URSI AP-RASC 2019, New Delhi, India; 09 - 15 March 2019

Landau damping of dust acoustic solitary waves in a superthermal dusty plasma

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Abstract

We have studied the Landau damping of dust acoustic solitary waves in a dusty plasma containing kappa-distributed, two-temperature superthermal ions. Geotail satellite observations have reported the presence of a hot and a relatively colder ion population in the magnetotail region of Earth. Moreover, negative charging of dust particles by the collection of background electrons gives rise to an electron depleted plasma. Our aim is to scrutinize the wave-particle interaction phenomenon and to highlight the effect of Landau damping due to cold, heavier, ions on the characteristics of dust acoustic waves in the specific region of Earth's magnetotail. For this investigation, we have used the kappa distribution to fit the velocity distribution of both the superthermal ion species. Our results clearly indicate that the superthemal character of the charged particles has a noteworthy influence on the Landau damping phenomenon and the nonlinear structures so formed. This investigation may help throw some light on the observed events of charged particle acceleration and extensive wave activity in specific regions of Earth's magnetotail.

1 Introduction

Natural occurrence of dusty plasmas in the solar system (Saturn's B-rings, dust streams from Jupiter etc.), Earth's atmosphere (ionosphere and magnetosphere) and cosmic environments (cometary tails, asteroids zones, interstellar medium etc.) have led plasma physicists to investigate the charged dust generated wave modes for a deep understanding of the basic physical processes occurring in space and astrophysical environments [1]. Predicted for the first time by Rao et al. [2], the dust acoustic waves are a form of sound waves propagating through the dust component of the plasma in which the mass of the charged dust provides inertia, whereas the restoring force is provided by the background electrons and ions. The negative charging of dust due to the accumulation of background electrons by neutral dust surface leads to an electron depleted dusty plasma.

The resonant wave-particle interactions in plasma give rise to the Landau damping phenomenon. Predicted first time for the Langmuir waves, Landau damping has since been recognized in different wave modes in plasma. Ott and Sudan [3] proposed a nonlinear theory of Landau damping in which they derived the famous Korteweg-de Vries (KdV) equation with an additional term indicating the Landau damping effects on the ion-acoustic waves propagating in plasma. Later on, different theoretical and experimental studies pertaining to nonlinear Landau damping in different plasma systems have been reported [4],[5]. Recently, the effects of Landau damping have been analyzed on low-frequency dust acoustic waves in the presence of dust charge fluctuations in a dusty plasma and it is observed that the variation in dust charge enhances the damping rate of the dust acoustic waves (DAWs) [6]. This study was also performed while considering Maxwellian velocity distribution of the background particles.

Due to the low density and high temperature of different space and astrophysical environments, the plasma particles obey certain non-Maxwellian distributions such as kappa distribution, instead of following the Maxwellian distribution. One such high-temperature ion population has been reported by the Geotail satellite in the magnetotail region of Earth. Two temperature ions having corresponding peaks at 7 keV and 0.7 keV indicate a hot ion population comprised of H^+ ions and a relatively colder ion population of heavy O^+ ions [7]. A kappa-type velocity distribution is characterized by a Maxwellian core and high energy tails indicating superthermal particles [9]. A number of researchers have studied the effect of superthermal kappa distribution on the characteristics of dust acoustic nonlinear structures. However, no investigation has been performed for the study of Landau damping of dust acoustic solitary waves in an electron depleted dusty plasma containing kappa-distributed ions.

The motivation of present investigation is to study the influence of Landau damping on the dust acoustic solitary structures in Earth's magnetotail region while taking into account the observations of Geotail spacecraft. Our aim is to emphasize the influence of superthermal character of charged particles on the resonance of dust acoustic waves with the heavy oxygen ions that are present in the plasma environment under consideration. The findings of the present study may be useful for a better understanding of wave-particle interaction phenomenon the magnetotail region of Earth.

The manuscript is organized as follows: Section 2 depicts the model equations for charged particles of different species considered in the present investigation and a derivation of KdV equation modified by an additional Landau damping term is presented. Sections 3 and 4 present respectively the results and concluding remarks of the present study.

2 Derivation of Landau damped KdV equation

In order to study the Landau damping phenomenon of dust acoustic waves in an electron-depleted dusty plasma containing negatively charged dust fluid and two-temperature kappa-distributed ions species, we consider a hybrid set of model equations. The dynamics of negatively charged inertial dust are governed by the fluid model equations and those of cold and hot ions are governed by the kinetic Vlasov equations. As the colder O^+ ions are considered more massive as compared to the hot H^+ ions, we shall investigate Landau damping of DAWs due to cold heavier ions. The condition of quasi-neutrality at equilibrium leads to the condition given by $n_{c0}+n_{h0}=Z_d n_0$ which yields $\mu+\nu=1$, where, $\mu=\frac{n_{c0}}{Z_d n_0}$, $\nu=\frac{n_{h0}}{Z_d n_0}$, Z_d is the charge and n_{c0}, n_{h0} and n_0 are the number densities of the cold ions, hot ions and dust particles respectively. The normalized fluid equations for the motion of the inertial dust particles are as follows:

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0, \qquad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\frac{\partial \phi}{\partial x}$$

$$\frac{{\lambda_d}^2}{L^2}\frac{\partial^2 \phi}{\partial x^2}=-\mu n_c-\nu n_h+n. \tag{1}$$

The normalized kinetic Vlasov equation for ion species and the expression for their number densities are given as:

$$\delta\frac{\partial f_s}{\partial t}+v\frac{\partial f_s}{\partial x}-\frac{m_c}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_s}{\partial v}=0, \qquad n_s=\sqrt{\frac{T_c m_s}{T_s m_c}}\int_{-\infty}^{\infty} f_s dv. \tag{2}$$

where $s=c,h$ for cold and hot ions respectively, and

- $\delta=\sqrt{\frac{Z_d m_c}{m_d}}$ represents the inertial effects due to cold ions and the Landau damping by heavier ion species.
- $\frac{n_1}{n_0}$ depicts the strength of nonlinearity in electrostatic disturbances for the given system.
- $\frac{{\lambda_D}^2}{L^2}$ depicts the measure of the wave dispersion arising due to the deviation from the charge quasi-neutrality. Here, L represents the characteristic scale length for variations in the different physical quantities, namely n,u,ϕ etc.

We have normalized the number densities of dust and the two temperature ion species (n,n_c,n_h) respectively by their unperturbed number densities (n_0,n_{c0},n_{h0}). Space and time variables are normalized by the dust Debye length, i.e., $\lambda_{D,d}=\left(\frac{K_B T_c}{4\pi n_0 Z_d e^2}\right)^{1/2}$ and inverse dust plasma frequency, i.e., $\omega_{p,d}^{-1}=(\frac{m_d}{4\pi n_0 {Z_d}^2 e^2})^{1/2}$ respectively. The speed and electrostatic potential are normalized respectively by the dust acoustic speed $C_d=\left(\frac{Z_d K_B T_c}{m_d}\right)^{1/2}$ and $\phi_0=\frac{K_B T_c}{e}$, where T_c is the colder ion species temperature, m_d and Z_d are the mass and charge of the inertial dust particles respectively. Our main interest is to study the interplay between the nonlinear, the dispersive and the Landau damping effects, so, we consider [3, 5]

$$\delta=\alpha_1\varepsilon, \qquad \frac{n_1}{n_0}=\alpha_2\varepsilon, \qquad \frac{\lambda_D^2}{L^2}=\alpha_3\varepsilon. \tag{3}$$

where α_j $(j=1,2,3)$ is a constant of the order unity and ε is a smallness parameter. From Eq. (2), the equations for the two ion species become

$$\alpha_1\varepsilon\frac{\partial f_s}{\partial t}+v\frac{\partial f_s}{\partial x}-m\frac{\partial \phi}{\partial x}\frac{\partial f_s}{\partial v}=0, \tag{4}$$

where $m=\frac{m_c}{m_s}$ and $s=c,h$ for cold and hotter ion population respectively. Now, we shall derive a characteristic nonlinear wave equation by making use of the hybrid model equations (Eq.(1) - Eq.(2)). We shall make use of the following stretched co-ordinates to obtain the KdV equation with an additional term indicating Landau damping of the dust acoustic solitary waves:

$$\xi=\varepsilon^{1/2}(x-V_{ph}t), \ \tau=\varepsilon^{3/2}t \tag{5}$$

where V_{ph} is the nonlinear wave speed (relative to the frame) normalized by C_d. The dependent physical quantities are expanded with respect to ε about the equilibrium state as

$$\begin{aligned}
n &= 1+\alpha_2\varepsilon n^{(1)}+{\alpha_2}^2\varepsilon^2 n_d^{(2)}+...\\
u &= \alpha_2\varepsilon u^{(1)}+{\alpha_2}^2\varepsilon^2 u^{(2)}+...\\
\phi &= \alpha_2\varepsilon\phi^{(1)}+{\alpha_2}^{(2)}\varepsilon^2\phi^{(2)}+...\\
n_s &= 1+\alpha_2\varepsilon n_s^{(1)}+{\alpha_2}^2\varepsilon^2 n_s^{(2)}+...\\
f_s &= f_s^{(0)}+\alpha_2\varepsilon f_s^{(1)}+{\alpha_2}^{(2)}\varepsilon^2 f_s^{(2)}+...
\end{aligned} \tag{6}$$

where ${f_s}^{(0)}$ represents the equilibrium distribution function for ion species and $s=c,h$ for cold and hot ions. The cold and hot ions having high temperatures are assumed to obey superthermal kappa distribution whose normalized distribution function has the form [9]:

$$f_s(v)=\frac{1}{(\pi\kappa_s)^{1/2}}\frac{\Gamma(\kappa_s+1)}{\Gamma(\kappa_s-\frac{1}{2})}\left(1+\frac{T_c m_s}{T_s m_c}\frac{v^2}{\kappa_s}\right)^{-(\kappa_s+1)}. \tag{7}$$

Substituting Eqs. (5)-(6) in the set of Eqs. (1) and (2) and then equating coefficients at different orders of ε to zero, we obtain the equations for the first and second order perturbed physical quantities. Following the procedure used in reference [8], from the first and second order equations, we obtain a KdV equation with an additional Landau damping term as:

$$\frac{\partial n}{\partial \tau}+X_1 P\int_{-\infty}^{\infty}\frac{\partial n}{\partial \xi'}\frac{\partial \xi'}{(\xi-\xi')}+X_2 n\frac{\partial n}{\partial \xi}+X_3\frac{\partial^3 n}{\partial \xi^3}=0. \tag{8}$$

Eq. (8) is the Landau damping modified KdV equation, where $n \equiv n^{(1)}$. The Landau damping, nonlinear and dispersion coefficients are

$$X_1 = \frac{\alpha_1 V_{ph}{}^4}{\sqrt{\pi}} \left(\frac{\mu}{\kappa_c{}^{3/2}} \frac{\Gamma(\kappa_c+2)}{\Gamma(\kappa_c-1/2)} + \frac{\nu\sigma^{3/2}}{\sqrt{m}\kappa_h{}^{3/2}} \frac{\Gamma(\kappa_h+1/2)}{\Gamma(\kappa_h-1/2)} \right);$$

$$\begin{aligned} X_2 &= \frac{\alpha_2 V_{ph}{}^3}{2} \Big(\frac{3}{V_{ph}{}^2} - 4V_{ph}{}^2 \Big(\frac{\mu}{\kappa_c{}^{3/2}} (\kappa_c+3/2)(\kappa_c{}^2-1/4) \\ &+ \frac{\nu\sigma^{3/2}}{\sqrt{m}\kappa_h{}^{3/2}} (\kappa_h+3/2)(\kappa_h{}^2-1/4) \Big) \Big); \\ X_3 &= \frac{\alpha_3 V_{ph}{}^3}{2}, \end{aligned} \tag{9}$$

where,

$$V_{ph} = \sqrt{\frac{1}{\frac{2\mu}{\kappa_c}(\kappa_c{}^2 - \frac{1}{4}) + \frac{2\sigma\nu}{\kappa_h}(\kappa_h{}^2 - \frac{1}{4})}}.$$

The coefficients X_2 and X_3 specify the magnitudes of nonlinear and dispersive terms whose delicate balance gives rise to solitary structures and the term X_1 represents the strength of Landau damping of dust acoustic solitary waves in plasma. κ_c and κ_h depict the kappa indices for the cold and the hot ion population. The solution of this Landau damped KdV equation has the form of soliton decaying with time as:

$$n = N\text{sech}^2 \left[\left(\xi - \frac{X_2}{3} \int_0^\tau N d\tau \right) / W \right], \tag{10}$$

where $N_0\left(1+\frac{\tau}{\tau_0}\right)^{-2}$ and τ_0 is found to be $\frac{1.37}{X_1}\sqrt{\frac{3X_3}{X_2 N_0}}$. It is clear that the inclusion of Landau damping effects in dynamics of dust acoustic solitary waves (DASWs) represented by Eq.(10) leads to the decay of peak solitary amplitude with time.

3 Results

We have implemented the reductive perturbation method and introduced the effects of wave-particle interaction in the famous Korteweg-de Vries (KdV) equation. A Landau damping modified KdV equation has been derived that governs the dynamics of dust acoustic solitary waves (DASWs) in a dusty plasma comprising two temperature ions obeying kappa distribution. The phenomenon of dust charging by the collection of electrons depletes the background electrons in plasma and gives rise to an electron depleted dusty plasma. We have performed this investigation while assuming a significant depletion of background electrons. Various coefficients of the Landau damping modified KdV equation are found to be functions of different physical parameters and hence, we shall perform a numerical investigation to study the influence of these parameters on the propagation properties of DASWs under the influence of Landau damping. For numerical analysis, the parameteric regime is considered as per the observations of Geotail satellite in the Earth's magnetotail as: $T_c = 0.7$ keV, $T_h = 7$ keV, $m_d = 4.6 \times 10^{-16}$kg, $Z_d = 4.6 \times 10^5$, $m_c = 2.65 \times 10^{-26}$kg for the O^+ ions and $m_h = 1.672 \times 10^{-27}$ kg for the H^+ ions [7].

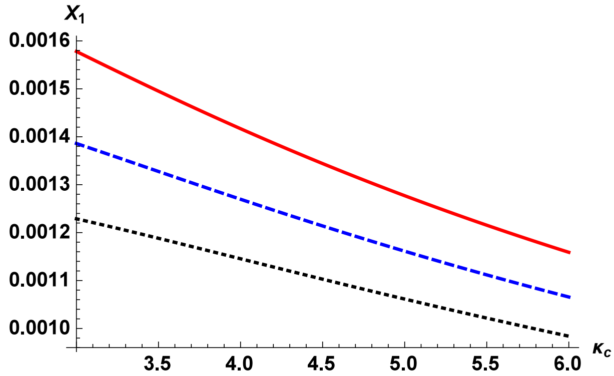

Figure 1. Variation of the Landau damping coefficient X_1 with kappa parameter for colder ions κ_c for different values of kappa parameter for hot ions κ_h. Red (solid) Curve: $\kappa_h = 3.25$; Blue (dashed) Curve: $\kappa_h = 3.5$; Black (dotted) Curve: $\kappa_h = 3.75$. Other parameters being μ=0.12, $\sigma = 0.1$, $\alpha_1 = 0.01$.

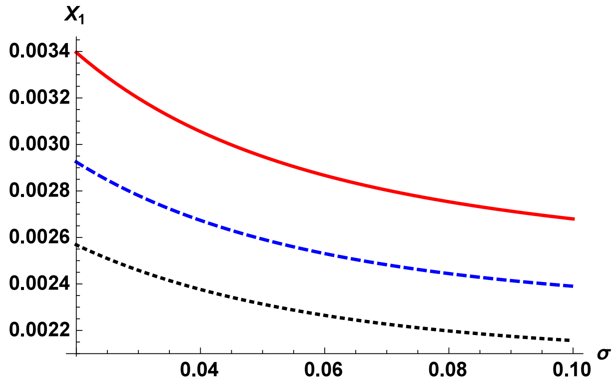

Figure 2. Variation of the Landau damping coefficient X_1 with temperature ratio of cold to hot ions σ with different values of cold ions to dust density ratio μ. Red (solid) Curve: $\mu = 0.1$; Blue (dashed) Curve: $\mu = 0.12$; Black (dotted) Curve: $\mu = 0.14$. Other parameters being κ_h=3.25, κ_c=4, $\alpha_1 = 0.01$.

The impact of variation in different physical parameters on the Landau damping coefficient is depicted in Fig.1 and Fig.2. The Landau damping of the dust acoustic waves becomes more prominent with an increase in the superthermality (decrease in κ parameter) of both the cold and the hot ions (shown in Fig.1) and suffers depreciation with an increase in the values of density ratio of cold ions to dust and the cold to hot ions temperature ratio (shown in Fig.2). This trend is almost similar to the results for the variation of normalized nonlinear wave speed (V_{ph}) with different physical parameters (figure not shown). One may easily note the small value of Landau damping of the DASWs due to kappa-distributed ions. However, it is asserted that even

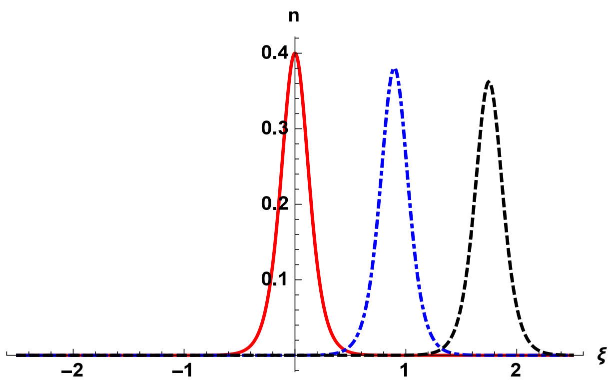

Figure 3. (Color online) Time evolution of the dust acoustic solitary structure under the influence of Landau damping due to kappa-distributed ions. Red (solid) Curve: $\tau = 0$; Blue (dashed) Curve: $\tau = 100$; Black (dotted) Curve: $\tau = 200$. Other parameters being μ=0.12, $\sigma = 0.1$, $\alpha_1 = 0.01$, $\alpha_2 = 0.5$, $\alpha_3 = 0.8$, $\kappa_c = 4$, $\kappa_h = 3.25$.

such an inappreciable value of the Landau damping coefficient may instill significant effects on the characteristics of dust acoustic nonlinear waves and structures propagating the plasma environment under consideration.

A subtle balancing of the nonlinear and dispersive effects in the medium tend to form localized hump-like structures called solitons. However, the introduction of Landau damping phenomenon in a plasma tends to affect the characteristics of these solitary structures. Fig.3 shows the time evolution of the analytical solution of the Landau damped KdV equation. The effect of the interaction of dust acoustic solitary wave with the heavier, cold O^+ ions in the plasma is indicated by Fig.3. It is seen that the wave-particle interaction reduces the peak amplitude of the solitary structure with the passage of time. Hence, the soliton is considered to be Landau damped even by the damping coefficient of small magnitude in Eq.(8). It is noteworthy that a variation in different physical parameters has a profound influence on the characteristics of dust acoustic solitary waves in the presence of Landau damping effects.

4 Conclusions

We have analyzed the effects of a kappa-type non-Maxwellian distribution of ions on the Landau damping of dust acoustic solitary waves in an electron depleted dusty plasma. The numerical analysis has been performed in context with the observations of Geotail spacecraft in the Earth's magnetotail region. It is noteworthy that even a small amplitude of Landau damping by colder O^+ ions may affect the characteristics of dust acoustic solitary structures while significantly decaying their amplitude. Enhancement in the superthermal character of the ion species leads to a more rapid damping of the dust acoustic solitary structures. The findings of this investigation exhibit a significant influence of the physical parameters, specifically the non-Maxwellian character of the kappa distribution function on the Landau damping phenomenon of the dust acoustic solitary waves in the Earth's magnetotail.

5 Acknowledgements

Y. G. acknowledges University Grants Commission for providing fellowship under Basic Scientific Research (BSR) scheme. This work was supported by DRS-II(SAP) No. F530/17/DRS-II/2015 (SAP-I) UGC, New Delhi.

References

[1] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics Publishing, Bristol, 2002), pp. 1–35.

[2] N.N. Rao, P.K. Shukla and M.Y. Yu, Planet, "Dust acoustic waves in dusty plasmas," *Planet. Space Sci.*, **38**, 4, April 1990, pp. 543–546, doi: 10.1016/0032-0633(90)90147-I.

[3] E. Ott, R. Sudan, "Nonlinear Theory of Ion Acoustic Waves with Landau Damping," *Phys. Fluids*, **12**, 11, November 1969, pp. 2388–2394, doi: 10.1063/1.1692358. in a magnetized nonthermal plasma with warm ions," *Phys. Plasmas*, **9**, 2, February 2002, pp. 465 (2002), doi: 10.1063/1.1427022.

[4] Y. Saitou and Y. Nakamura, "Ion-acoustic soliton-like waves undergoing Landau damping," *Phys. Letters A*, **343**,5, June 2005, pp. 397–402, doi: 10.1016/j.physleta.2005.06.035.

[5] A. Barman and A.P. Misra, "Landau damping effects on dust-acoustic solitary waves in a dusty negative-ion plasma," *Phys. Plasmas*, **21**, 7, July 2014, pp. 073708(1-10), doi: 10.1063/1.4890571.

[6] A. Sikdar and M. Khan, "Effects of Landau damping on finite amplitude low-frequency nonlinear waves in a dusty plasma," *J. Theor. Appl. Phys.*, **11**, 2, June 2017, pp. 137-142, doi: 10.1007/s40094-017-0248-x.

[7] M.N. Nishino, T. Terasawa, M. Hoshino, "Geotail Observations of the Cold Plasma Sheet on the Duskside Magnetotail," *COSPAR Colloq. Ser.* **16**, September 2005, pp. 28–33, doi: 10.1016/S0964-2749(05)80005-8.

[8] Yashika Ghai, N. S. Saini, and B. Eliasson, "Landau damping of dust acoustic solitary waves in nonthermal plasmas," *Phys. Plasmas*, **25**,1, January 2018, pp. 013704(1–11), doi: 10.1063/1.5011005.

[9] Hellberg, M. A., R. L. Mace, T. K. Baluku, I. Kourakis and N. S. Saini, "Comment on "Mathematical and physical aspects of kappa velocity distribution" [Phys. Plasmas 14, 110702, (2007)]," *Phys. Plasmas*, **16**, 9, September 2009, pp. 094701(1-5), doi: 10.1063/1.3213388.