Sensitivity Enhancement by Parity-Time Symmetry in Wireless Telemetry Sensor Systems

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Abstract

We propose here a new paradigm, leveraging the concept of parity-time (PT) symmetry in modern physics, to enhance the sensitivity and quality factor (Q-factor) of inductor-capacitor (LC) passive wireless sensors. We demonstrate that when the electromagnetically coupled sensor and reader is suitably tailored to satisfy the space-time reflection symmetry, this PT-symmetric wireless sensing system can provide significantly greater modulation levels, sensing resolution, and sensitivity when compared with conventional loop-antenna readers. This finding may benefit a number of wireless sensing, detection, and imaging systems, particularly for loss- and noise-immune miniature RFID sensor, wearables, and micromachined implants.

1. Introduction

Self-powered wireless sensing is a growing trend towards contactlessly and continuously monitor the physical and chemical characteristics in medical, industrial, and environmental applications, where traditional wired sensors suffer from complex wiring and difficult maintenance. Nowadays, many passive wireless sensors based on simple, tunable LC-oscillator have been proposed for sensing the pressure, strain, humidity, temperature, and human-body conditions (e.g. intraocular, intravascular, and intracranial pressures [1]-[2]). In general, a loop antenna is usually used to interrogate these devices via the inductive coupling, as shown in Fig. 1. Such telemetry system can be analyzed with a simple, but yet effective, equivalent circuit model shown in Fig. 1, where the loop antenna and the passive sensor are modeled as an inductor and an RLC tank, respectively.

So far, most of the research efforts have been concentrated on the design, fabrication and optimization of the passive sensor itself [1]-[2], while very limited effort has been made to develop a robust and high-performance read-out device. However, wireless measurements for passive sensors still present several challenges not yet addressed. One of the primary challenges for modern miniature wireless sensors, such as those based on microelectromechanical systems (MEMS), lies in how to achieve accurate and sensitive measurements. It is known that the sensitivity and signal-to-noise ratio (SNR) of the passive wireless microsensors are often hindered by the low modal Q-factor of sensor, owning to the limited device dimensions, associated with skin-effect resistive losses, and material losses from the package and background (e.g. human body at radio frequencies (RF)).

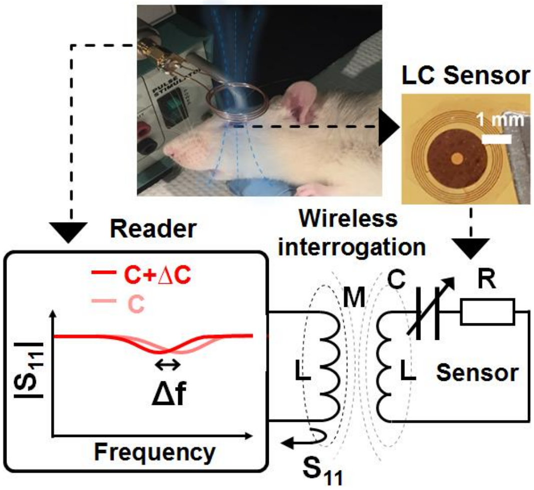

Figure 1. (a) Schematic and equivalent circuit diagram of the wireless passive LC sensor. Here an MEMS pressure sensor, based on a pressure-tuned micromachined varactor, is interrogated by a loop antenna, and therefore the eye-pressure variation of animal is detected by monitoring the resonance frequency shift.

In this work, we present a novel PT-symmetric wireless sensing system, as shown in Fig. 2, capable of considerably enhancing the Q-factor and sensitivity of general LC-based wireless sensors. The concept of PT symmetry was originated within the framework of quantum mechanics, giving rise to many exciting physical properties. For instance, a non-Hermitian PT-symmetric Hamiltonian may counterintuitively have real eigenfrequencies [3]. Here, we intend to extend the concept of PT-symmetry to the RF telemetry systems, with a particular focus on the wireless sensing technique. Figure 2 shows that equivalent circuit of PT-symmetric wireless sensing system, which remains invariant after the parity (spatial inversion) $\mathcal{P}$ and time-reversal $\mathcal{T}$ transformations [3]. The proposed PT-symmetric wireless sensing system employs the active reader (effectively an –RLC tank) to wirelessly interrogate the passive wireless sensor (effectively an RLC tank circuit). In the following, we will first discuss the theory of

PT-symmetry in RF electronic circuits, and then demonstrate our idea with a practical MEMS-based LC sensor, which is designed and fabricated to sense the intraocular pressure (IOP) of human eyes [2].

2. PT-Symmetric Telemetry Systems

PT-symmetry, first proposed in quantum mechanics by Bender in 1998, has recently been extended to laser and optical experiments with balanced gain and loss, due to the formal similarities between the Maxwell and Schrödinger wave equations [4]-[6]. In the radio engineering society, telegrapher's equation comprising distributed elements are commonly used to describe electrical phenomena, resulting from Maxwell's equations within the realm of electromagnetism. Hence, PT-symmetry may also be realized in the RF frequency range, in the form of transmission line or lumped-element circuitry. Figure 2 shows a PT-symmetric RF circuitry, which consists of a pair of tightly coupled RLC (e.g. sensor) and –RLC (e.g. reader) tanks, respectively responsible for attenuation and amplification of RF signals, so as to balance loss and gain in the PT system [4]-[6]. Applying the Kirchhoff's law to the coupled circuits in Fig. 2, the circuit can be expressed using the Liouvillian formalism as:

$$\mathrm{L}\Psi = \frac{d\Psi}{d\tau};$$

$$\mathrm{L} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{1-\kappa^2} & \frac{\kappa}{1-\kappa^2} & -\frac{1}{\gamma\left(1-\kappa^2\right)} & -\frac{\kappa}{\gamma\left(1-\kappa^2\right)} \\ \frac{\kappa}{1-\kappa^2} & -\frac{1}{1-\kappa^2} & \frac{\kappa}{\gamma\left(1-\kappa^2\right)} & \frac{1}{\gamma\left(1-\kappa^2\right)} \end{pmatrix} \quad (1)$$

where $\Psi \equiv \left(q_1, q_2, \dot{q}_1, \dot{q}_2\right)^T$, q_1 (q_2) corresponds to the charge stored on the capacitor on the −RLC (RLC) tank, $\tau \equiv \omega t$, the rescaled mutual inductance $\kappa = M/L$, $\omega_0 = 1/\sqrt{LC}$, and the gain-loss parameter (or non-Hermiticity parameter) $\gamma = R^{-1}\sqrt{L/C}$; all frequencies are measured in units of ω_0. It is evident that the Liouvillian expression, Eq. (1), remains the same under a combined parity $\mathcal{P}$ (i.e., $q_1 \leftrightarrow q_2$) and time reversal $\mathcal{T}$ (i.e., $t \to -t$) transformation. It can be shown that the matrix $-j\mathrm{L}$ can be interpreted as the Schrödinger equation with non-Hermitian effective Hamiltonian H_{eff}, which is symmetric with respect to generalized $\mathcal{PT}$ transformations:

$$\left[\mathcal{PT}, H_{eff}\right] = 0, \quad \text{(2a)}$$

$$\text{where } \mathcal{P} = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_x \end{pmatrix}, \quad \mathcal{T} = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} \end{pmatrix}\mathcal{K}, \quad \text{(2b)}$$

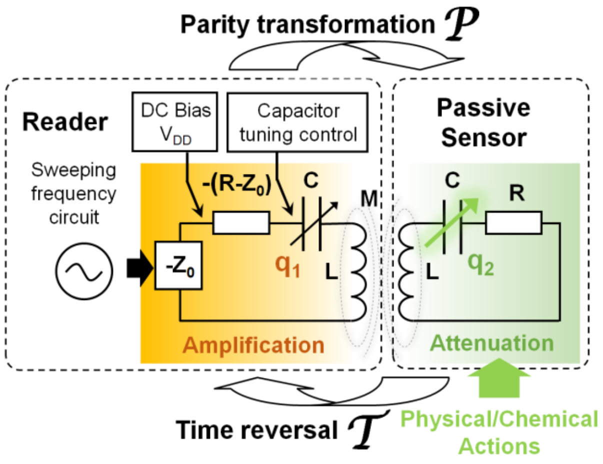

Figure. 2. PT-symmetric wireless sensing system, consisting of a passive LC sensor (equivalent to a RLC tank) inductively coupled to an active –RLC reader, with the whole system follows the space-time reflection symmetry.

σ_x is the Pauli matrix, $\mathbf{I}$ is the 2×2 identity matrix, and $\mathcal{K}$ conducts the operation of complex conjugation. Such system has four normal mode frequencies given by:

$$\omega_{1,2,3,4} = \pm\sqrt{\frac{2\gamma^2 - 1 \pm \sqrt{1-4\gamma^2+4\gamma^4\kappa^2}}{2\gamma^2\left(1-\kappa^2\right)}}. \quad (3)$$

Figure 3(a) presents the complex eigenfrequencies evolve with γ, showing three distinct regimes of behavior. Similar to its quantum counterparts [3]-[4], eigenspectrum of the PT-symmetric telemetric system displays an exceptional point γ_{EP}, where the system would transit from the exact symmetry phase with real eigenfrequencies to the broken symmetry phase with complex-valued eigenfrequencies. The eigenfrequencies branch out into the complex plane below the exceptional point:

$$\gamma_{\mathrm{EP}} = \frac{1}{\kappa}\sqrt{\frac{1+\sqrt{1-\kappa^2}}{2}}. \quad (4)$$

In the parametric region $\gamma \in \left[\gamma_{EP}, \infty\right]$, eigenfrequencies are purely real ($\gamma \in \mathbb{R}$), as shown in Fig. 3(a), and $\mathcal{PT}\Phi_k = \Phi_k$ such that the PT-symmetry is exact. If one seeks only purely real solutions, there is redundancy because positive and negative eigenfrequencies of equal magnitude are essentially identical. The motion in this exact PT-symmetric phase is oscillatory at the two eigenfrequencies (resonance frequencies). The region $\gamma \in \left[\gamma_c, \gamma_{EP}\right]$ is known

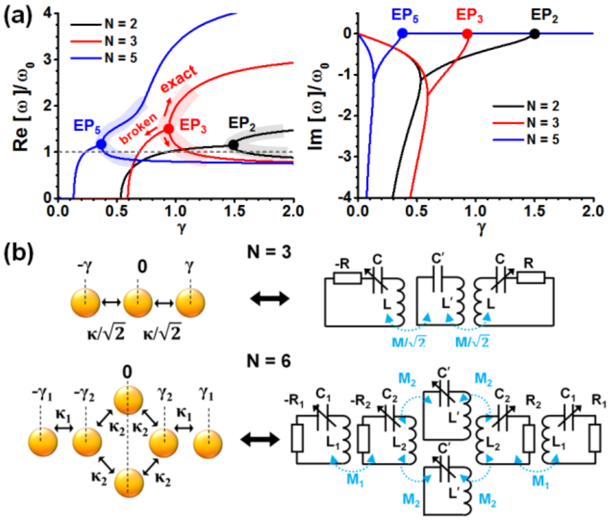

Figure 3. (a) Normalized real (left) and imaginary (right) eigenfrequency against non-Hermiticity parameter γ (which is related to sensor's impedance) for the PT-symmetric wireless sensing system in Fig. 2. (b) Examples of higher-order PT-symmetric wireless sensing system, consisting of multiple tightly-coupled active/passive RLC tank circuits.

as the broken PT-symmetric phase, where eigenfrequencies become two complex conjugate pairs ($\gamma \in \mathbb{C}$), as shown in Fig. 3(a). In such region, $\mathcal{PT}\Phi_k \neq \Phi_k$ and thus the PT-symmetry is broken. A second crossing between the pairs of degenerate frequencies (and another branching) occurs at the lower critical point:

$$\gamma_c = \frac{1}{\kappa}\sqrt{\frac{1-\sqrt{1-\kappa^2}}{2}}. \quad (5)$$

In the sub-critical region $\gamma \in \left[0, \gamma_c\right]$, ω_k become purely imaginary and therefore the modes have no oscillatory part and simply blow up or decay away exponentially, which is of little interest for telemetry sensing applications. We note that nearby the singular exceptional point, eigenfrequencies experience the exotic bifurcation effect, resulting a dramatic resonance frequency shift with respect to the perturbation of sensor's impedance (or effectively γ). For instance, in a PT-symmetric wireless capacitive sensor, where $\gamma \propto C^{-1/2}$ is dynamically tuned, one may achieve a large drift of resonant frequency in response to capacitive changes beyond the inverse-square law.

The EP-related threshold for γ can be further reduced by introducing the concept of higher-order PT-symmetry, inspired by the Bose–Hubbard model, describing the interacting bosons in a quantum system. The higher-order PT-symmetric RF circuits and wireless sensors may be particularly useful for enhancing the Q-factor and sensitivity for miniature implants and wearables, which typically possess large resistance and low modal Q-factor. Figure 3(b) shows examples of the higher-order PT-symmetric system formed by N coupled resonators, following a recursive bosonic quantization law; here the parameter γ represents the non-Hermiticity parameter and κ is the hopping constant between the two sites. Assuming an electronic molecule comprised of N resonators, eigenmodes of such circuits obey the evolution equation $d\Psi / d\tau = -j\mathbf{H}_{eff}\Psi$, where Ψ is the modal column vector of dimensionality N and $\mathbf{H}_{eff}$ is an $N \times N$ non-Hermitian matrix that plays the role of an effective Hamiltonian. For simple non-interacting two-site Bose-Hubbard Hamiltonian, we will analytically calculate the optimal design parameters, such as γ and κ, using the classical analogs to quantum coherent and displaced Fock states. Figure 3(a) also presents the eigenfrequencies against γ for non-interacting PT-symmetric Bose-Hubbard Hamiltonians. It is clearly seen that in the higher-order PT systems (N > 2), the exceptional points can have downshifted to lower γ region, when the coupling strength $\kappa = 1/\sqrt{(N+1)/2}$. As a result, even a passive sensor with low intrinsic Q-factor, corresponding to a low γ, can operate in the exact symmetry phase, where real eigenfrequencies render sharp and narrowband resonances,

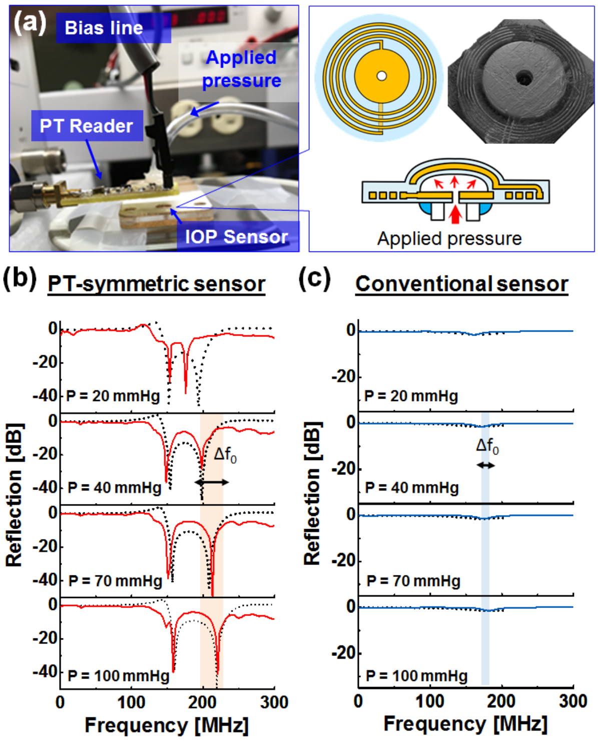

Figure 4. (a) Measurement setup for the PT-symmetric wireless pressure sensor. Measured reflection spectra for the (b) PT-symmetric and (b) conventional wireless sensing scheme, under different applied pressures.

as well as highly sensitive frequency responses. Moreover, the effect of singularity-enhanced splitting of eigenfrequencies can be more dramatic in the higher-order PT systems. As can be seen in Fig. 3(a), eigenfrequencies of higher-order PT-symmetric system become ultrasensitive to sensor's impedance disturbances around the exceptional point.

3. Experimental Demonstration with the PT-Symmetric Wireless Pressure Sensor

We have designed and microfabricated a MEMS-based wireless intraocular pressure sensor [Fig. 4(a)], consisting of a variable parallel-plate capacitor (varactor tuned by pressure), connected in series to a planar microcoil. There is also an effective resistance accounting for total power dissipations. Such passive micromachined pressure sensor is equivalent to an RLC tank. When an active reader based on an –RLC tank circuit is used to wirelessly interrogate the passive sensor (RLC tank), the PT-symmetric telemetry circuit can be realized, as discussed in the previous section. These two amplifying and attenuating resonators may interestingly achieve the balanced gain and loss in the PT system. Figure 4(a) shows the wireless MEMS-based pressure sensor that is magnetically coupled to the active reader comprising a planar microstrip inductor, varactors, and a negative resistance converter based on the Colpitts-type oscillator [7].

Figures 4(a) and 4(b) present evolutions of reflection spectrum for the MEMS-based intraocular pressure (IOP) sensor in Fig. 4(a), which is interrogated by the active PT reader and the conventional loop antenna, respectively. To simulate the effect of IOP variations in human eyes, the MEMS-based sensor was connected to an air compressor, with the internal pressure (20 mmHg - 100 mmHg) precisely controlled by a regulator. Here, the effective resistance $R = 150\ \Omega$ and the inductance of the microcoil $L = 0.3\ \mu\text{H}$ were extracted from the measurement results and are confirmed by the full-wave simulation. The coupling strength was fixed, $\kappa = 0.5$, and the dimensionless non-Hermiticity $\gamma = R^{-1}\sqrt{L/C}$ was varied by the MEMS varactor C (which is a function of applied pressure). These design parameters render the system operating in the exact PT-symmetric phase, with γ barely above the threshold γ_{EP}, so as to take advantages of the singularity-induced ultrahigh sensitivity. It is seen from Fig. 4(b) that the PT-symmetric telemetry system can have two sharp, high-Q reflection dips associated with a pair of real eigenfrequncies, as can be seen in Fig. 3(a). On the other hand, it is seen from Fig. 4(c) that the conventional passive loop-antenna reader ($L = 0.3\ \mu\text{H}$) shows a broad resonance linewidth, implying low sensing resolution and low modulation depth. Here, we find a good agreement between measurements results (solid lines) and full-wave simulations (dashed lines) in Figs. 4(b) and 4(c). We should note here that around the exceptional point of PT system, the bifurcation of eigenfrequencies occurs [Fig. 3(a)], leading to dramatic resonance frequency (eigenfrequency) shift. On the contrary, the conventional coil-antenna reader shows less observable shift in resonance frequency. Overall, it is evident that a wireless sensor system engineered into the PT-symmetric dimer can provide superior sensitivity and detectivity when compared with conventional passive readers because it achieves not only higher spectral resolution and greater modulation depth, but also more sensitive frequency responses.

5. Conclusions

We have proposed a new wireless sensing platform based on the concept of PT-symmetry for enhancing the Q-factor, sensing resolution, and sensitivity of wireless sensors based upon the passive LC oscillator. We have theoretically and experimentally demonstrated that when the equivalent circuit of the wireless sensing system is tailored to a particular PT-symmetric topology, sharp and deep reflection dips can be obtained, which are very sensitive to perturbation of sensor's impedance. Our technique may overcome the long-standing open challenge of implementing a miniature micro- or nano-sensor with high Q-factor and high sensitivity, and will open up new exciting avenues for loss-immune, high-performance sensors, thanks to contactless gain-loss interactions and eigenfrequency bifurcations resulting from the singularity in PT-symmetric systems.

6. References

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