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Sparse signal detection with spatial diversity using multi-rate sampling

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Abstract

We propose a spectrum sensing system consisting of K spatially distributed cognitive radios linked to a fusion centre. Each cognitive radio samples a wideband sparse multi-band signal, where each band comes from a distant device, at a random rate below Nyquist. Estimations of the signal support are done locally on undersampled data, then are processed at the data fusion center which finally outputs the support of the original signal. The proposed system is resilient and offers substantial gains in terms of detection by exploiting spatial diversity. The overall sampling rate remains moderate and the local sampling rates are far below that of Nyquist, relaxing the sampling rate constraints on ADCs, a traditional limiting factor in wideband sampling.

1 Introduction

Due to an ever-growing demand of radio resources, competition for access to the electromagnetic medium is stiff. Regulation bodies lack new frequency bands to fulfill users' requests for more spectrum resources. Yet, studies show that large portions of the spectrum – both licensed and unlicensed – are not used everywhere, at all times. This has led to the 1993 introduction of the Cognitive Radio (CR) paradigm by Mitola [1]. An aspect of the CR paradigm is opportunistic communications, where a CR can detect and exploit locally unused segments of the licensed spectrum, without causing interference to primary users of licensed spectrum band. To detect that a carrier is free of transmission at a given time, the CR performs spectrum sensing. This operation usually involves scanning a wide part of the spectrum, entailing high Nyquist sampling rates. This is problematic because there is a limit to analog-to-digital converters (ADCs) sampling rates. It also leads to dealing with large numbers of samples, which requires more powerful – and more power-hungry – computing resources at the receivers. Fortunately, such wideband signals are usually sparse in frequency, and can be sampled at rates far below that of Nyquist [2]. As such, our aim is to reliably detect the location of a sparse signal with as few samples as possible, using sub-Nyquist sampling techniques.

Previous works on sub-Nyquist sampling (sampling below the Nyquist rate) include Multi-Coset Sampling [3], a non-periodic sampling scheme, and the Modulated Wideband Converter (MWC) [4], where the signal is multiplied by periodic mixing functions before filtering and sampling. In both methods, the signal support is recovered by solving an optimisation problem. One limitation of these systems is that they rely on dedicated and rather complex hardware. They also include solving numerical systems subject to unstabilities. Other previous works explore the idea of a Multi-Rate Sampler (MRS) [5], which samples the input signal at different sub-Nyquist rates in different branches. The spectral components of the signal are then folded at different frequencies, except for the aliases located at the actual signal's bands, all located at the same frequencies. We extend this system to a distributed system where the different branches are different CRs, located at different places, linked to a fusion centre which agregates and processes sampled data (see Figure 1). The proposed system provides two main improvements, which are the beneficial use of spatial diversity and the simplification of the hardware present in multi-channel/multi-sampler devices.

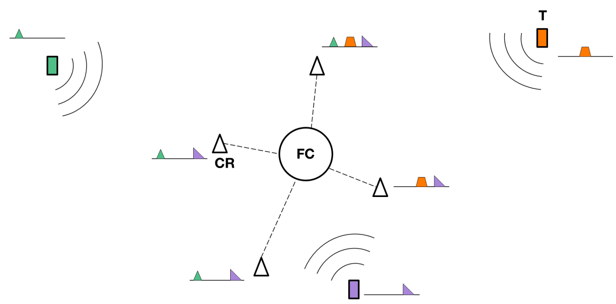

Figure 1. Distributed sub-Nyquist sampling system exhibiting spatial diversity. Bands are shown unaliased at reception for clarity. T is a *transmitting device* and FC is the *fusion centre*.

The remainder of the paper is organised as follows. Section 2 describes the proposed system model. Section 3 presents some results pertaining to our system. Section 4 explores the future refinements and improvements of the proposed system. Section 5 concludes the paper.

2 Proposed system model

Our proposed system is composed of M devices which transmit in the frequency range $[f_l, f_u]$ and K cognitive radios (CR) located at diverse locations linked in a star topology to a fusion centre with which the CRs exchange information. Our goal is to recover the support along the frequency axis of a continuous-time band-limited multi-band signal of bandwidth $BW = f_u - f_l$. A multi-band signal is a signal whose support is a finite union of intervals. The occupancy rate of a multi-band signal $s(t)$ is $\lambda = \frac{\mu(\text{supp}(s))}{BW}$ where $\mu(I)$ is the Lebesgue measure of I (for a union of distinct intervals, the Lebesgue measure is the sum of the lengths of the intervals) and $\text{supp}(s)$ is the support of signal $s(t)$. Our inputs for the system are K, f_l and f_u.

Let $s_j(t)$ be the continuous-time band-limited transmitted signal by the j-th device. The received signal at each CR is, when converted to baseband:

$$y_i(t) = \sum_{j=0}^{M-1} (s_j \star h_{ji})(t) + n_i(t) \tag{1}$$

where $h_{ji}(t)$ is the channel response between the j-th device and the i-th CR, and $n_i(t)$ is an additive white Gaussian noise (AWGN) of variance σ_i^2.

Upon reception at each CR, the signal is converted to baseband and goes through an analogue low-pass filter at frequency BW. Then, each CR samples the signal at a different rate $f_{Si}, 1 \leq i \leq K$. f_{Si} is below the Nyquist rate $f_{Nyq} = 2 \cdot (f_u - f_l)$. Let $\rho_i = \frac{f_{Nyq}}{f_{Si}} \geq 1$ be the reduction ratio between the Nyquist rate and the i-th CR's sampling rate. This process is known as undersampling. Next, samples are put together in batches. Batches have the same constant duration across all CRs, the duration δ of a batch being defined as $\delta = \frac{N_i}{f_{Si}}$, where N_i is the number of samples in any given batch of samples at the i-th CR. The samples in a batch form a sample vector $\boldsymbol{y_i}$.

Next, each CR performs an FFT on the last sample vector $\boldsymbol{y_i}$ to give $\boldsymbol{\hat{y}_i}$. As the sampling rate is below Nyquist, the FFT bins cover a ρ_i-times smaller span than the bandwidth BW of the original signal. Noise and all signal components are folded onto $[-f_{Si}/2, f_{Si}/2]$. The occupancy rate of the signal sampled at the i-th CR, λ_i is comprised between λ (maximum folding of bands of interest) and $\lambda \cdot \rho_i$ (no folding of bands of interest onto other bands of interest).

Classic spectrum sensing is then performed on each $\boldsymbol{\hat{y}_i}$. In this paper we consider energy detection for its simplicity. However determination on γ is not detailed. For each frequency bin, the squared value $|\hat{y}_i(f)|^2$ is compared to a threshold γ. If $|\hat{y}_i(f)|^2 \geq \gamma$, signal is said to be present and the output is 1. Else, no signal is detected and the output is 0.

Performing energy detection on each FFT bin results in a vector $\boldsymbol{l_i}$ composed of N_i binary values called the likelihoods (of presence) vector. They are now sent to the fusion centre (FC) for further processing and determination of the original frequency support.

The FC replicates the different likelihood vectors $\boldsymbol{l_i}$ over the whole bandwidth BW. The expanded likelihood vectors now have the same length $N = \lfloor BW \cdot \delta \rfloor$ and are concatenated into a $K \times N$ matrix L called the likelihood matrix. Each expanded likelihood vector is a row of L. The nonzero expanded likelihood vector values are the candidates for original signal support. The support is found by exploiting the fact that the candidates are located at different parts of the folded spectrum (due to having been sampled at different rates) except for the ones that are at the same location as in the original signal support [5]. For each column of L, the FC applies a voting rule which determines the support of the original signal. Some well-known voting rules are AND and OR (a logical AND/OR is applied to all values in a given column of L to give the output for the FFT bin corresponding to the column) [6].

3 Simulation results

For each device-to-CR link, we model an emission on the channel as a Bernoulli trial of parameter p_r. This means that any given transmitted band is received at any given receiver with probability p_r. This simple assumption can represent a wide variety of cases. For instance, a channel with favorable line-of-sight propagation where transmission powers are high corresponds to a high probability of reception p_r. A moderately favorable channel (some shadowing, noise, or a long distance between device and CR) will be represented with a intermediary p_r. In a difficult environment, p_r will be low. We suppose the delay from transmitter to receiver to be 0. The received signal (1) becomes:

$$y_i(t) = \sum_{j=0}^{M-1} b_{ij} \cdot s_j(t) \tag{2}$$

where

$$b_{ij} = \begin{cases} 1 & \text{with probability } p_r \\ 0 & \text{with probability } 1 - p_r. \end{cases} \tag{3}$$

We set $K = 10$ CRs and $M = 4$ transmitting devices. We focus on the 2.4 GHz ISM band (whose bandwidth BW is 83.5 MHz). We suppose that each device transmits on a randomly chosen 500 kHz band at all times. The occupancy rate is thus $\lambda = \frac{4 \times 0.5}{83.5} \approx 2.4\%$.

The sampling rates f_{Si} are drawn from a uniform distribution $U(\frac{f_{Nyq}}{12}, \frac{f_{Nyq}}{8})$, which means that the reduction rate ρ_i ranges from 8 to 12. The mean sampling rate is $\frac{f_{Nyq}}{10}$ and on average, the overall sampling rate (defined as the geometric sum of all CRs' sampling rates divided by the number of CRs) is f_{Nyq}. This is the Nyquist regime. This is about 40 times the Landau rate, *i. e.* the lower-bound for sampling without loss of information. The batch duration δ is set to 1 ms. For a given batch, the samples of each CR form the sample vector $\boldsymbol{y_i}$ of length $N_i = \delta \cdot f_{Si}$ on which the FFT is applied.

The form of the received signal (2) renders the energy detection step trivial as the threshold γ can be set to an arbitrarily low non-zero value. Conceptually, difficulties pertaining to energy detection techniques arise when the SNR of the received signal is low. We choose to model the SNR within the aforementioned probability of reception p_r.

Once energy detection is performed on each $\hat{\boldsymbol{y}}_i$, the different likelihood vectors $\boldsymbol{l_i}$ are sent to the fusion center, expanded, padded if necessary, and concatenated in the L matrix. The voting rules used are of type "at least $K - \kappa$" [6]:

$$\Lambda[n] = \begin{cases} 1 & \text{if } \sum_{i=0}^{K-1} L_{in} \geq K - \kappa \\ 0 & \text{otherwise} \end{cases} ,0 \leq n \leq N-1 \quad (4)$$

where Λ is the array of the combined likelihoods of presence. The support of the original signal s is then the support of Λ.

To evaluate our system, the nonzero values of Λ are compared to the frequency support of $\boldsymbol{s}$. The proportion of detected bands, or the detection rate, $P_D = \frac{\mu(\text{supp}(\Lambda \cap s))}{\mu(\text{supp}(s))}$ and proportion of false alarms, or the false alarm rate, $P_{FA} = \frac{\mu(\text{supp}(\Lambda - s))}{\mu(\text{supp}(s))}$ are computed. The varying parameters are p_r, κ and ρ_i. Each experiment has been averaged over 1000 cycles.

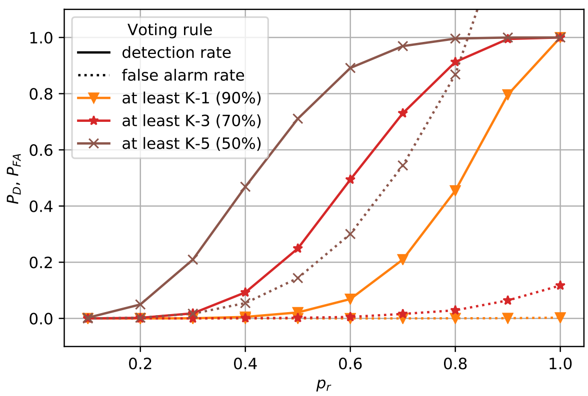

Figure 2. Proportion of detected bands P_D and of false alarms P_{FA} versus probability of reception of a given band at a given CR p_r under several voting rules in the Nyquist regime ($\rho = 10$, $K = 10$).

Figure 2 displays the detection rate P_D and the false alarm rate P_{FA} as a function of the probability of reception of each band at each CR p_r varying from 0.1 to 1 for different voting rules. The first observation is that in the case of an ideal reception of each band at each CR ($p_r = 1$), the system correctly detects the support ($P_D \approx 1.0$) with, for some voting rules, a rate of false alarms as low as 0. Note that this case corresponds to an absence of spatial diversity, since all CRs receive the same bands.

When p_r is below 1, the different CRs receive different bands and spatial diversity comes into play. For a non-spatially diverse system, we necessarily have $P_D \leq p_r$. The proposed system goes beyond this limit. Consider $p_r = 0.8$ for voting rule "at least 70%" in Figure 2: 92.2% of bands are correctly detected (a 15% gain over the best non-spatially diverse systems), while the probability of false alarm is kept at the reasonable level of 3.5%. Spatial diversity is exploited and provides gains in adverse environments, compared to non-spatially diverse systems. For a proportion of false alarms of about 5%, the order of magnitude of the gains is 10%.

The system can also work in the "true" sub-Nyquist regime (overall average sampling rate of the system below f_{Nyq}), but the device-to-CR link has to be excellent. Under good propagation conditions, reducing the sampling rate has little consequences on the proportion of detected bands P_D, but the proportion of false alarms P_{FA} dramatically increases. Indeed, reducing the sampling rate causes the narrower sub-sampled bandwidths to be replicated more, thus proposing more candidates for the support, which in turn increases the false alarm rate. However, if one is not overly concerned about the false alarm rate, reducing the sampling rate far below Nyquist is plausible — up to the point where the sub-Nyquist spectra $\hat{\boldsymbol{y}}_i$ are completely full of aliases. Figure 3 suggests that for $P_D = 0.95$ (under $p_r = 0.95$), the local sampling rates can be reduced to $\frac{f_{Nyq}}{16}$ (overall sampling rate of $\frac{K}{\rho} f_{Nyq} = 0.625 f_{Nyq}$, a gain of 37% over an overall sampling rate equal to f_{Nyq}) while keeping the false alarm rate P_{FA} below 10%. Note that to keep P_{FA} low, the voting rule has to be fairly strict. Since candidates are numerous, there needs to be a large consensus across all CRs. This explains why the link quality needs to be excellent: if all CRs do not receive the same bands, a wide consensus is hard, not to say impossible, to reach.

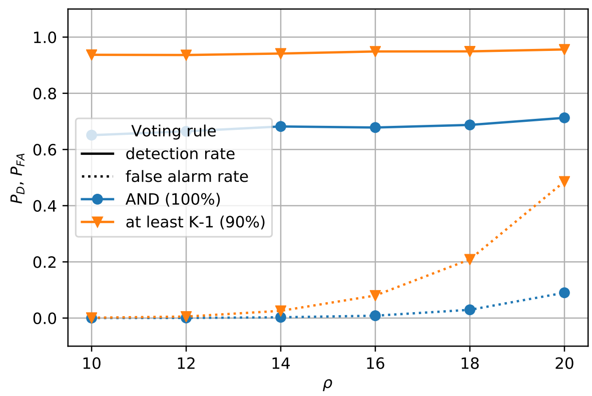

Figure 3. Proportion of detected bands P_D and of false alarms P_{FA} versus average sampling rate reduction ρ, with an excellent link ($p_r = 0.95$).

Conversely, if we allow for an overall sampling rate above that of Nyquist (while remaining below Nyquist at each CR), the resilience of the system is significantly improved. Consider Figure 4: for an overall sampling rate of $2.5 f_{Nyq}$, the detection rate under voting rule "at least $K-7$" is 85% for $p_r = 0.3$ (false alarm rate: 5.7%) which represents a gain of 114% over non-spatially diverse systems. Indeed, high sampling rates means little spectrum folding. The candidates have a higher chance of being the actual signal frequencies. The requirements on the consensus needed to keep P_{FA} low are more lax, meaning that fewer CRs have to receive the bands for a consensus to be reached, which in turn allows for a lower p_r.

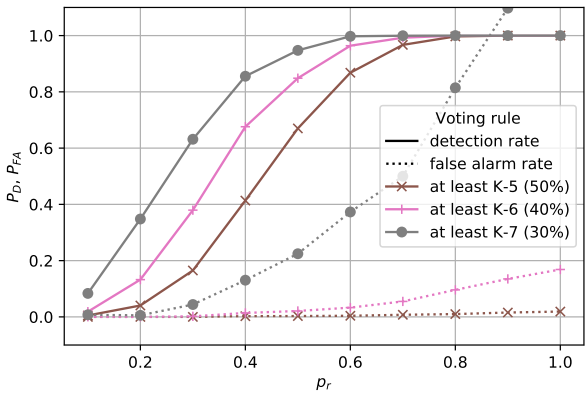

Figure 4. Proportion of detected bands P_D and of false alarms P_{FA} versus probability of reception p_r in the above-Nyquist regime ($\rho = 4$, $K = 10$).

4 Future improvements

Future improvements will be along three areas: modeling, signal detection, and fusion.

The channel model is extremely basic and refining the model would be beneficial in understanding the potential of the proposed system.

Energy detection-based systems are simple but their performance is strongly degraded in low SNRs. Sub-Nyquist sampling systems suffers from noise folding [7] and thus are particularly vulnerable to this phenomenon. Cyclostationarity-based approaches provide better results in low SNRs at the expense of an increased complexity. A question of utmost interest is whether cyclostationarity-based spectrum sensing could be used in distributed multi-rate sampling systems to detect signal presence in the folded spectrum.

A current limitation of the proposed system is the choice of the voting rule. There is no "one-fits-all": the voting rule needs to be carefully chosen for each case. A strict voting rule will result in a low detection rate; a lax voting rule will result in a high false alarm rate. Further works will focus on precisely identifiying of the criteria that impact the choice of the best voting rule for a given situation. Other fusion methods will be considered, among which AI-based ones.

5 Conclusion

We proposed a new spectrum sensing system based on spatially distributed cognitive radios sampling at random uniform sub-Nyquist rates, thus exploiting aliasing and spatial diversity. Interesting trade-offs are put forward and explored. The hardware complexity of the samplers is reduced compared to pre-existing solutions. In particular, constraints on the ADC sampling rates are relaxed. The proposed system works in sub-Nyquist, Nyquist and above-Nyquist regimes. In the sub-Nyquist regime, the system provides a significant reduction of the overall sampling rate (up to 40% from the Nyquist rate for a sparse signal), but the link quality has to be excellent. In the Nyquist regime, spatial diversity is made use of to provide detection gains of about 10%. In the above-Nyquist regime, detection gains are far higher, making the system more resilient in case of poor link qualities. Further refinements are needed to get a deeper and fuller understanding.

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