# $S_{11}$ Calibration Method for a Coaxial Line with Three Reference Materials and no Short Termination Condition for Dielectric Measurement in Liquids

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## Abstract

This paper proposes an $S_{11}$ calibration method for a coaxial-feed-type open-ended cut-off waveguide with a vector network analyzer (VNA) from three reference materials with no short termination condition. The equation for $S_{11}$ calibration from three arbitrary impedance standards was first derived, and the electrostatic capacitance unique to the jig was then quantified. The theoretical values for input impedance at the front surface of the sample material were then determined by substituting the capacitance value into the equivalent circuit at frequencies of 0.50, 0.75 and 1.0 GHz. The calibrated $S_{11}$ value for the front surface of the sample material was then determined. The validity and measurement accuracy of the proposed method were verified via comparison of this $S_{11}$ calibration value with that calculated using EM analysis. The results numerically indicated the validity of $S_{11}$ calibration for the coaxial-feed type open-ended cut-off waveguide using VNA from the three reference materials with no short termination condition based on the proposed procedure. The effectiveness of the method was also verified from actual measurement of $S_{11}$ in the frequency range from 0.5 to 3.0 GHz, thereby confirming its effectiveness.

## 1. Introduction

In this study, a method for the calibration of $S_{11}$ for a coaxial-feed type cut-off circular waveguide based on the insertion of three reference materials with no short termination condition using a vector network analyzer (VNA) was proposed. The purpose is for application to the liquid dielectric measurement method [1] – [2] with no short termination as previously proposed by the author. A calibration equation for $S_{11}$ based on arbitrary impedance standards was first derived. Next, theoretical values for input impedance assuming the insertion of reference materials were defined. An equation to calculate the capacitance of the coaxial line section was then derived. Moreover, the capacitance of the sample insertion section was also determined. The calculated values of input impedance based on the above capacitances and equivalent circuit under various liquid insertion conditions were thus compared with values calculated via the electromagnetic (EM) analysis via the mode-matching (MM) method. Next, the calculated value of input impedance, assuming the insertion of three reference materials, was substituted into the $S_{11}$ calibration equation. The input impedance value for the front surface of the sample material via the EM analysis was also substituted into the $S_{11}$ calibration equation assuming the measurement of actual reference materials and unknown materials. In the results, the $S_{11}$ value for the jig with the coaxial-feed type cut-off circular waveguide was calibrated for the front surface of the sample. Measurement accuracy with $S_{11}$ calibration via VNA using this method was also clarified from comparison with the value calculated using EM analysis at frequencies of 0.50, 0.75 and 1.0 GHz. Numerical analysis indicated that $S_{11}$ values for the coaxial line were successfully calibrated with only three reference materials and no short termination condition with the impedance standard based on the proposed method.

## 2. Calibration theory for $S_{11}$ with three reference materials

Calibration with general SOL (short, open, loaded) conditions [3] – [5] is not performed for $S_{11}$ with the coaxial-feed-type cut-off circular waveguide proposed here. Specifically, a reference material is used instead of load termination as a calibrator [6]. $S_{11}$ is also not determined with the coaxial tip short-circuited, but is measured with a second reference material inserted. The theoretical value of $S_{11}$ on the front surface of the sample with three termination conditions is first determined from equivalent circuit assuming the insertion of the three reference materials. Next, the actual $S_{11}$ value by inserting these materials is measured using VNA. The $S_{11}$ value for the front surface of the sample with insertion of the unknown material is then calibrated via substitution of these values into the equation described later. The equation for calibration of the coaxial line $S_{11}$ value via the proposed method is derived as outlined below. Eq. (1) is first satisfied [6] from the measuring instrument in Figure 2, which corresponds to the analytical model of Figure 1. Here, 2a, 2b, d and $\varepsilon_{rA}$ are the outer-conductor inner diameter of the coaxial line, the inner-conductor outer diameter, the sample insertion length and the permittivity of the supported dielectric, respectively.

$$\dot{\Gamma}_m = \frac{a_2}{b_2} = \frac{\dot{\rho}_m - \dot{E}_{DF}}{\dot{E}_{SF} \cdot \dot{\rho}_m + \dot{E}_{RF} - \dot{E}_{SF} \cdot \dot{E}_{DF}} \qquad (1)$$

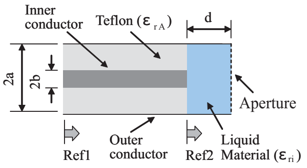

Fig. 1: Analytical model

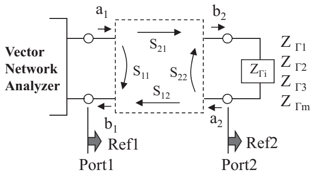

Fig. 2: Analytical model equivalent circuit

Moreover, $\Gamma_m$ is the reflection coefficient for the front of the sample material (Ref. 2) after calibration, and $\rho_m$ is the measured value of the reflection coefficient for Ref. 1 before calibration. $\Gamma_m$ in Eq. (1) can be determined from the three unknowns of $E_{DF}$, $E_{SF}$ and $\gamma$ as in the following equation:

$$\dot{E}_{RF} - \dot{E}_{SF} \cdot \dot{E}_{DF} \equiv \gamma \qquad (2)$$

Thus, these three unknowns are determined by solving the simultaneous equations under the three termination conditions with respect to Eq. (1). The following equation for determining $E_{SF}$, $E_{DF}$ and $E_{RF}$, which are constituent elements of Eq. (1), can be thus derived from the above relationship:

$$E_{SF} = \frac{\dot{\rho}_2 - \dot{\rho}_3 + \gamma \cdot \left(\dot{\Gamma}_3 - \dot{\Gamma}_2\right)}{\dot{\Gamma}_2 \cdot \dot{\rho}_2 - \dot{\Gamma}_3 \cdot \dot{\rho}_3} \qquad (3)$$

$$E_{DF} = \dot{\rho}_1 - \dot{\Gamma}_1 \cdot \left(E_{SF} \cdot \dot{\rho}_1 + \gamma\right) \qquad (4)$$

$$E_{RF} = \dot{E}_{DF} \cdot \dot{E}_{SF} + \gamma \qquad (5)$$

$\gamma$ is calculated using

$$\gamma = \frac{\left(\dot{\rho}_2 - \dot{\rho}_1\right) \cdot \left(\dot{\Gamma}_2 \cdot \dot{\rho}_2 - \dot{\Gamma}_3 \cdot \dot{\rho}_3\right) + \left(\dot{\rho}_2 - \dot{\rho}_3\right) \cdot \left(\dot{\Gamma}_1 \cdot \dot{\rho}_1 - \dot{\Gamma}_2 \cdot \dot{\rho}_2\right)}{\left(\dot{\Gamma}_3 - \dot{\Gamma}_2\right) \cdot \left(\dot{\Gamma}_2 \cdot \dot{\rho}_2 - \dot{\Gamma}_1 \cdot \dot{\rho}_1\right) + \left(\dot{\Gamma}_1 - \dot{\Gamma}_2\right) \cdot \left(\dot{\Gamma}_3 \cdot \dot{\rho}_3 - \dot{\Gamma}_2 \cdot \dot{\rho}_2\right)} \qquad (6)$$

Here, $\rho_1$, $\rho_2$ and $\rho_3$ are the reflection coefficients for Ref. 1 measured with the insertion of reference liquids, $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ are the theoretical value of reflection coefficients for Ref. 2, respectively. The three samples are distinguished with insertion depending on the subscripts 1, 2 and 3 for $\Gamma$ and $\rho$. At this time, a coaxial-feed-type cut-off circular waveguide was used as a jig for calibrating $S_{11}$. Accordingly, the open-ended coaxial line shown in Fig. 1 can be represented as the equivalent circuit shown in Fig. 3, where $C_f$ and $\varepsilon_{ri} \cdot C_0$ are defined as the capacitance at the tip of the coaxial line and the sample insertion section. The following equation is thus determined [6].

$$\dot{\Gamma}_i = \frac{1 - j\omega C_0 \cdot \dot{\varepsilon}_{ri} \cdot Z_0 - j\omega C_f \cdot Z_0}{1 + j\omega C_0 \cdot \dot{\varepsilon}_{ri} \cdot Z_0 + j\omega C_f \cdot Z_0} \qquad (7)$$

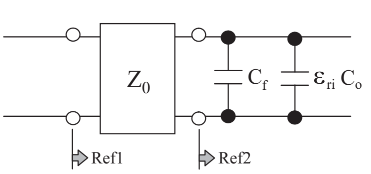

Fig. 3: Analytical model equivalent circuit

Advance calibration of $S_{11}$ using a VNA is needed for dielectric measurement. However, in permittivity estimation [1], [2], $S_{11}$ cannot be calibrated from a short termination condition because the calibration plane is inside the cavity when a coaxial-feed type cut-off circular waveguide is used for the jig. Accordingly, input impedance with insertion of the second reference material was defined in this study as the impedance standard with no short termination condition. Accordingly, jig $S_{11}$ values were calibrated using three reference materials (including air) with this method.

## 3. Quantification of electric capacity at the jig

The capacitance of the coaxial tip is then quantified for calibration of the coaxial line $S_{11}$ value. Two capacitances are connected to the front end of the transmission line as shown in Fig. 3. In this case, it is assumed that $C_f \equiv \varepsilon_{rA} \cdot C_0$ is satisfied for $\varepsilon_{ri} = 1.0 - j\ 0.0$. Based on the result, $C_f$ is expressed as follows:

$$C_f \equiv \frac{\varepsilon_{rA}}{j^2 \cdot \omega \cdot \left(1 + \varepsilon_{rA}\right) \cdot X_{\Gamma i}} = -\frac{\varepsilon_{rA}}{\omega \cdot \left(1 + \varepsilon_{rA}\right) \cdot X_{\Gamma i}} \qquad (8)$$

$C_f$ value for the coaxial line section was then determined by substituting the value of $Z_{\Gamma i}$ calculated using the MM method and $\varepsilon_{rA}$ into Eq. (8). Next, the capacitance $C_0$ at the cavity section was determined from the capacitance $C_f$ and $\Gamma_i$ ($Z_{\Gamma i}$) as determined from the above procedure. The following equation is determined by manipulating Eq. (7) :

$$C_0 = \frac{1}{\dot{\varepsilon}_{ri}} \cdot \left[ \frac{1}{j\omega \cdot Z_0} \left( \frac{1 - \dot{\Gamma}_i}{1 + \dot{\Gamma}_i} \right) - C_f \right] \qquad (9)$$

Here, pure water ($\varepsilon_{r1}$), methanol ($\varepsilon_{r2}$) and air ($\varepsilon_{r3} = 1.0$) were chosen to be three reference materials. The capacitance $C_0$ at the cavity section is then calculated by substituting the values of $Z_0$, $\varepsilon_{ri}$ and $Z_{\Gamma i}$ using the MM method into Eq. (9). However, as $C_f$ and $C_0$ are lumped constants in practice, the values need to be set to a single value for each reference material over all frequency bands. The input impedance calculation error from Eq. (7) based on the average of three frequencies as the values of $C_f$ and $C_0$ was thus verified. Specifically, $C_f = 2.1972 \times 10^{-14}$ F was first set to the value of $C_f$. Next, the value of $C_0$ was also set to $C_0 = 2.8152 \times 10^{-14}$ F for pure water, $C_0 = 2.7840 \times 10^{-14}$ F for methanol and $C_0 = 1.0718 \times 10^{-14}$ F for air. Here, values based on the Debye dispersion equation [7], [8] at a liquid temperature of 25°C were used to represent the complex permittivity of each material. The theoretical values of input impedance for Ref. 2 with substitution of $C_f$, $C_0$ and $\varepsilon_{ri}$ into Eq. (7) were then compared with those calculated using the EM analysis via the MM method. As shown in Tables 1, 2 and 3, the differences were around 1.03% for the real part and under 0.4% for the imaginary part with pure water as a reference material, 1.5 and 0.3% with methanol, and under 0.006% for air. In addition, calculation of $Z_{\Gamma i}$ was performed with 2a = 4.10 mm, 2 b = 1.30 mm, d = 5.00 mm and $\varepsilon_{rA} = 2.05$.

TABLE 1: INPUT IMPEDANCE BASED ON EQ. (7) WITH $C_F$=2.192 × $10^{-14}$ F AND $C_0 = 2.8152 \times 10^{-14}$ F (PURE WATER, 25°C)

| Freq.<br>$Z_{\Gamma i}$ | 0.50 GHz | 0.75 GHz | 1.0 GHz |
|---|---|---|---|
| $Z_{\Gamma i}$ determined using Eq. (7) [Ω] | 3.4295<br>– j 142.518 | 3.4294<br>– j 95.0270 | 3.4293<br>– j 71.2702 |
| $Z_{\Gamma i}$ determined using the MM method [Ω] | 3.4651<br>– j 142.953 | 3.4651<br>– j 95.0617 | 3.4651<br>– j 71.0267 |
| Difference [%] (real, imaginary) | -1.03<br>-0.304 | -1.03<br>-0.0365 | -1.03<br>+0.343 |

TABLE 2: INPUT IMPEDANCE BASED ON EQ. (7) WITH $C_F$=2.192 × $10^{-14}$ F AND $C_0$ = 2.7840 × $10^{-14}$ F (METHANOL, 25°C)

| Freq. $Z_{\Gamma i}$ | 0.50 GHz | 0.75 GHz | 1.0 GHz |
|---|---|---|---|
| $Z_{\Gamma i}$ determined using Eq. (7) [Ω] | 44.041<br>– j 343.152 | 43.983<br>– j 229.863 | 43.909<br>– j 173.545 |
| $Z_{\Gamma i}$ determined using the MM method [Ω] | 44.696<br>– j 343.329 | 44.642<br>– j 229.677 | 44.574<br>– j 173.087 |
| Difference [%] (real, imaginary) | -1.47<br>-0.00516 | -1.48<br>+0.00810 | -1.49<br>+0.265 |

TABLE 3: INPUT IMPEDANCE BASED ON EQ. (7) WITH $C_F$=2.192 × $10^{-14}$ F AND $C_0$ = 1.0718 × $10^{-14}$ F (AIR, 25°C)

| Freq. $Z_{\Gamma i}$ | 0.50 GHz | 0.75 GHz | 1.0 GHz |
|---|---|---|---|
| $Z_{\Gamma i}$ determined using Eq. (7) [Ω] | 0.00<br>– j 9737.2250 | 0.00<br>– j 6491.4834 | 0.00<br>– j 4868.6125 |
| $Z_{\Gamma i}$ determined using MM method [Ω] | 0.00<br>– j 9737.8377 | 0.00<br>– j 6491.6156 | 0.00<br>– j 4868.4231 |
| Difference [%] (Imaginary) | – 0.00630 | – 0.00204 | + 0.00389 |

## 4. Verification of calibration accuracy

The theoretical value $Z_{\Gamma i}$ of input impedance for Ref. 2 with the insertion reference materials was defined in the previous section. Finally, these values were substituted into Eq. (1), and the reflection coefficient $\Gamma_m$ (Ref. 2) was calibrated. The calibration value of the reflection constant based on this procedure $\Gamma_m$ was then compared with the value calculated using the MM method. For this purpose, it was first assumed that pure water, methanol and air as reference materials and pure water and methanol as unknown materials were inserted into the jig. The calibrated values of the reflection constant $\Gamma_m$ using Eqs. (1) – (6) were verified via the following procedure:

1. Input impedance $Z_{\Gamma i}$ (Ref. 2) calculated by substituting the values of $C_f$, $C_0$ and $\varepsilon_{ri}$ assumed for the use of the three reference materials into Eq. (7) is defined as the theoretical value.
2. Input impedance determined by moving the electrical length of $Z_{\Gamma i}$ value (Ref. 2) calculated using the MM method assuming the insertion of three reference materials back to the connection plane of the jig (Ref. 1) is defined as the measurement value $Z_{\rho i.}$
3. Input impedance determined by moving the electrical length of $Z_{\Gamma m}$ value calculated using the MM method (Ref. 2) to the connection plane of the jig (Ref. 1) assuming unknown material is defined as the measurement value $Z_{\rho m}$.
4. The calibration value of $\Gamma_m$ (Ref. 2) is determined as the input impedance by substituting the values of items 1, 2 and 3 into Eqs. (1) – (6).
5. The measurement accuracy of the calibrated value $\Gamma_m$ based on Eqs. (1) – (6) in item 4 is checked by comparing the calculated input impedances in items 2 and 3.

After determination of the theoretical $Z_{\Gamma i}$ value, the calibration value of $\Gamma_m$ (Ref. 2) was determined by substituting these values into Eqs. (1) – (6) as detailed in item 4. The calibrated value $\Gamma_m$ of the reflection coefficient (Ref. 2) was then compared with the value calculated using the MM method as the input impedance $Z_{\Gamma m}$ as detailed in item 5 at frequencies of 0.50, 0.75 and 1.0 GHz and a liquid temperature of 25°C. The results of these studies are shown in Table 4. The error of input impedance based on Eqs. (1) – (6) was within 1.04% for the real part and 0.40% for the imaginary part with pure water as the unknown material and 1.51% for the real part and 0.26% for the imaginary part with methanol. These details indicate that input impedance can be calibrated at a calculation accuracy within 1.5% even with various liquids assumed as unknown materials using VNA with the proposed method.

TABLE 4: VERIFICATION OF CALIBRATION ACCURACY ASSUMING THE INSERTION OF AN UNKNOWN MATERIAL BASED ON EQ. (5)

| Frequency Situation | | 0.50 GHz | 0.75 GHz | 1.0 GHz |
|---|---|---|---|---|
| Pure water 25°C | $Z_{\rho m}$ determined using the MM method | 1.9194<br>– j 101.034 | 1.9524<br>– j 63.2719 | 2.0001<br>– j 43.1029 |
| | $Z_{\Gamma m}$ determined using the MM method | 3.4651<br>– j 142.953 | 3.4651<br>– j 95.0617 | 3.4651<br>– j 71.0267 |
| | $Z_{\Gamma m}$ determined using Eq. (1) | 3.4295<br>– j 142.518 | 3.4294<br>– j 95.0270 | 3.4293<br>– j 71.2702 |
| | Difference [%] | 1.04<br>0.305 | 1.04<br>0.0365 | 1.04<br>-0.342 |
| Methanol 25°C | $Z_{\rho m}$ determined using the MM method | 13.2154<br>– j 183.050 | 13.3086<br>– j 119.590 | 13.3582<br>– j 87.4329 |
| | $Z_{\Gamma m}$ determined using the MM method | 44.696<br>– j 343.329 | 44.642<br>– j 229.677 | 44.574<br>– j 173.087 |
| | $Z_{\Gamma m}$ determined using Eq. (1) | 44.041<br>– j 343.152 | 43.983<br>– j 229.483 | 43.909<br>– j 173.545 |
| | Difference [%] | 1.49<br>0.0516 | 1.50<br>0.0845 | 1.51<br>– 0.264 |

## 5. Frequency characteristics of measured input impedance after $S_{11}$ calibration

Next, the $S_{11}$ value for the coaxial line fed by an SMA connector was calibrated with pure water, methanol and air as reference materials using VNA over the frequency band of 0.5 – 3.0 GHz. Room temperature during calibration and measurement was controlled at 25 ± 1.0°C. First, the value of $S_{11}$ measured after calibration in an open condition was compared with that calculated using the mode-matching technique for input impedance. The results shown in Figs. 4 and 5 indicate that the measured values of input impedance after calibration with the three reference materials corresponded closely to those calculated using the mode-matching method for both the real and the imaginary parts in all frequency bands. Accordingly, it was found that $S_{11}$ values can be perfectly reproduced if the value at the time of calibration is measured even after calibration with the reference material in the jig. The measured input impedance with the insertion of pure water after calibration of $S_{11}$ with the three reference materials was also compared with the value calculated using the mode-matching technique. The results shown in Figs. 6 – 7 indicate that the measured value after calibration corresponded closely to that calculated using the mode-matching method over all frequency measurement bands because pure water and methanol were chosen as reference materials for calibration.

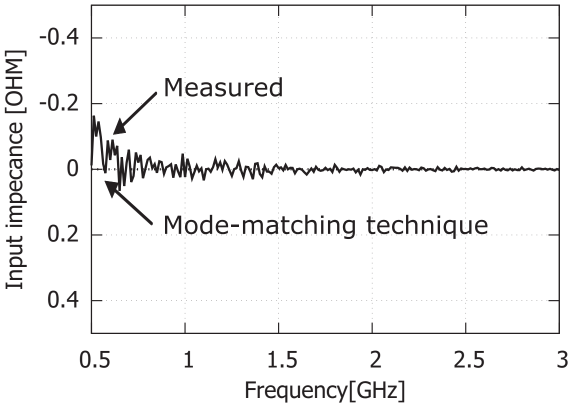

Figure 4. Measured input impedance after calibration (air, real)

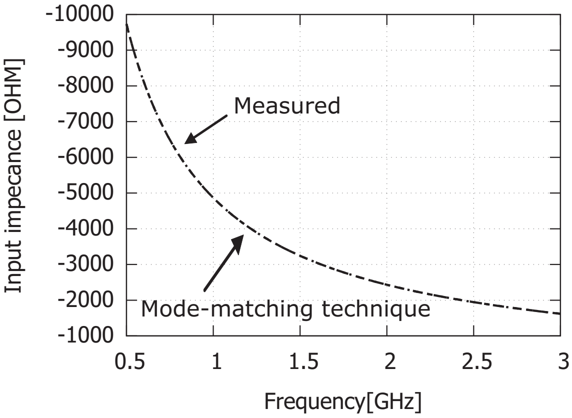

Figure 5. Measured input impedance after calibration (air, imaginary)

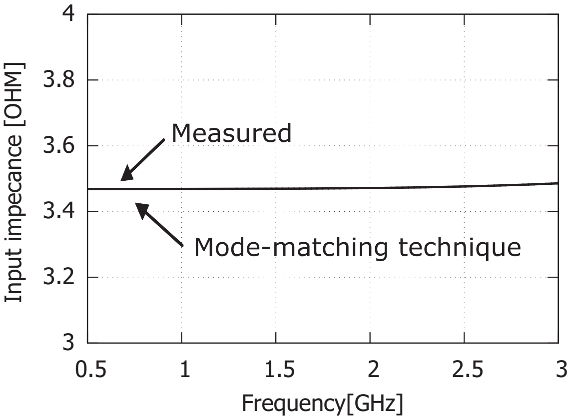

Figure 6. Measured input impedance after calibration (pure water, real)

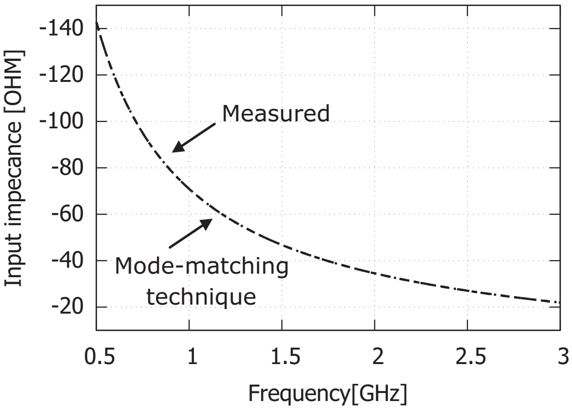

Figure 7 Measured input impedance after calibration (pure water, imaginary)

## 6. Conclusion

This study was performed to examine an $S_{11}$ calibration method without a basis of short termination with arbitrary impedance standards and three reference materials. For this purpose, an equation to determine the capacitance of the coaxial line section and the sample insertion section was derived. The input impedance calculated from equivalent circuit analysis with substitution of capacitance for each part was defined as the theoretical value of the input impedance required for calibration. $S_{11}$ measurement accuracy with calibration based on the input impedance of three sample insertion conditions was verified. The results numerically verified that $S_{11}$ can be calibrated using only three reference materials and no short termination condition. The accuracy of $S_{11}$ measurement after calibration based on this procedure with three sample insertion conditions using input impedance as an electrical standard was verified via actual measurement in the frequency range of 0.50 – 3.0 GHz. The results indicated that $S_{11}$ can be calibrated with only three reference materials and no short termination condition as an impedance standard. In future work, it will be necessary to determine the true permittivities of liquids to support $S_{11}$ calibration under this method.

## 7. References

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