URSI AP-RASC 2019, New Delhi, India; 09 - 15 March 2019

Maximizing absorption for dielectric scatterers

Ari Sihvola* and Dimitrios C. Tzarouchis
Aalto University School of Electrical Engineering, Department of Electronics and Nanoengineering, Espoo, Finland
e-mail: ari.sihvola@aalto.fi, dimitrios.tzarouchis@aalto.fi

Abstract

The absorption characteristics of a dielectric, isotropic sphere in the plasmonic regime are analyzed. Relations are derived for the position of the maximum absorption efficiency for the first electric dipole resonance mode. The dissipation results are connected with the natural frequencies of the sphere as well as to the circuit-inspired matching analogy for optimum absorption.

1 Introduction

The scattering by an object is a central problem in electromagnetics. How are electric and magnetic fields perturbed when a plane wave hits an irregularity? Such inhomogeneity can be in the form of a sharp boundary with a sudden change in the constitutive parameters, or a gradual densing of the background environment.

The radiating power of the incident field—the Poynting vector of the wave—gets decreased due to the scattering by the object, and also owing to the absorption by the dielectric losses that the incident wave incites in the scattering volume. In this presentation, we will focus on the absorption aspects of scatterers, and in particular the way how the optimal (maximum) losses of a fixed-parameter object can be acquired for a given setting of the wave–scatterer constellation.

We can benchmark the scattering, absorption, and extinction properties of a scatterer through the analysis of the electromagnetic interaction of a spherical (lossy) sphere with an incident plane wave. The quantifying factors are the efficiencies (relative cross sections) of the particle as it interacts with the incoming electromagnetic wave.

2 Mie scattering, absorption, extinction

The scattering problem (Figure 1) shows a propagating plane wave encountering an object (here isotropic, homogeneous, dielectric sphere). The incident fields become scattered, absorbed, and diffracted. For a sphere, the scattering perturbation can be calculated analytically using the Lorenz–Mie analysis [1, 2]. A dissipative object also absorbs energy, in addition to scattering. The sum of these two effects is the extinction. These effects are measured by the scattering, absorption, and extinction cross sections, and in the normalized form as the efficiencies [3].

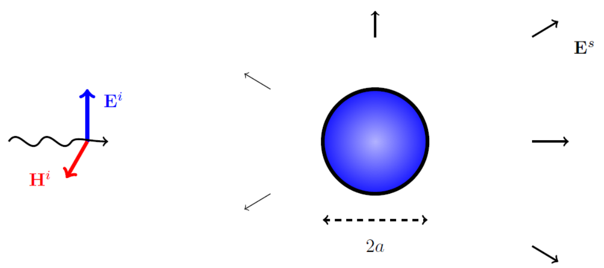

Figure 1. An incident electromagnetic wave hits a dielectric sphere. Electromagnetic energy is scattered and absorbed.

The efficiencies depend on the relative permittivity ε and the size of the scatterer. The absolute size is normalized in the following way:

$$x = ka \tag{1}$$

where k is the free-space wave number and a the radius of the sphere.

The (dimensionless) scattering efficiency is the total scattering cross section normalized by the geometrical cross section of the sphere, and the three efficiencies can be calculated from the series

$$Q_{\rm sca} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2+|b_n|^2\right) \tag{2}$$

$$Q_{\rm ext} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\mathrm{Re}\{a_n+b_n\} \tag{3}$$

$$Q_{\rm abs} = Q_{\rm ext} - Q_{\rm sca} \tag{4}$$

where the Mie coefficients a_n (electric multipoles) and b_n (magnetic multipoles) contain spherical Bessel and Hankel functions and their derivatives [3]. For a lossless sphere (ε is real), absorption efficiency $Q_{\rm abs}$ vanishes.

Figure 2 shows the strong variation of the extinction (scattering) efficiency of a lossless sphere as function of its relative permittivity and size parameter. Note the strong plasmonic resonances (due to electric multipoles) between

$-5 < \varepsilon < 0$, and the curving ranges of the dielectric resonances (due to electric and magnetic multipoles) for large positive values of ε.

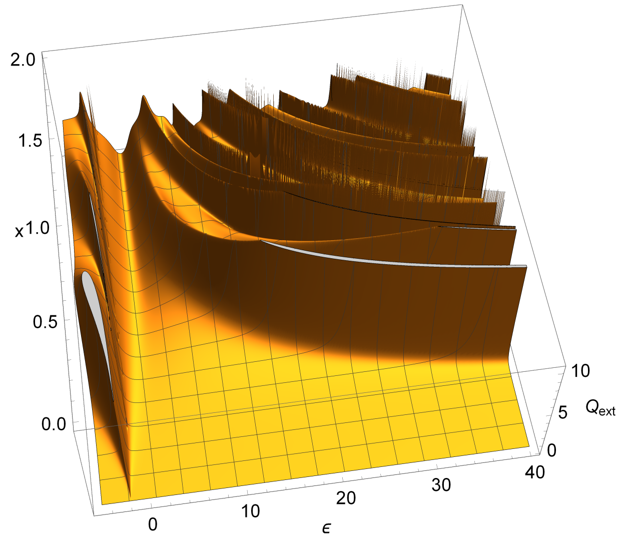

Figure 2. An attempt to visualize the behavior of the extinction efficiency of a lossless dielectric sphere as function of its size parameter x and relative permittivity ε. For the plasmonic region ($\varepsilon < 0$), the electric multipole magnitudes grow without limit as the size x decreases. The dielectric resonances curve to large positive permittivity values as the size decreases. Note that the amplitude roof in the picture is $Q_{\text{ext}} = 10$ and higher-order resonances are too sharp to be resolved.

3 Electric dipole resonance and maximum absorption

Focusing on the plasmonic resonances, and in particular, the resonance due to the electric dipole (Mie coefficient a_1), it is known from electrostatic reasoning that the "Mossotti catastrophe" for subwavelength spheres takes place at $\varepsilon = -2$. However, as the size of the sphere increases, Padé approximants of a_1 [4] show that the resonance (red)shifts with the following dependence on the size parameter x:

$$\varepsilon_{\text{res}} = -2 - \frac{12}{5}x^2 \tag{5}$$

Let us next raise the question: assuming a dissipative material (complex permittivity $\varepsilon = \varepsilon' - j\varepsilon''$), what is the maximum absorption for a sphere with a given size parameter?

To answer this, numerical exploration is needed. Assuming small to moderate losses, the position of the absorption peak is very accurately the same as for scattering (given by Equation (5)). Numerically it seems that the maximum of Q_{abs} for the electric dipole resonance is approximately

$$Q_{\text{abs,max}} \approx 12x/\varepsilon'' \tag{6}$$

which holds quite well for size parameter values below $x \approx 0.1$ and large enough loss values. However, if ε'' is very small, the behavior reverses itself, and the relation becomes linear with ε'' but to the inverse fifth power of x:

$$Q_{\text{abs,max}} \approx 3\varepsilon'' x^{-5} \tag{7}$$

In fact, the result (6) is rather counter-intuitive: with decreasing loss factor the maximum absorption increases! Anyway, Figure 3 shows how well these approximations work when the imaginary part of the permittivity changes from 10^{-6} to 1, for a sphere with size parameter $x = 0.1$.

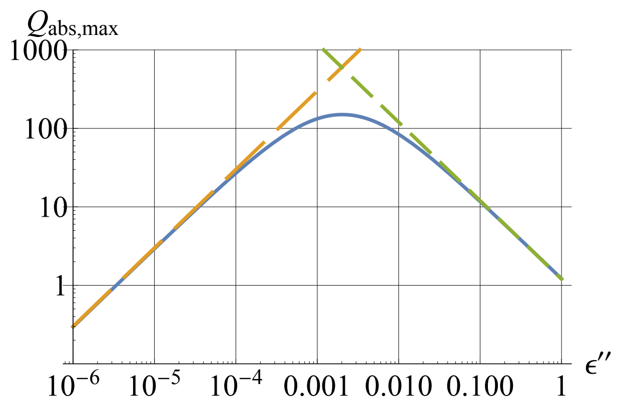

Figure 3. The maximum of absorption efficiency (the dipole peak at around -2.024) for a dielectric sphere of size parameter $x = 0.1$ with varying intrinsic losses (solid blue line). Note how the behavior changes from that of Equation (7) (dashed orange) to (6) (short-dashed green) when the loss factor increases.

The achievable maximum for the absorption efficiency can also be looked from the point of view of varying the size. Figure 4 shows $Q_{\text{abs,max}}$ as function of size parameter and with fixed loss factor. Combining both views, Figure 5 shows the maximum as function of both parameters.

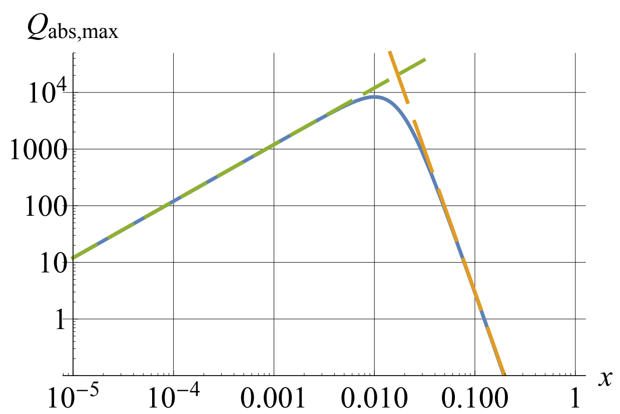

Figure 4. The same as in Figure 3, for fixed loss ($\varepsilon'' = 10^{-5}$), as function of the size parameter.

What, then, is the maximum of the maxima? As can be seen, depending on which parameter is kept constant, the maximum takes place at a different position. Furthermore, the magnitude of $Q_{\text{abs,max}}$ is also different:

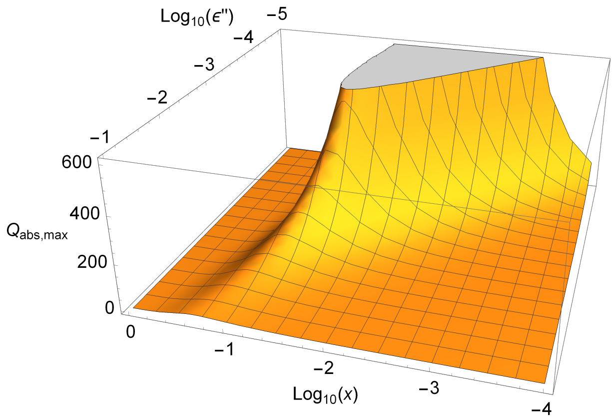

Figure 5. The maximum absorption efficiency (computed at permittivity $-2-2.4x^2-\mathrm{j}\,\varepsilon''$ which corresponds to the maximum) as function of size parameter x and loss ε''.

For fixed size parameter x:

$$Q_{\mathrm{abs,max}} = 1.5x^{-2} \quad \text{at} \quad \varepsilon'' = 2x^3 \tag{8}$$

and for fixed imaginary part of the permittivity ε'':

$$Q_{\mathrm{abs,max}} = 18\cdot(10\varepsilon'')^{-2/3} \quad \text{at} \quad x = (\varepsilon''/10)^{1/3} \tag{9}$$

4 Discussion

These results concerning the maximum absorption efficiency can be connected with the concept of radiative reaction. This is a damping mechanism accounting for scattering loss that can be seen to arise from the Padé expansion of the Mie coefficients [4], and also from the circuit-inspired analysis of the scattering process [5]. Indeed, as shown in [4], the maximum absorption for the first plasmonic resonance follows the relation

$$\varepsilon'' \approx 2x^3 \tag{10}$$

as in Equation (8) above. The justification for the result (10) followed a matching principle between the radiative damping and material dissipation.

Another point of view for looking at the matching principle is to search for a pole for the a_1 coefficient, in other words the *natural frequencies*. Figure 6 shows the absolute value of the a_1 Mie coefficient for $x = 0.1$ in the region where ε'' is negative (active medium). The pole takes place for $\varepsilon \approx -2.024 - \mathrm{j}0.002$ which matches the earlier conditions (5), (8), and (10). An interesting observation is the resonant behavior of the three efficiencies around this pole position, displayed in Figure 7.

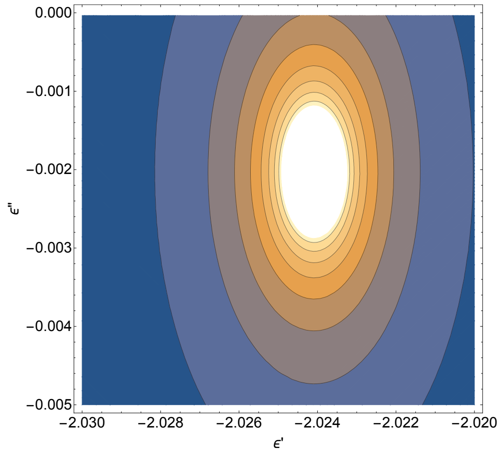

Figure 6. Contour plot of absolute value of the electric dipole coefficient $|a_1|$ around the first pole (note that $\varepsilon'' < 0$ corresponds to active medium). Size parameter $x = 0.1$.

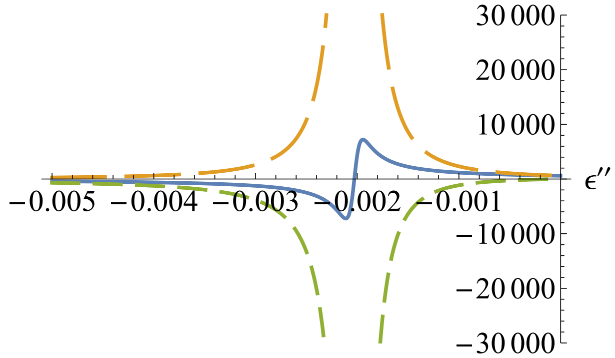

Figure 7. The extinction (solid blue), scattering (dashed orange), and absorption (short-dashed green) efficiencies of a plasmonic sphere for $\varepsilon' = -2.024$ as function of the imaginary part of the permittivity ε'' ($\varepsilon'' < 0$ corresponds to active medium). The size parameter is $x = 0.1$. Note the negative absorption efficiency (negative dissipation), but a positive scattering efficiency which together shape a "double resonance" for the extinction efficiency.

References

[1] L. Lorenz, “Lysbevægelser i og uden for en af plane Lysbolger belyst Kugle,” *Kongelige Danske Videnskabernes Selskabs Skrifter*, **6**, pp. 2–62, 1890.

[2] G. Mie, “Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen", *Annalen der Physik*, **25**, pp. 377–445, 1908.

[3] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, Wiley: New York, 1983.

[4] D.C. Tzarouchis, P. Ylä-Oijala, and A. Sihvola, “Unveiling the scattering behavior of small spheres,” *Physical Review B,* **94**, 140301(R), 2016.

[5] D.C. Tzarouchis, P. Ylä-Oijala, and A. Sihvola, “Resonant scattering characteristics of homogeneous dielectric sphere,” *IEEE Transactions on Antennas and Propagation,* **65**, 6, pp. 3184–3191, June 2017.