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Power Allocation Games for Overlaid Radar and Communications

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Abstract

Electromagnetic spectrum is a scarce resource that is becoming increasingly contested and congested with the allocation of available spectrum to newer communications technologies. Several radio-frequency operators especially radars are now required to share their spectrum with communications. In this work, we investigate the spectrum sharing between a radar and a communications system from a game-theoretic perspective. Since both systems compete for the same resource, their interaction can be modeled as a two-person zero-sum game. Specifically, we analyze transmit power allocation in a symmetric game where a player is unaware of the strategies adopted by the competitor. Our power allocation optimization constraints include preset signal-to-interference-plus-clutter-plus-noise ratio and maximum interference tolerance.

1 Introduction

In recent years, sensing systems that share the spectrum with wireless communications and yet operate without any significant performance losses have captured significant research interest [1, 2, 3]. The interest in such spectrum sharing systems is driven by an increasing contention over the demand for the radio-frequency (RF) spectrum by both radar and communications systems. Since RF spectrum is a finite resource, it is essential to find novel solutions so that the two systems can cohabitate the spectrum.

Several important research efforts are currently underway for efficient radio spectrum utilization [4]. The National Science Foundation (NSF) has sponsored the Enhancing Access to the Radio Spectrum (EARS) project that is studying coexistence of various stakeholders for a flexible access to the licensed and unlicensed spectrum [5]. The Defense Advanced Research Projects Agency (DARPA) Shared Spectrum Access for Radar and Communications (SSPARC) program is focused on designing S-band military radars for spectrum sharing with military communications [6].

At present, two major paradigms have emerged to enable spectrum sharing: *spectral coexistence* [7, 2, 8, 9] and *spectral co-design* [10]. The former focuses on devising strategies to mitigate the interference adaptively for either communications or radar under the assumption that the two systems coexist as separate systems. The latter approach requires development of new joint remote sensing and communications where a single unit is employed for both functions while also utilizing the opportunistic access to the spectrum. New software-defined systems integrate these various systems to minimize circuitry and maximize flexibility. In this paper, we focus on spectral coexistence.

Several novel approaches have been considered for spectral coexistence (see, e.g. [11, 12] for an overview). Typically, these approaches allow *spectral cooperation*, i.e., some information exchange between the two systems, but with minimal change in the system hardware and processing. The radar-centric architectures usually assume fixed interference levels from communications and design the system for required performance level in a radar task, for example high probability of detection under a false alarm constraint [13]. Similarly, the communications-centric systems improve performance metrics like the error vector magnitude and bit/symbol error rate for interference from radar by ensuring sufficient signal-to-interference-and-noise ratio [7]. The receiver processing techniques in spectral coexistence range from conventional notch filters at hostile frequencies and beamformers steering nulls towards interfering transmitters to designing transmit waveforms that avoid resources used by the other coexisting systems. Later design solutions use convex optimization of radar performance metrics for given spectral constraints. The objective functions in such (convex and nonconvex) optimization problems consider signal-to-noise-ratio (SNR) [14], transmit energy in stopband [13], sidelobe levels [15], information theoretic metrics [16, 17], cognition of interference from the other system [18, 19], and many spectral constraints [14].

Recently, there is some interest in analyzing the spectral coexistence problem from a game theory perspective [20, 21]. Game theory mathematically studies decision making in an environment of conflict and cooperation between rational players, i.e., players who selfishly try to maximize their interests. The spectral coexistence problem fits well into a typical game theory formulation because the two players - radar and communications - compete for finite resources and have an adversarial relationship. Their interaction can, therefore, be modeled as a non-cooperative, two-person zero-sum (TPZS) game.

While application of game theory to communications has a rich history (see, e.g. [22] and references therein), its use in radar problems is relatively recent. A significant advance was due to [23] which analyzed the interaction of a jammer with a multiple-input-multiple-output (MIMO) radar as a TPZS game. Similar games were later formulated to find the most optimal constant-false-alarm-rate detector [24], transmit scheme in a polarimetric MIMO radar [25], code design in radar networks [26], frequency-hopping [27], and target tracking [28, 29].

In the specific context of spectral coexistence, recent studies such as [21, 30, 31] consider game-theoretic power allocation strategies for a distributed multiple-radar environment. In this paper, contrary to these prior works, we focus on games exclusively involving a radar and a communications unit. Our goal is to formulate the coexistence problem as a game which attempts to find optimal transmit power for both systems in the presence of interference from each other. We now introduce the system models.

2 System Models

Consider a common observation time, in which the discrete-time samples of radar received data are denoted by $\mathbf{x}_R[n]$, and that of the communications system be $\mathbf{x}_C[n]$. The total data received from both systems are represented by a concatenated vector $\mathbf{x} = [\mathbf{x}_R; \mathbf{x}_C]$. This assumes ideally that both can be observed simultaneously and instantly, i.e. in near real-time relay/feedback without additional noise.

2.1 Radar

The operational objective of a radar is to maximize the probability of detection of a target. This is fundamentally a binary hypotheses testing problem. Let $\mathscr{H}_0$ represent the hypothesis that no target is present, and $\mathscr{H}_1$ the hypothesis that a target is present; and let $p_{\mathbf{x}|\mathscr{H}_k}$, $k = 0, 1$ be the data distribution under each hypothesis. Given a data measurement $\mathbf{x}$, the goal of binary hypothesis testing is to find a decision rule $\phi(x)$ that maps every measurement to a decision declaring one of the hypotheses to be true. Usually $\phi(\cdot)$ has a value of zero or unity, i.e. it is an indicator function, where unity indicates a decision for hypothesis $\mathscr{H}_1$.

The Neyman-Pearson (NP) criterion is well-suited for the radar problem as it seeks to maximize the probability of target detection (P_d) while constraining the probability of false alarm (P_{fa}). NP suggests choosing $\phi(\cdot)$ to maximize the Lagrangian objective function

$$\begin{aligned}\mathscr{P}_{NP} &= P_d + \lambda(\alpha - P_{fa}) \\ &= \int \phi(\mathbf{x}) p_{\mathbf{x}|\mathscr{H}_1} d\mathbf{x} + \lambda \left(\alpha - \int \phi(\mathbf{x}) p_{\mathbf{x}|\mathscr{H}_0} d\mathbf{x} \right). \quad (1)\end{aligned}$$

where $\lambda, \alpha < 1$.

By inspection, the optimal NP decision rule is

$$\phi_{NP}(\mathbf{x}, \lambda) = \begin{cases} 1, & p_{\mathbf{x}|\mathscr{H}_1} > \lambda p_{\mathbf{x}|\mathscr{H}_0} \\ 0, & p_{\mathbf{x}|\mathscr{H}_1} < \lambda p_{\mathbf{x}|\mathscr{H}_0}. \end{cases} \quad (2)$$

When both density functions are continuous and $p_{\mathbf{x}|\mathscr{H}_0}$ nonzero, then decision rule can be summarized as the following statistical test compared to threshold:

$$T(\mathbf{x}) \triangleq \frac{p_{\mathbf{x}|\mathscr{H}_1}}{p_{\mathbf{x}|\mathscr{H}_0}} \underset{\mathscr{H}_1}{\overset{\mathscr{H}_0}{\lesseqgtr}} \lambda \quad (3)$$

that is known as the likelihood ratio test (LRT). Under the NP criterion, no other decision rule can do better.

2.2 Communications

The goal of a comm system is data transfer from one point in space to another at the highest rate with low errors. Let the transmitted data be $\mathbf{s}$ and the received data be $\mathbf{x}$. Mutual information, which quantifies the bits of information conveyed between a source and its measurement, is defined as

$$I(\mathbf{s}; \mathbf{x}) = E_{\mathbf{x},\mathbf{s}} \left\{ \log \frac{p_{\mathbf{x}|\mathbf{s}}}{p_{\mathbf{x}}} \right\} = \int \int p_{\mathbf{x},\mathbf{s}} \log \frac{p_{\mathbf{x}|\mathbf{s}}}{p_{\mathbf{x}}} d\mathbf{x} d\mathbf{s}. \quad (4)$$

The maximal rate of information is obtained from the maximum achievable value of mutual information; i.e.,

$$C = \max_{p_{\mathbf{s}} \in \mathscr{S}} I(\mathbf{s}; \mathbf{x}), \quad (5)$$

the channel capacity, where the maximization is over the channel excitation distribution $p_{\mathbf{s}}$ constrained to a desired set $\mathscr{S}$. With proper coding the maximal data rate achievable such that arbitrarily low decoding error is possible is given by C above.

3 Power Allocation Game

Let the radar and communications transmit power be P_R and P_C, respectively. We define the complex-Gaussian-distributed gains for the various discrete-time channel impulse responses with zero mean and variances σ_t^2, σ_i^2, σ_c^2, σ_f^2, and σ_r^2, respectively, as follows:

$h_T \sim \mathscr{CN}(0, \sigma_t^2)$ for radar transmitter to the target and back to the radar receiver

$h_I \sim \mathscr{CN}(0, \sigma_i^2)$ for radar transmitter to clutter and back to the radar receiver

$h_C \sim \mathscr{CN}(0, \sigma_c^2)$ for radar transmitter to the

communications receiver
$h_F \sim \mathcal{CN}(0, \sigma_f^2)$ for communications transmitter to the communications receiver
$h_R \sim \mathcal{CN}(0, \sigma_r^2)$ for radar transmitter to the target and clutter and then to the communications receiver. The discrete-time transmit signals for the radar and communications are $x_T[n]$ and $s[n]$, respectively.

Given a specific range-cell under test in a radar system, we have the two hypotheses as

$$\mathcal{H}_0 : x_R[n] = h_I[n]\sqrt{P_R}x_R[n] + h_C[n]\sqrt{P_C}x_C[n] + w[n] \quad (6)$$
$$\mathcal{H}_1 : x_R[n] = h_T[n]\sqrt{P_R}x_R[n] + h_I[n]\sqrt{P_R}x_R[n] + h_C[n]\sqrt{P_C}x_C[n] + w[n],$$

where $n = 0, \cdots, N-1$ and $w[n] \sim \mathcal{CN}(0, \sigma_w^2)$ is the noise term. Here, the signal-to-interference-plus-clutter-plus-noise-ratio (SICNR) is

$$\mathrm{SICNR}_R = \frac{\sigma_t^2 P_R}{\sigma_c^2 P_C + \sigma_i^2 P_R + \sigma_w^2} \quad (7)$$

The radar may also have a maximum power, maximum interference and minimum SICNR constraints so that

$$0 \le P_R \le P_{R,\max}, \quad (8)$$
$$\sigma_c^2 P_C \le T_{C,\max}, \quad (9)$$
$$\text{and } \mathrm{SICNR}_R \ge \mathrm{SICNR}_{R,\min}, \quad (10)$$

where $P_{R,\max}$, $T_{C,\max}$, and $\mathrm{SICNR}_{R,\min}$ are pre-defined constants.

The signal at the communications receiver is

$$s[n] = h_F[n]\sqrt{P_C}x_C[n] + h_R[n]\sqrt{P_R}x_R[n] + v[n], \quad (11)$$

where $n = 0, \cdots, M-1$ and $v[n] \sim \mathcal{CN}(0, \sigma_v^2)$ is the noise term. The signal-to-interference-plus-noise-ratio (SINR) is

$$\mathrm{SINR}_C = \frac{\sigma_f^2 P_C}{\sigma_r^2 P_R + \sigma_v^2} \quad (12)$$

The communications receiver may have the maximum power, maximum interference and minimum SINR constraints as

$$0 \le P_C \le P_{C,\max}, \quad (13)$$
$$\sigma_r^2 P_R \le T_{R,\max}, \quad (14)$$
$$\text{and } \mathrm{SINR}_C \ge \mathrm{SINR}_{C,\min}, \quad (15)$$

where $P_{C,\max}$, $T_{R,\max}$, and $\mathrm{SINR}_{C,\min}$ are pre-defined constants.

Let the game be the triplet $\mathcal{G} = \langle \mathcal{K}, \mathcal{S}, \mathcal{U} \rangle$ where $\mathcal{K}$ is the set of players with cardinality $|\mathcal{K}| = K$, $\mathcal{S} = S_1 \times \cdots \times S_K$ is the space comprising of strategies $\{S_i\}_{i=1}^K$ of all players, and $\mathcal{U} = \{u_1, \cdots, u_K\}$ is the set of utility functions of each player which map their strategies to a real line, i.e., $u_i : S_i \to \mathbb{R}$, $i = 1, \cdots, K$. In our spectral coexistence problem, $|\mathcal{K}| = 2$ and $K = 1$ and 2 corresponds to radar and communications, respectively. Further $S_1 = [0, P_{R,\max}]$ and $S_2 = [0, P_{C,\max}]$.

The utility functions are given by the difference of payoff (maximization of SICNR) and the cost functions (minimization of power). Therefore,

$$u_1 = \ln(\mathrm{SICNR}_R - \mathrm{SICNR}_{R,\min}) - (\mu_1 \sigma_t^2 P_R + \gamma_1 \sigma_r^2 P_R) \quad (16)$$

$$\text{and } u_2 = \ln(\mathrm{SINR}_C - \mathrm{SINR}_{C,\min}) - (\mu_2 \sigma_f^2 P_C + \gamma_2 \sigma_c^2 P_C), \quad (17)$$

where μ_i and γ_i, $i = 1, 2$ are to be determined. The power allocation can be determined as a non-cooperative game which requires solving for the following:

$$\underset{P_R}{\text{maximize}} \; u_1$$
$$\text{subject to } 0 \le P_R \le P_{R,\max}, \quad (18)$$
$$\sigma_r^2 P_R \le T_{R,\max}, \quad (19)$$
$$\mathrm{SICNR}_R \ge \mathrm{SICNR}_{R,\min}, \quad (20)$$

and

$$\underset{P_C}{\text{maximize}} \; u_2$$
$$\text{subject to } 0 \le P_C \le P_{C,\max}, \quad (21)$$
$$\sigma_c^2 P_C \le T_{C,\max}, \quad (22)$$
$$\mathrm{SINR}_C \ge \mathrm{SINR}_{C,\min}. \quad (23)$$

The goal is to find the Nash equilibrium solution $\mathbf{P_i}^* = (P_i^*, P_j^*)$, where $i \ne j$ take values as 'R' and 'C' such that

$$u(P_i^*, P_j^*) \ge u(P_i, P_j^*), \text{ for all } i, j. \quad (24)$$

In this talk, we would present numerical simulations and analysis toward achieving Nash equilibrium. Note that we did not yet consider the mutual information or rate as a metric in the communications system. In future, this could be useful criterion to relax the problem to meet the equilibrium conditions.

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