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Wideband Electromagnetic Scattering Computations for Smooth Conducting 2D Cylinders Using the RAS-AWE Method

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Abstract

This paper investigates the potential of using the Random Auxiliary Sources (*RAS*) method with the Asymptotic Waveform Evaluation (*AWE*) approach to compute a wideband response of the electromagnetic scattering from smooth conducting 2D structures. The effects of the various parameters of the solution are studied in a statistical approach. Also, the results of the proposed method are presented for different geometries in comparison to the original RAS method and the Method of Moments (*MoM*).

1 Introduction

Accelerating the computation of the electromagnetic (*EM*) scattering of arbitrary objects is an ongoing research process carried out by several research groups [1]. Large open boundary problems are the most attracting problems to further accelerate their computations [1, 2]. The MoM can be considered among the methods that provide fast solutions for such problems due to the need to segment the surfaces of the problem instead of the entire volume [3]. Besides, incorporating the AWE with MoM enables the procedure to compute a wide band response for the MoM solution using only one matrix inversion, which is a significant acceleration technique [1, 4].

The Asymptotic Waveform Evaluation (*AWE*) [5] is typically a method to evaluate the response at different points close to the original solution. This method has been first used in conjunction with the MoM in [4] for EM problems. Since then, this method has been used for several EM applications [1]. Recently, several advancements to the AWE approach has been reported in [6] employing different expansion functions to widen the achievable bandwidth.

In addition, the RAS method, introduced in [7, 8, 9], provides fast solutions to the EM scattering problems of arbitrary objects in comparison to the MoM. The RAS method is based on the surface equivalence principle. The equivalent problems are based on using randomly distributed sources within the object boundary. The sources produce fields that satisfy the original boundary conditions. The main reason behind the accelerated response of the RAS method is the need for a smaller number of unknowns compared to MoM to provide the unique solution for problems. Several studies about this method have been presented in [8] for the 2D problems as well as [9] for the 3D problems. However, the potential of using the AWE in conjunction with the RAS method has not been presented yet, which is the main purpose of this work.

This work presents a study of the potential of using the AWE concept the RAS method to accelerate the wideband frequency computations for the EM scattering of 2D smooth conducting structures under uniform plane wave illumination at oblique incidence. The formulation of this approach is presented. Also, the effects of different parameters on the performance of the AWE results are reported. Nevertheless, the proposed method is verified with the original RAS method and the MoM using several evaluations over a frequency band to bench mark the performance. Therefore, the main contribution of this work can be listed as follows:

- Introducing the RAS-AWE formulation.
- Studying the effect of the RAS-AWE parameters on the achievable bandwidth in a statistical approach.
- Verifying the proposed method against the original RAS method and the MoM for different geometries.

2 Formulation

The problem of interest can be presented in Fig. 1 showing an arbitrary 2D scatterer illuminated by an obliquely incident uniform plane wave with angles α^i, θ^i, and ϕ^i to include all the possible incidence situations of such scheme as defined in [10], where α^i is the polarization angle of the electric field incident with the plane of incident, θ^i and φ^i are the elevation and the azimuthal angle of the incident plane wave, respectively.

The procedure for the RAS method requires introducing randomly distributed infinitesimal sources with unknown moments within the scattererâĂŹs boundary as sources for the scattered fields in the equivalent exterior problem [8]. Applying the boundary conditions on the scatterer's boundary of the total tangential field results in an overdetermined matrix equation, which is solved by the complex least square method [11] forming the standard system of

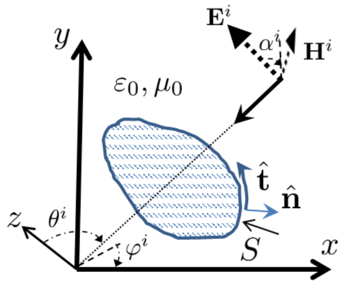


Figure 1. The 2D EM scattering problem illustration with arbitrary plane wave excitation with oblique incidence (2.5D).

linear equations, which is eligible for the AWE implementation. Thus, the RAS matrix squared equation can be written in the form

$$\mathbf{Z}^{*T}\mathbf{Z}.\mathbf{I} = \mathbf{Z}^{*T}\mathbf{V}, \tag{1}$$

where $\mathbf{Z}$ is the overdetermined matrix equation, $\mathbf{I}$ is the unknown moments of the randomly distributed infinitesimal sources, $\mathbf{V}$ is the excitation vector, ^{*T} denotes the matrix conjugation and transposition. The AWE process starts by expanding the unknown currents to a Taylor series as a function of the wavenumber $k = \omega\sqrt{\mu_0\varepsilon_0}$ viz

$$\mathbf{I}(k) = \sum_{n=0}^{Q} m_n(k-k_0)^n, \tag{2}$$

with k_0 is the wavenumber at the solution frequency and

$$m_n = \left[\mathbf{Z}^{*T}\mathbf{Z}\right]^{(-1)}(\mathbf{A}-\mathbf{B}), \tag{3}$$

with

$$\mathbf{A} = \sum_{i=0}^{n} \frac{\left[\mathbf{Z}^{(n-i)}\right]^{*T}\mathbf{V}^{(i)}}{(n-i)!i!}$$
$$\mathbf{B} = \sum_{k=1}^{n}\sum_{i=0}^{k} \frac{\left[\mathbf{Z}^{(k-i)}\right]^{*T}[\mathbf{Z}]^{(k)}m_{n-k}}{(k-i)!i!} \tag{4}$$

where the $^{(i)}$ is the i^{th} derivatives of the matrices. Since the radius of convergence of the Taylor series is known to be of limited bandwidth [1, 4], the unknown current expansion is then fitted to the Padé rational polynomial function to improve its bandwidth as used in [1, 4] as

$$\mathbf{I}(k) = \frac{\sum_{i=0}^{L} a_i(k-k_0)^i}{1+\sum_{j=1}^{M} b_j(k-k_0)^j};\ L+M=Q. \tag{5}$$

3 Effects of the Different Parameters

Since the RAS method uses random infinitesimal sources in each program run, the sources distributions are not unique despite having a unique EM solution. Therefore, the effect of this randomness has to be taken into account to study the effect of the different parameters in a statistical approach.

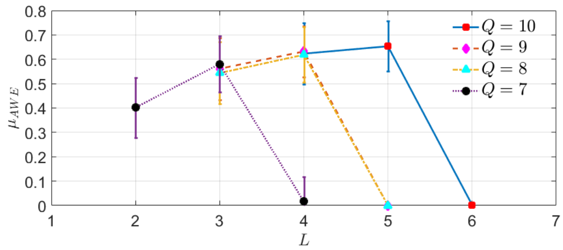


Figure 2. The effect of changing L on the average achievable bandwidth. The results are computed for a $1\lambda_0$ PEC cylinder. The error bars indicate the standard deviation.

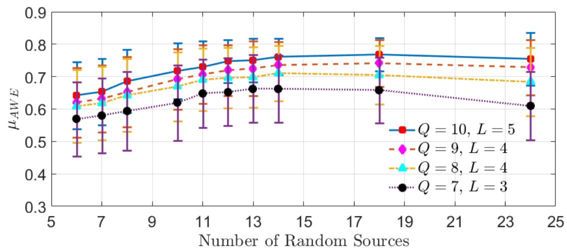


Figure 3. The effect of changing the number of unknowns on the average achievable bandwidth. The results are computed for a $1\lambda_0$ PEC cylinder.

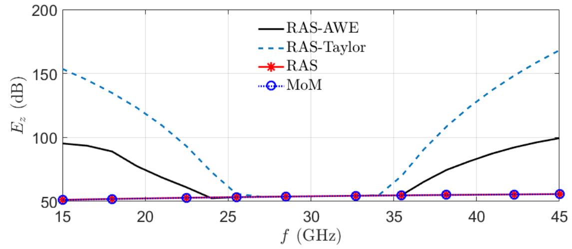
Furthermore, the performance results presented in this section are generated through a 1000 time run for each case to be able to observe the average of the obtained bandwidth (μ_{AWE}).

The effect of changing the order of numerator over the achievable bandwidth is observed in Fig. 2 indicating that optimum value of L is floor($Q/2$). In addition, the effect of changing the number of unknowns on the achievable bandwidth is shown in Fig. 3. The presented curve indicates that the optimum choice for the number of unknowns in almost double the recommendations in [8] to better operate for the RAS-AWE requirements.

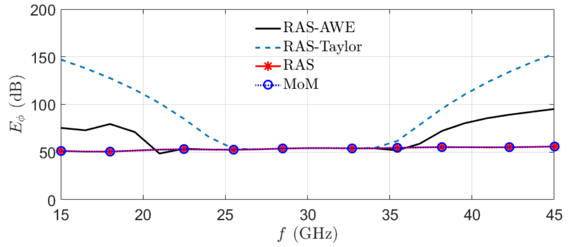
4 Results and Discussions

The first verification case is for a $2\lambda_0$ diameter PEC scattering cylinder illuminated by a uniform plane wave with angles $\alpha^i = 45°$, $\theta^i = 45°$, and $\phi^i = 45°$. The wideband response of the monostatic scattered fields is presented in Fig. 4 using both the AWE and Taylor expansions. The results show the bandwidth advantage obtained from a single solution of the RAS-AWE over the RAS-Taylor method. It is worth mentioning that the same normalization factor is used in the scattered fields as [8, 10]. Moreover, the results are compared to the accurate evaluations of the RAS method and the MoM. The performance benchmark of this test case is presented in Table 1 showing the superior performance of the proposed method over the RAS and MoM.

The other verification case is for a PMC $1\lambda_0$ diameter super quadratic cylinder that satisfies the equation $((x/0.5)^5 +$



(a)



(b)

Figure 4. The monostatic normalized scattered electric field of the EM scattering problem of a $2\lambda_0$ diameter PEC cylinder using $Q = 10$. (a) The z component. (b) The ϕ component.

Table 1. The performance benchmark of the EM scattering problem of a $2\lambda_0$ diameter PEC cylinder.

Number of Frequency Points	21
RAS-AWE Time	3.9 s
RAS Time	6.739 s
MoM Time	99.702 s

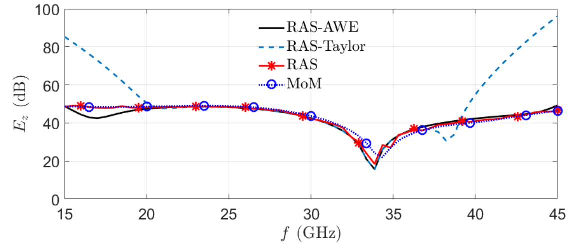
Table 2. The performance benchmark of the EM scattering problem of a $1\lambda_0$ diameter PMC superquadratic cylinder.

Number of Frequency Points	61
RAS-AWE Time	3 s
RAS Time	13.6 s
MoM Time	46.9 s

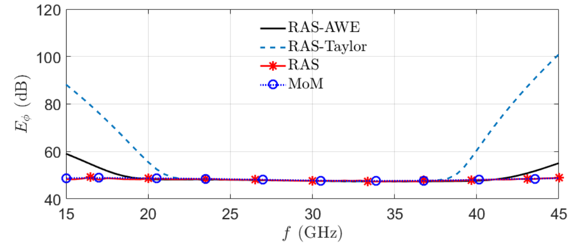
$(y/0.5)^5 = 1)$ illuminated by the same uniform plane wave as the previous case. The wideband monostatic scattered fields are presented in Fig. 5 in comparison to the RAS and MoM. The RAS-AWE shows a noticeable agreement with the exact RAS and MoM solution for a much wider frequency band than the previous case. Also, the predominance benchmark is reported in Table 2. It should be mentioned that the achievable bandwidth reduces as the structure size increases for the same Taylor series order, which explains the difference in the obtained bandwidth between the presented test cases.

5 Conclusions

The asymptotic waveform evaluation has been implemented on the 2D RAS method. The potential of using the proposed method for the different settings has been studied in a statistical approach. In addition, several verification cases have been presented in comparison to the original RAS method and the MoM promoting the significant acceleration in computation time of the proposed method over its predecessors.



(a)



(b)

Figure 5. The monostatic normalized scattered electric field of the EM scattering problem of the PMC $1\lambda_0$ diameter superquadratic cylinder using $Q = 10$. (a) The z component. (b) The ϕ component.

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