\omega) = \int_0^{H_0} \frac{-r^2}{2\pi\varepsilon_0}\times\frac{j\omega}{c^2R^3}\times \\
I(\mathrm{z}',\omega)\times\exp(-\,\mathrm{j}\,\omega\, R/c)\,\mathrm{dz}'
\end{aligned} \qquad (5).
$$

in which, r and z are the cylindrical coordinates of the observation point, R is the distance between each element of current along the channel from the observation point ($R=\sqrt{r^2+\left(z'-z\right)^2}$), $I(\mathrm{z}',\mathrm{t})$ is the Fourier transform of the current along the channel, c is the speed of light and ε_0 is the permittivity of free space.

First, we calculate the radiated electric field at far distance ranges in free space for all observation angles in the frequency domain using (5), from which one can simply derive $E_z(d/2,\theta,\omega)$. The effect of propagation through the absorbing medium is accounted for using the following correction factors:

$$
\begin{aligned}
&\mathrm{K_{amp}}(\theta,\omega) = \frac{\int_0^{\mathrm{H}\_reflection} \frac{1}{\cos(\theta(\mathrm{h}))}\times dh}{\frac{d}{2\sin(\theta_0)}} \\
&\mathrm{K_{phase}}(\theta,\omega) = \exp\left[\mathrm{j}\left(\frac{\omega}{c}\right)\times\left(\frac{d}{2\sin(\theta_0)}\right)\right]
\end{aligned} \qquad (6).
$$

in which $H\_reflection$ corresponds to $x_i = \frac{d}{2}$, and θ_0 is the illumination angle.

2.6 Superposition in the frequency-angle space

Once the solution is obtained in the frequency-angle space, the total vertical electric field at the observation point can be evaluated superimposing all the contributions using

$$
\begin{aligned}
E_z(\mathrm{d},\mathrm{z}=0,\mathrm{t}) = \mathrm{F}^{-1}(\sum_\theta\sum_\omega E_z(d/2,\theta,\omega) \\
\times K_{amp}(\theta,\omega)\times K_{phase}(\theta,\omega) \\
\times R_p(\theta,\omega)\times AL(\theta,\omega)\times \mathrm{PD}(\theta,\omega))
\end{aligned} \qquad (7).
$$

3. Validation using FDTD simulation

Full-wave FDTD simulations [5] are used as reference to test the validity of the proposed approach. The vertical electric field radiated by a lightning discharge is evaluated at a distance of 380 km from the lightning channel. The channel-base current represented by the sum of two Heidler's functions is presented in Figure 5. This waveforms corresponds to a recorded current of a subsequent return stroke of an upward flash recorded at the Säntis Tower [5]. The MTLE model with parameters presented in Section 2.5 was used to specify the current distribution along the lightning channel. Figure 6 shows the obtained results using the proposed approach, and those obtained using the FDTD method (the FDTD simulation parameters are described in [5]).

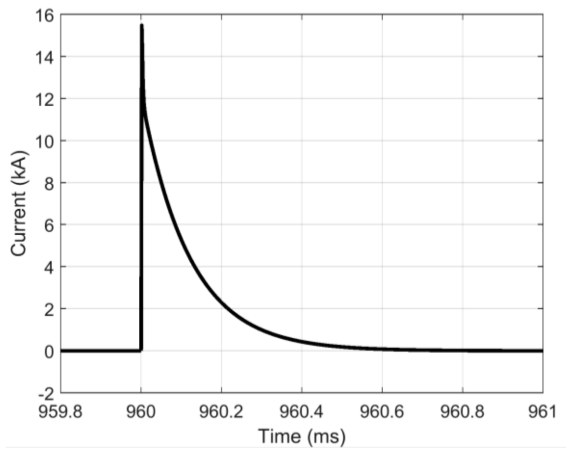

Figure 5. Channel-base current of a subsequent return stroke recorded at the Säntis Tower represented using Heidler's functions (adapted from [5]).

It can be seen from Figure 6 that the field predicted by the proposed semi-analytical approach is in reasonably good agreement with the FDTD results. The observed differences in the arrival time of the skywave can be explained by the fact that the earth curvature was considered in the FDTD simulations, but neglected in the semi-analytical approach. In addition, differences in the risetime and in the initial pulse width can be explained by the fact that in the FDTD simulations, the ground finite conductivity was accounted for (10^{-3} S/m).

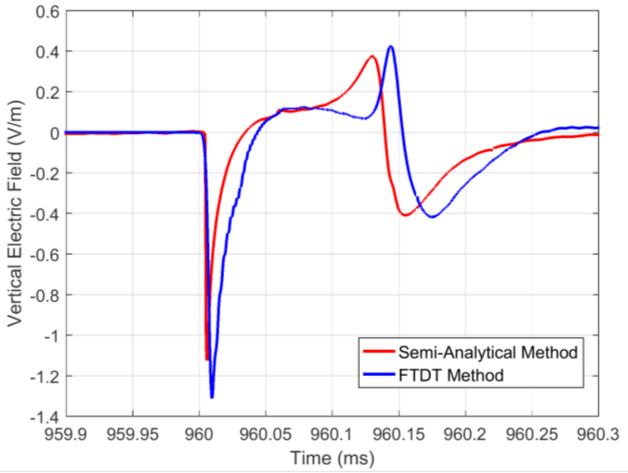

Figure 6. Comparison of Simulated vertical electric field using the semi-analytical approach and FDTD method.

4. Conclusions

We presented a semi-analytical simplified approach based on the ray tracing method to estimate the radiated electric field associated with lightning return strokes, taking into account ionospheric reflections.

The field transfer function at each frequency and each illumination angle was determined by applying the generalized Snell's law of refraction, and the antenna radiation pattern concept was used to obtain the source illumination efficiency, from which the radiated field was derived.

The proposed method was validated using as reference full-wave FDTD simulations.

5. References

1. S. A. Cummer, "Modeling electromagnetic propagation in the earth-ionosphere waveguide," IEEE Transactions on Antennas and Propagation, Vol. 48, No. 9, pp. 1420-1429, 2000.
2. U. S. Inan, S. A. Cummer, and R. A. Marshall, "A survey of ELF and VLF research on lightning - ionosphere interactions and causative discharges," J. Geophys. Res., 115, A00E36, 2010.
3. A. R. Jacobson, X.-M. Shao, and R. Holzworth, "Full-wave reflection of lightning long-wave radio pulses from the ionospheric Dregion: Numerical model," J. Geophys. Res., 114, A03303, 2009, doi:10.1029/2008JA013642.
4. P. C. Chang, J. G. Walker, and K. I. Hopcraft, "Ray tracing in absorbing media," Journal of Quantitative Spectroscopy and Radiative Transfer 96, no. 3 (2005): 327-341.
5. M. Azadifar, D. Li, F. Rachidi, M. Rubinstein, G. Diendorfer, W. Schulz, V. A. Rakov, M. Paolone, and D. Pavanello, "Analysis of Lightning-Ionosphere Interaction Using Simultaneous Records of Source Current and 380-km Distant Electric Field," submitted to Journal of Atmospheric and Solar-Terrestrial Physics, 2016.
6. Y. Baba, and Vladmir A. Rakov, "Applications of the FDTD method to lightning electromagnetic pulse and surge simulations," Lightning Protection (ICLP), 2014 International Conference o. IEEE, 2014.
7. M. Khosravi-Farsani, R. Moini, S. H. H. Sadeghi, and F. Rachidi, "On the validity of approximate formulas for the evaluation of the lightning electromagnetic fields in the presence of a lossy ground," IEEE Trans. Electromagn. Compat., vol. 55, no. 2, pp. 362–370, Apr. 2013.
8. A. Mimouni, F. Rachidi, and Z. Azzouz, "A finite-difference time domain approach for the evaluation of electromagnetic fields radiated by lightning strikes to tall structures," J. Electrostatics, vol. 66, no. 9, pp. 504–513, Sep. 2008.
9. D. Li, Q. Zhang, T. Liu, and Z. Wang, "Validation of the Cooray Rubinstein (C-R) formula for a rough ground surface by using three dimensional (3-D) FDTD,"J. Geophys. Res., vol. 118, no. 22, pp. 12749–12754, Nov. 2013.
10. R. Moini, B. Kordi, G. Z. Rafi, and V. A. Rakov, "A new lightning return stroke model based on antenna theory," J. Geophys. Res., vol. 105, no. D24, pp. 29693–29702, Dec. 2000.
11. J. Paknahad, K. Sheshyekani, M. Hamzeh, and F. Rachidi, "Lightning electromagnetic fields and their induced voltages on overhead lines: The effect of a non-flat lossy ground," in Proc. 32nd Int. Conf. Lightning Protection, Shanghai, China, 2014, pp. 591–594.
12. C. A. Nucci, C. Mazzetti, F. Rachidi, and M. Ianoz, "On lightning return stroke models for LEMP calculations," presented at the 19th Int. Conf. Lightning Protection, Graz, Austria, 1988.
13. F. Rachidi and C. A. Nucci, "On the Master, Uman, Lin, Standler and the modified transmission line lightning return stroke current models," J. Geophysical Res.: Atmospheres, vol. 95, pp. 20389–20393, 1990.
14. C. A. Balanis, "Antenna Theory, Analysis and Design," John Wiley & Sons, Inc., 2nd ed. 1982.
15. J. R. Wait, and K. P. Spies, "Characteristics of the Earth-ionosphere waveguide for VLF radio waves," US Dept. of Commerce, National Bureau of Standards: for sale by the Supt. of Doc., US Govt. Print. Off, 1964.
16. R. F. Harrington, "Time-Harmonic Electromagnetic Fields," Wiley-IEEE Press, ISBN: 978-0-741-20806-8.
17. M. A. Rubinstein, and M. A. Uman. "Methods for calculating the electromagnetic fields from a known source distribution: Application to lightning." IEEE Transactions on Electromagnetic Compatibility 31.2 (1989): 183-189.