

Estimator-Correlator based Spectrum Sensing with PU Signal Uncertainty in Full Duplex CRNs

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Abstract

In this paper, a promising spectrum sensing technique is proposed using Neyman Pearson (NP) based energy detection scheme for cognitive radio (CR) systems, where the secondary user is running in full duplex (FD) mode under residual self-interference (SI). First, NP based likelihood ratio test is carried out in additive white Gaussian noise (AWGN) environment and the closed form expressions for performance metrics are derived analytically. Next, minimum mean square error based estimator correlator for spectrum sensing under AWGN scenario is proposed, where PU signal having uncertain/arbitrary covariance structure is considered affirming a more realistic scenario in a heterogeneous network. Finally, the proposed detection scheme is extended to a practical Rayleigh fading scenario in FD CR under residual SI, where closed form analytical expressions for performance metrics are also derived. The simulated and analytical results are found in good compliance with each other.

1 Introduction

The demand for high data rate communications leads to intensive deployment of advance wireless technologies in several applications such as wireless hand-held devices, smart cities, and smart homes, etc., which results in drastic depletion of spectrum resources causing spectrum scarcity problem [1, 2]. To overcome this drawback, cognitive radio (CR) has appeared as an efficient technology to enhance the spectrum utilization [3]. In networks (CRNs), spectrum sensing followed by dynamic spectrum access are the two mandatory processes [4]. In the first process, secondary users (SUs)/unlicensed users sense the primary users (PUs)/licensed users activity and accordingly decide their transmission opportunities while in the second process, SUs transmit on the unused spectrum without interfering to PU.

Generally, in most CRNs, the SU operates on half duplex mode that utilizes two separate channels for sensing and transmission [5]. Full duplex (FD) system bearing the potential of almost increasing the spectrum efficiency twice as it facilitates simultaneous bidirectional communications using the same frequency channel [6, 7]. However, due to simultaneous transmission and reception, there may be excessive interference leakage which is called as self interfer-

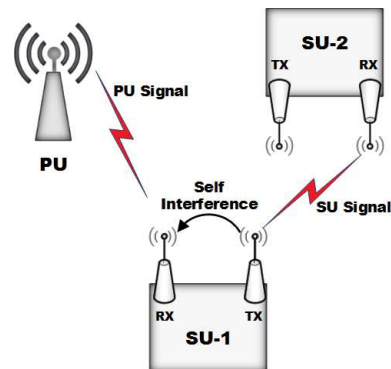


Figure 1. Sensing architecture of FD CRN

ence (SI). To suppress that, the researchers have proposed various techniques in analog, propagation and digital domains, which makes FD technology feasible in CRNs [8]. In FD CR systems, SUs perform the simultaneous sensing and transmission, enhancing the capacity of the robust spectrum sensing in FD enabled CRNs which is discussed in [9, 10]. The most common affluxion to the spectrum sensing is the energy detection method which has low complexity [11], but it is highly susceptible to noise variance uncertainties. In [12], the authors show that the Neyman Pearson (NP) based spectrum sensing technique exhibits a better performance. Researchers have extensively worked on spectrum sensing under noise uncertainties [13, 14], but signal uncertainty is another limiting factor that results in additional detection performance degradation [15].

In this research, a novel detection technique considering PU signal with arbitrary covariance matrix under Rayleigh fading scenario is proposed. In this work, first NP based LRT is performed for PU signal detection under residual SI occurring due to FD capabilities of SU and the analytical expressions for the P_d and P_{fa} are derived under additive white Gaussian noise (AWGN) scenario. Later, spectrum sensing for PU signal having arbitrary covariance matrix representing a more feasible and practical uncertainty model is presented, where the NP detector correlates the received signal sample with an estimate of the transmitted signal sample, obtained by minimum mean square error (MMSE) estimation technique. The study is further stretched out by examining the effect of Rayleigh fading imposing a realistic scenario for signal detection. For the above implemented case also, closed form expressions for P_d and P_{fa} are obtained and found closely matched with the simulated results.

2 System model

Fig. 1 depicts a FD communication system for SUs in CRN under residual SI, where SU-1 is simultaneously sensing the availability of PU link and communicating with SU-2. The received signal, $y[n]$ under H_0 and H_1 at SU-1 are given by;

$$\begin{aligned} H_0 : y[n] &= z[n] + w[n] \quad n = 1, \dots, N \\ H_1 : y[n] &= x[n] + z[n] + w[n], \quad n = 1, \dots, N \end{aligned} \quad (1)$$

In (1), $w[n]$ and $z[n]$ are white noise and unavoidable residual SI, assumed to be Gaussian random variables with mean ($\mu = 0$) and variances (σ_w^2) and (σ_z^2) respectively. $x[n] = hs[n]$ is also assumed to have Gaussian distribution $\mathcal{N}(0, \sigma_x^2)$ [9], where, $s[n]$ is PU signal and h is channel gain. Granted, the contiguity of both the antennas for transmission as well as reception on the same device, this premise is justified [9, 11]. In the following sections, we present the LRT detection with NP technique in AWGN scenario. Then, we proceed further to a more practical scenario where, we extend the proposed detection algorithm considering PU signal with arbitrary covariance to a Rayleigh fading scenario.

3 Basic energy detection scheme using NP technique under residual SI

Considering the previous section, probability distribution of the received signal $y[n]$ at SU under hypothesis H_0 and H_1 are as follows:

$$\begin{aligned} H_0 : y[n] &\sim \mathcal{N}(0, \sigma_z^2 + \sigma_w^2) \\ H_1 : y[n] &\sim \mathcal{N}(0, \sigma_x^2 + \sigma_z^2 + \sigma_w^2). \end{aligned} \quad (2)$$

For ease of calculation, we assume $\sigma_{H_0}^2 = \sigma_z^2 + \sigma_w^2$ for null hypothesis, and $\sigma_{H_1}^2 = \sigma_x^2 + \sigma_z^2 + \sigma_w^2$ for alternate hypothesis. A NP detector decides H_1 when the likelihood ratio is greater than a threshold value (γ), given as:

$$L(y) = \frac{\mathcal{P}(y; H_1)}{\mathcal{P}(y; H_0)} > \gamma. \quad (3)$$

Thus, the decision statistic for H_1 to be true becomes;

$$\begin{aligned} T(y) &= \ln(L(y)) = \ln\left(\frac{\mathcal{P}(y; H_1)}{\mathcal{P}(y; H_0)}\right) > \ln(\gamma), \\ &= \ln \left\{ \frac{\frac{1}{[2\pi\sigma_{H_1}^2]^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma_{H_1}^2} \sum_{n=0}^{N-1} y^2[n]\right\}}{\frac{1}{[2\pi\sigma_{H_0}^2]^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma_{H_0}^2} \sum_{n=0}^{N-1} y^2[n]\right\}} \right\} > \ln(\gamma), \end{aligned} \quad (4)$$

After retaining only the data dependent terms and with scaling, the test statistic is rewritten as:

$$T(y) = \sum_{n=0}^{N-1} y^2[n] > \gamma'. \quad (5)$$

From the right tail approximation, the performance metrics i.e. P_{fa} and P_d are given by;

$$P_{fa} = 2Q\left(\sqrt{\frac{\gamma'}{\sigma_{H_0}^2}}\right), \quad \text{and} \quad P_d = 2Q\left(\sqrt{\frac{\gamma'}{\sigma_{H_1}^2}}\right). \quad (6)$$

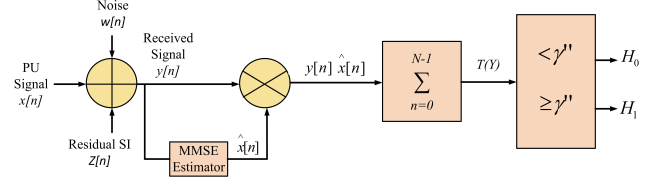


Figure 2. Block schematic of proposed estimator-correlator based energy detection in FD-CR under residual SI

For the fixed value of P_{fa} , the threshold value γ' is given as:

$$\gamma' = \sigma_{H_0}^2 \left\{ Q^{-1}\left(\frac{P_{fa}}{2}\right) \right\}^2. \quad (7)$$

4 Proposed estimator correlator based energy detection schemes

4.1 Energy detection for PU signal with arbitrary covariance matrix structure in AWGN

The block schematic of estimator-correlator for the detection of signal in FD CRN is shown in Fig. 2. Under the assumed scenario, the hypothesis H_0 and H_1 are given as:

$$\begin{aligned} H_0 : y[n] &\sim \mathcal{N}(0, \sigma_z^2 + \sigma_w^2) = \mathcal{N}(0, \sigma_{H_0}^2 \mathbf{I}) \\ H_1 : y[n] &\sim \mathcal{N}(0, \mathbf{C}_x + (\sigma_z^2 + \sigma_w^2) \mathbf{I}) = \mathcal{N}(0, \mathbf{C}_x + \sigma_{H_0}^2 \mathbf{I}). \end{aligned} \quad (8)$$

Here, the PU signal is to be estimated depending on the received data vector \mathbf{y} , and \mathbf{x} & \mathbf{y} are jointly Gaussian with zero mean. The MMSE estimate of the PU signal is interpreted as:

$$\hat{\mathbf{x}} = \mathbf{C}_{xy} \mathbf{C}_{yy}^{-1} \mathbf{y} \quad (9)$$

where, $\mathbf{C}_{xy} = E[\mathbf{x}\mathbf{y}^T] = \mathbf{C}_x$ and $\mathbf{C}_{yy} = E[\mathbf{y}\mathbf{y}^T] = \mathbf{C}_x + \sigma_{H_0}^2 \mathbf{I}$. Thus, the obtained MMSE estimate of the PU signal is given by;

$$\hat{\mathbf{x}} = \mathbf{C}_x (\mathbf{C}_x + \sigma_{H_0}^2 \mathbf{I})^{-1} \mathbf{y} \quad (10)$$

Now, the NP detector decides H_1 if

$$L(y) = \frac{\frac{1}{[(2\pi)^{\frac{N}{2}} |\mathbf{C}_x + \sigma_{H_0}^2 \mathbf{I}|^{\frac{1}{2}}]} \exp\left\{-\frac{1}{2} \mathbf{y}^T (\mathbf{C}_x + \sigma_{H_0}^2 \mathbf{I})^{-1} \mathbf{y}\right\}}{\frac{1}{[2\pi\sigma_{H_0}^2 \mathbf{I}]^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma_{H_0}^2} \mathbf{1}^T \mathbf{y} \mathbf{y}^T \mathbf{1}\right\}} > \gamma. \quad (11)$$

By taking logarithm and retaining only the data dependent terms, the obtained test statistic is represented as:

$$\begin{aligned} T(y) &= \mathbf{y}^T \frac{1}{\sigma_{H_0}^2 \mathbf{I}} \left(\frac{1}{\sigma_{H_0}^2} (\mathbf{C}_x + \sigma_{H_0}^2 \mathbf{I}) \mathbf{C}_x^{-1} \right)^{-1} \mathbf{y}, \\ &= \mathbf{y}^T \mathbf{C}_x (\mathbf{C}_x + \sigma_{H_0}^2 \mathbf{I})^{-1} \mathbf{y}. \end{aligned} \quad (12)$$

Using equation (10), the above equation is reformulated as:

$$T(y) = \mathbf{y}^T \hat{\mathbf{x}} > \gamma''. \quad (13)$$

In scalar format, the decision statistic can be given by;

$$T(y) = \sum_{n=0}^{N-1} y[n] \hat{x}[n]. \quad (14)$$

The above decision test statistic clearly shows that, the correlation of the received signal with an estimate of the PU signal is used for the decision making.

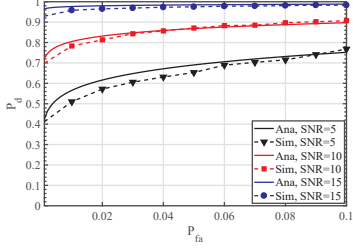


Figure 3. ROC plot with $N = 10$ at different SNR values for the proposed algorithm under Rayleigh fading scenario.

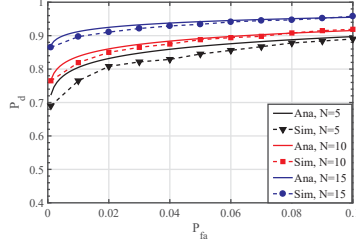


Figure 4. ROC plot at SNR = 10 dB with different values of N for the proposed algorithm under Rayleigh fading scenario.

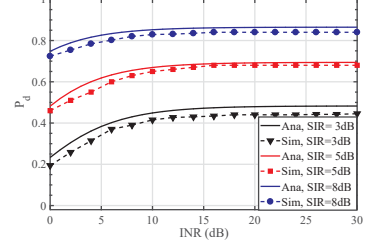


Figure 5. P_d vs. INR with $N = 10$ at different SIR values for the proposed algorithm under Rayleigh fading scenario.

4.2 Energy detection under Rayleigh fading scenario

In this subsection, we have considered a more practical scenario where the PU signal is affected with Rayleigh fading. Here, the received signal is formulated as:

$$y[n] = h[n]s[n] + z[n] + w[n]. \quad (15)$$

The PU signal channel $h[n]$ is considered as a non-causal linear system that has the known preamble signal $s_p[n]$. The received signal $y[n]$ is modeled as a linear combination of transmitted samples s_p before and after n . We have,

$$y[n] = s_p(n-k)h(k) + s_p(n-k+1)h(k-1) + \dots + s_p[n+k-1]h[-k+1] + z[n] + w[n]. \quad (16)$$

To find $h[k]$, based on the assumed Bayesian linear model the received signal vector \mathbf{y} is expressed as:

$$\mathbf{y} = \mathbf{A}\boldsymbol{\theta} + \mathbf{z} + \mathbf{w}, \quad (17)$$

where, $\mathbf{z} = (z[0], z[1], \dots, z[N-1])^T$, $\mathbf{w} = (w[0], w[1], \dots, w[N-1])^T$, $\boldsymbol{\theta}$ is a $k \times 1$ random vector and \mathbf{A} is $N \times k$ known observation matrix given as:

$$\mathbf{A} = \begin{bmatrix} s_p(-k) & \dots & s_p(0) & \dots & s_p(k-1) \\ \vdots & & \vdots & & \vdots \\ s_p(n-k) & \dots & s_p(n) & \dots & s_p(n+k-1) \end{bmatrix}. \quad (18)$$

Now, considering equation (17), we have,

$$\mathbf{x} = \mathbf{A}\boldsymbol{\theta} \approx \mathcal{N}(0, \mathbf{A}\mathbf{C}_\theta\mathbf{A}^T), \quad (19)$$

As stated earlier, both the residual SI and noise are assumed to be white, using equation (10) we compute the MMSE estimate of the PU signal \mathbf{x} as:

$$\hat{\mathbf{x}} = \mathbf{A}\mathbf{C}_\theta\mathbf{A}^T(\mathbf{A}\mathbf{C}_\theta\mathbf{A}^T + \sigma_{H_0}^2\mathbf{I})^{-1}\mathbf{y}. \quad (20)$$

If the PU spectrum is considered flat under Rayleigh fading scenario then we can consider $\mathbf{C}_\theta = \sigma_s^2\mathbf{I}$ and further, the test statistic is modeled as:

$$T(\mathbf{y}) = \sigma_s^2\mathbf{y}^T\mathbf{A}\mathbf{A}^T(\sigma_s^2\mathbf{A}\mathbf{A}^T + \sigma_{H_0}^2\mathbf{I})^{-1}\mathbf{y}. \quad (21)$$

With using matrix inversion lemma: $(\mathbf{A} + \mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{D}\mathbf{A}^{-1}\mathbf{B} + \mathbf{C}^{-1})\mathbf{D}\mathbf{A}^{-1}$, and let $\mathbf{A} = \sigma_{H_0}^2\mathbf{I}$, $\mathbf{B} = \sigma_s^2\mathbf{A}$, $\mathbf{C} = \mathbf{I}$ and $\mathbf{D} = \mathbf{A}^T$, the test statistic is represented as:

$$T(\mathbf{y}) = \sigma_s^2\mathbf{y}^T\mathbf{A}\mathbf{A}^T \left[\frac{1}{\sigma_{H_0}^2}\mathbf{I} - \frac{1}{\sigma_{H_0}^4}\sigma_s^4\mathbf{A} \left(\frac{\sigma_s^2\mathbf{A}^T\mathbf{A}}{\sigma_{H_0}^2} + \mathbf{I} \right)^{-1} \mathbf{A}^T \right] \mathbf{y}. \quad (22)$$

for large value of N , $\mathbf{A}\mathbf{A}^T \approx (\frac{N}{2}\mathbf{I})$. Thus,

$$T(\mathbf{y}) = \frac{C}{N}\mathbf{y}^T\mathbf{A}\mathbf{A}^T\mathbf{y}, \quad (23)$$

where, $C = \frac{N\sigma_s^2}{2 + \sigma_{H_0}^2}$. Hence, P_{fa} and P_d obtained are;

$$P_{fa} = pr(T(\mathbf{y}) > \gamma''' : H_0) = \exp\left\{-\frac{\gamma'''}{C\sigma_{H_0}^2}\right\}, \quad (24)$$

and

$$P_d = pr(T(\mathbf{y}) > \gamma''' : H_1) = \exp\left\{-\frac{\gamma'''}{2\sigma_{H_1}^2}\right\}. \quad (25)$$

Assuming P_{fa} fixed to some constant value, the threshold value γ''' is obtained as:

$$\gamma''' = C\sigma_{H_0}\ln\left(\frac{1}{P_{fa}}\right). \quad (26)$$

5 Simulation performance

This section focuses on investigating the detection performance analysis for the proposed algorithms in FD CR system shown in Fig. 1 through MATLAB simulation. The parameters that we use to evaluate the performance of aforementioned proposed algorithms are number of samples N , signal-to-interference ratio (SIR), interference-to-noise ratio (INR), and signal-to-noise ratio (SNR). The simulation performance of the proposed sensing techniques is carried out using 5000 numbers of monte carlo simulation. Figure 3 to Fig. 6 summarizes the performance study of the proposed energy detector for PUs signals having arbitrary covariance matrix in Rayleigh fading scenario. In addition, we compare the performance of the proposed sensing algorithm having arbitrary PU signal covariance matrix under AWGN and Rayleigh fading scenario with the benchmark scheme and the compared detection performances are presented in Fig.7 and in Fig. 8.

It is inferred from Fig. 3 and Fig. 4, that, target detection probability ($P_d \geq 0.9$) for the applicability of the proposed sensing algorithm in upcoming FD CRN in 5G wireless communication systems can be achieved with $N = 10$ and SNR = 10 dB. Figure 5 and Fig. 6 also indicate the consequence of residual SI and they show that when SI increases, SIR decreases and P_d also deteriorates. From the comparative studies illustrated by Fig. 7 and Fig. 8, it is clearly

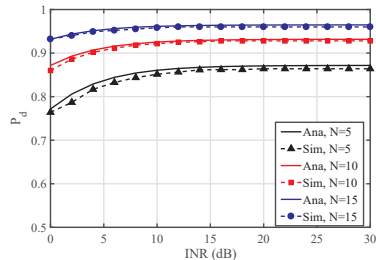


Figure 6. P_d vs. INR at $SIR = 8$ dB with different values of N for the proposed algorithm under Rayleigh fading scenario.

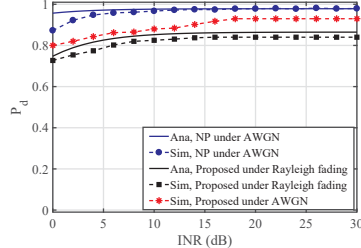


Figure 7. P_d vs. INR comparison of basic NP and proposed detection algorithm under AWGN and Rayleigh fading scenario with $P_{fa} = 0.01$ and $N = 10$.

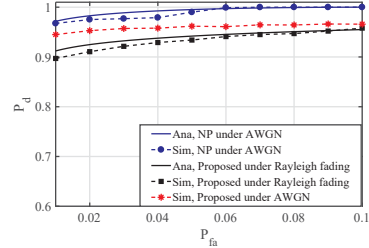


Figure 8. P_d vs. P_{fa} comparison of basic NP and proposed detection algorithm under AWGN and Rayleigh fading scenario with $SNR = 10$ dB and $N = 20$.

concluded that the proposed estimator correlator detector gives sensing performance near to the benchmark optimum NP detector approving its applicability in future wireless communications.

6 Conclusion

An accurate and efficient spectrum sensing is the fundamental requirement of FD CRNs for future 5G wireless applications. In this work, we investigated the estimator-correlator based energy detector for spectrum sensing with PU signal uncertainty in FD CRNs under both AWGN and Rayleigh fading scenario. The simulated and the analytical results are found close to each other. Performance evaluations of proposed algorithm clearly show that an optimal detection performance could be achieved at a moderately low SNR value of 10 dB using 10 samples and it also gives sensing performance near to the benchmark optimal NP detector performance approving its applicability in next generation wireless communication systems.

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