Optimization of sensor disposition for accurate localization in a practical environment

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Abstract

This paper proposes an optimization method of sensor disposition for geometrical localization with a probability based algorithm. This method can be applied to any practical environment. The proposed method makes use of the likelihood function of the location and the Cramer-Rao lower bound to obtain the expected accuracy distribution, and provide optimized sensor disposition in accordance with application scenarios. The experiment with IEEE 802.11 standards based devices demonstrates that the optimization of sensor disposition reduces the mean error by 50 percent.

1. Introduction

Estimating and tracking the location of a target device is currently a crucial technology for many applications such as wireless sensor networks, cellular networks, and the Internet-of-Things (IoT). A huge amount of effort has been invested to develop technologies for localizing a radio transmitter geometrically [1]-[3]. Geometrical localisation usually takes two steps. In the first step, geometrically related information is measured by such techniques as received-signal-strength-intensity (RSSI), time-of-arrival (TOA), and time-difference-of-arrival (TDOA). In the second step, the location is estimated on the basis of trilateration, triangulation, fingerprinting, particle filter, or a probability based algorithm. A probability based algorithm can exploit the least square (LS) method or the maximum likelihood (ML) method, combined with the maximum a posteriori (MAP) method.

The radio environments of localization are various, e.g. indoor or outdoor, tiny room or whole city, and line-of-sight (LOS) scenarios such as those in a rural region or non-line-of-sight (NLOS) scenarios such as those in a crowded city. Propagation characteristics and measurement limitations differ in accordance with these environments, so the localization scheme and sensor disposition should be chosen on the basis of each environment. In practice, an estimation area is complicated and restricted to a certain room, area, or shape. The measurement error distribution depends on the target's relative location to the boundary and sensor disposition. Thus, sensor disposition has a huge effect on estimation accuracy. While many authors have investigated the effects of sensor disposition and relative target position [4], [5], their assumption of the estimation area is limited to an isotropic plain.

This paper aims to provide an analytical method of optimizing sensor disposition in a specified restricted area. The proposed method utilizes the likelihood function of the target location and Cramer-Rao lower bound (CRLB) as a function of the target's relative position, and calculates the expected accuracy of each target location. Then an optimized sensor disposition is obtained in accordance with a specified application scenario. This method is about a probability based algorithm and can be applied to any RSSI, TOA, and TDOA technique.

The rest of the paper is organized as follows. Section 2 describes the mathematical formulation of the proposed method. In Section 3, experimental results are presented. Then Section 4 summarizes this paper.

2. Optimization of sensor disposition

2.1 Likelihood function of location

The localization technique of this paper is that an unknown location of a target node is estimated by exploiting measured data at sensor nodes. The location of sensor nodes are known in advance. Here, $\phi = (x, y, z)^T$ is the location of the target node, and $\phi_n = (x_n, y_n, z_n)^T$ is the location of the n-th sensor node ($n = 1, 2, \cdots, N$). Both the target device and sensor node are assumed to be within a location estimation area U with the size of $X_0 \times Y_0 \times Z_0$. $M(\phi) = [m_1(\phi), m_2(\phi), \dots, m_N(\phi)]^T$ is the measured data set, and $m_n(\phi)$ denotes the measured data of the n-th sensor node when the target node location is ϕ.

A. RSSI technique

In the RSSI technique, a statistical propagation model on RSSI in the location estimation has to be established. A model that can be applied to various situations is provided as a two-layered model [6], [7]

$$\tilde{m}_n(\phi) = \alpha \cdot d_n(\phi)^{-\beta}$$
$$d_n(\phi) = \|\phi - \phi_n\| \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm operator over a vector. The parameter α is a constant proportional to a

received power at a certain distance, and the parameter β is a power attenuation factor. These parameters are specific to an environment and radio frequency and need to be determined by a prior measurement campaign or by an extended algorithm that can estimate all parameters together, e.g. an expectation maximization method, a particle filter, or an iterative algorithm [7].

In practice, RSSI measurements vary randomly because of multipath fading. Rayleigh fading is viewed as a reasonable model for urban environments when there is no dominant propagation along a line of sight, and its statistical measurement distribution model is expressed as

$$p(m_n|\phi) = \frac{1}{\widetilde{m}_n(\phi)} e^{\left(-\frac{m_n}{\widetilde{m}_n(\phi)}\right)} \qquad (2).$$

(2) is the probability of that n-th sensor obtain m_n when the target is at ϕ. When the received powers m_n are independent in each sensor, the joint likelihood function for the measurement set of multiple sensors is expressed by:

$$l(\phi) = p(m_1, m_2, \ldots, m_N|\phi) = \prod_{n=1}^{N} p_n(m_n|\phi) \qquad (3).$$

The ϕ that maximize $l(\phi)$ in (3) is the estimated location when using the RSSI technique.

B. TOA technique

In the TOA technique, the time of arrival is measured at each single sensor. Assuming that a signal is emitted at time 0, the measured data $m_n(\phi)$, which is the time of arrival at the n-th sensor node, can be written as:

$$m_n(\phi) = \frac{d_n(\phi)}{c} + v_n \qquad (4)$$

where c is the speed of light, and v_n is the measurement error [2], [5]. When $v_n \sim N(0, \sigma_n^2)$ can be modelled as additive white Gaussian noise with the mean of zero and the standard deviation of σ_n, the probability density function for a single TOA measurement is written as:

$$p(m_n|\phi) = \frac{1}{\sqrt{2\pi}\sigma_n} exp\left[-\frac{m_n(\phi)-\frac{d_n(\phi)}{c}}{2\sigma_n^2}\right] \qquad (5).$$

The joint likelihood function for all sensors can be derived by substituting (5) to (3).

C. TDOA technique

In the TDOA technique, the difference in the distances of two sensors from the target node is calculated by two TOA measurements. The measurement equations become:

$$t_{ij}(\phi) = m_i - m_j + v_{ij} \qquad (6)$$

$$i, j = 1, 2, \cdots, M \; ; \quad i > j$$
$$v_{ij} \sim N(0, \sigma_i^2 + \sigma_j^2)$$
$$d_{ij}(\phi) = \|\phi_i - \phi_j\|$$

where independent distributed Gaussian noise is assumed. Note that the measurement noise $v_{ij} = v_i + v_j$ is composed of the noises at two sensors and has the covariance $\sigma_i^2 + \sigma_j^2$.

A single TDOA measurement defines a hyperboloid of possible target locations with the two sensors as foci. From N measurement sets, *(N-1)* non-redundant TDOAs can be obtained, and there are *N(N-1)/2* distinct TDOAs. The likelihood function of complete data sets can be expressed as [4]:

$$l(\phi) = \frac{1}{(2\pi)^{N/2}\sqrt{det(\boldsymbol{C})}} exp\left[-\frac{\left[t(\phi)-\frac{d(\phi)}{c}\right]^T}{2\boldsymbol{C}}\right] \qquad (7)$$

$$\boldsymbol{t}(\phi) = \left[t_{12}, t_{13}, \cdots, t_{ij}, \cdots, t_{(N-1)N}\right]^T$$
$$\boldsymbol{d}(\phi) = \left[d_{12}, d_{13}, \cdots, d_{ij}, \cdots, d_{(N-1)N}\right]^T$$

where $\boldsymbol{C}$ is the covariance matrix of the measurement set.

2.2 Expected accuracy distribution

The proposed method evaluates the localization accuracy by the expected values of mean square error. When measurement values can be considered as unbiased estimators that distribute around true values, the mean square error is equal to the variance. The minimum error variance is provided by using CRLB [4], [5]. It is calculated from the likelihood function of ϕ as follows:

$$A(\phi) = \sigma_{CRLB}^2 = \frac{1}{E\left[\left(\frac{\partial \; log \; l(\phi)}{\partial\phi}\right)^2\right]}$$

$$= \frac{1}{E\left[\left(\frac{\partial\Sigma \; log \; p(m_n|\phi)}{\partial\phi}\right)^2\right]} \qquad (8).$$

Because (8) is a function of the target's location ϕ, this provides the expected accuracy distribution for estimation area *U*. Besides, the n-th measured data depends on the sensor position ϕ_n; thus, (8) can be regarded as the function of sensor disposition $\boldsymbol{\Phi} = [\phi_1, \phi_2, \cdots, \phi_N,]$.

2.3 Proposed sensor disposition optimization

Eq. (8) indicates that the optimized sensor disposition can be calculated by minimizing a specific metric related to $A(\phi)$. The optimization target value is a statistical value such as mean, maximum, or percentile error over estimation area. For the case of mean, the objective function becomes:

$$f(\boldsymbol{\Phi}) = \underset{\phi}{mean} \; A(\phi, \boldsymbol{\Phi}) \qquad (9)$$

The optimized sensor dispositions are the ones that minimize (9), and this problem can be written as:

$$\underset{\boldsymbol{\Phi} \in \boldsymbol{U}}{\text{minimize}}\, f(\boldsymbol{\Phi}) \qquad (10).$$

For the minimization, one can exploit general nonlinear programming method such as the conjugate gradient method, the quasi-Newton method, multiplier method, or successive quadratic programming method.

Figure 1 shows the cumulative distribution function (CDF) of σ_{CRLB} at all places in a certain room. When the optimization target value is mean, σ_{CRLB} at CDF ≈ 0.5 is smaller than others, while σ_{CRLB} at CDF>0.8 are larger than others. This means that the accuracy is best at a general place, while at several places the error is larger. In contrast, for the case of max, σ_{CRLB} at CDF=1.0 are smaller than others while larger at CDR<0.95. This means that performance is stable at any place, while accuracy is not as good as others. Thus, the optimization target value should be chosen in accordance with the application scenario.
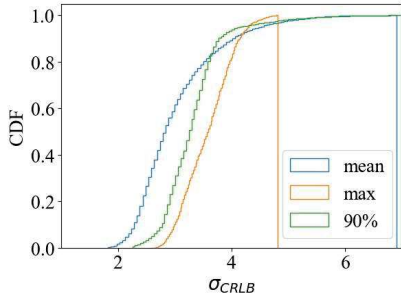

Figure 1. Example of CDF of $\boldsymbol{\sigma_{CRLB}}$.

3. Experimental results and discussion

3.1 Setup and application scenario

We evaluate the performance of sensor disposition optimization. Assume an application scenario for localizing a Wi-Fi enabled smartphone in a meeting room and determining the seating arrangement. The RSSI technique is selected because it is suitable for indoor localization. All sensor nodes and target devices are based on IEEE 802.11 standards. A Raspberry Pi with a USB Wi-Fi dongle is used as a sensor node and receives Wi-Fi packets and measures RSSI using Wireshark.

The experiment was conducted in a meeting room whose size is 4.7 m × 7.28 m (Figure 2). A table, 1.5 m × 4.5 m in size, is placed at the center of the room. Eight smartphones are used as target devices. Four different models are used in order to guarantee versatility in the smartphone model. Two smartphones of each model are placed both on and under the table to mimic practical position during a meeting. The number of sensors, N, is four, and the performance of two sets of sensor disposition, i.e. with and without optimization are compared. The sensors without optimization are deployed on the wall as a reference disposition. With an aim to determine the seating arrangement, the optimization target area is set around the table with a margin of 0.4 m. Moreover, accuracy priority is equal for all seats; thus, the optimization target value is set as mean.
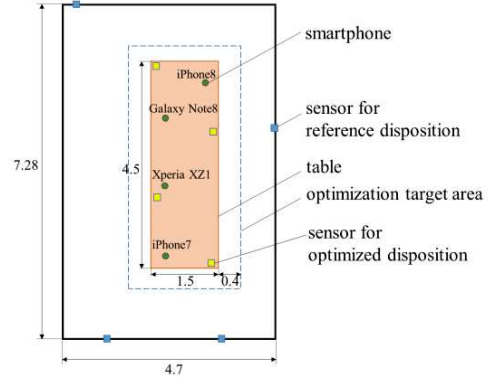

Figure 2. Experimental setup.

3.2 Sensor disposition optimization

Figure 3 shows a propagation characteristic of smartphones for an indoor environment. For versatility in indoor localization, these parameters were previously obtained in a different room with three different smartphones (iPhone 5, Xperia Z5, and Galaxy S7 Edge). The propagation constants in (1) were obtained as $\alpha = 1.41 \times 10^{-6}, \beta = 1.29$.
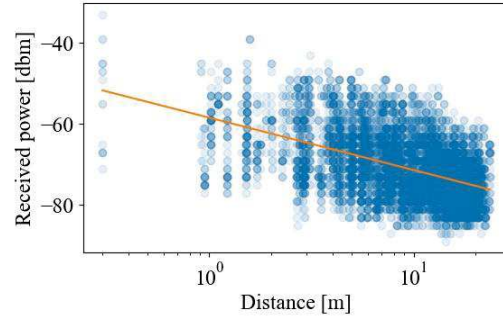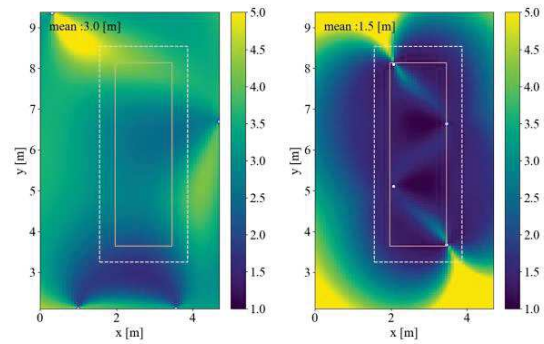

Figure 3. Received singal power in relation to distance.

(a) Reference disposition   (b) Optimized disposition

Figure 4. Expected accuracy distribution.

By substituting these parameters into (1) to (3), the joint likelihood function can be obtained. The optimised sensor disposition can be obtained by (10), using the quasi-Newton method for optimization. Figure 4 shows the expected accuracy distribution of the optimized and reference sensor dispositions. The standard deviation σ_{CRLB} of (8), i.e. expected error, is expressed by color. The mean errors within U of the reference and optimized sensor dispositions are 3.0 m and 1.5 m, respectively. This result means that the optimization can reduce mean error by 50%.

3.3. Experimental evaluation

Both optimized and reference disposition sensors are deployed at the same time. The RSSI of Wi-Fi packets from the smartphones was measured for 1 hour. Because each packet contains a MAC address and sequence number, this information can be utilized as an identifier to distinguish each packet. Thus, the localization is executed for every individual received packet.

The samples of individual localization results are shown in Figure 5. The device, the Xperia, is placed under the table. Each estimated location for every packet is plotted along with the mean result and real location. The optimization can reduce both mean error and dispersion. The mean
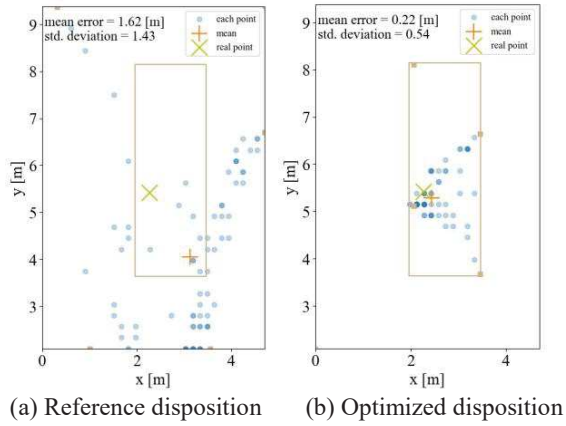
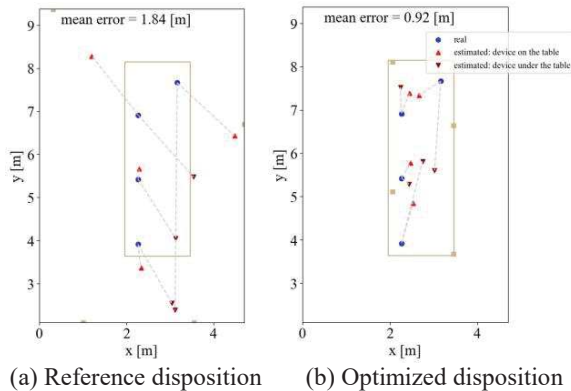estimated locations of all devices are shown in Figure 6. The mean estimation errors of the reference and optimized dispositions were 1.84 m and 0.92 m, respectively. The mean error can be reduced by 50%, the same as predicted in Section **3.2**. Although the absolute values of mean error are different between expected values and measured results, the rate of improvement is the same. This indicates that the proposed method is beneficial in improving the accuracy.

4. Conclusion

This paper proposed an optimization method of sensor disposition for geometrical localization with a probability based algorithm. In the experiment with IEEE 802.11 standards based devices and the RSSI based technique, the optimization of sensor disposition reduced both mean error and dispersion of localization results. The mean error was reduced by 50%, which is in good agreement with the expected error. The proposed method can be applied to any RSSI, TOA, and TDOA technique.

5. Acknowledgements

(a) Reference disposition (b) Optimized disposition

Figure 5. Individual estimated locations example.

(a) Reference disposition (b) Optimized disposition

Figure 6. Mean estimated locations of all devices.

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