



## **The Mixed Finite-Element Time Domain Method for Overcoming Low-Frequency Breakdown**

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The finite-element time domain (FETD) method has been widely applied to electromagnetic engineering applications, including microwave circuits, electromagnetic scattering analysis, integrated circuits and package design. It is advantageous over the finite difference time domain method for complex structures because of its modeling versatility by utilizing an unstructured mesh.

For electrically fine but complex structures, however, the explicit time integration in the FETD method will incur a very large number of time steps because of the stability condition. In this case, the temporal discretization is better replaced by an implicit time integration method such as the implicit Newmark-beta method to achieve an unconditionally stable time stepping solution (e.g., S. D. Gedney and U. Navsariwala, “An unconditionally stable finite element time-domain solution of the vector wave equation,” *IEEE Microw. Guided Wave Lett.*, vol. 5, no. 10, pp. 332–334, Oct. 1995). Nevertheless, when solving complex electrically small problems, the spatial discretization is far smaller than the wavelength, the implicit FETD matrix becomes singular and numerical instability occurs. This issue is known as the low-frequency breakdown, and is attributed to the increasingly decoupling between the electric and magnetic fields as the frequency approaches zero. In the frequency domain, this problem is manifested by the presence of the “non-physical DC modes” in the frequency-domain finite element method, and has been addressed by the mixed finite element method (F. Kikuchi, “Mixed and penalty formulations for finite element analysis of an eigenvalue problem in electromagnetism,” *Comput. Methods Appl. Mech. Eng.*, vol. 64, no. 1–3, pp. 509–521, 1987) and other methods.

In this work, we have developed a mixed finite-element time domain (FETD) method to eliminate the low-frequency breakdown phenomenon in the conventional finite element method for electrically fine structures. By incorporating Gauss’s law and current continuity equation into the FETD method, this mixed FETD method uses the implicit Newmark-beta algorithm for time integration to reduce the number of time steps. We will show several numerical examples to demonstrate that this mixed FETD method does not suffer from the low frequency breakdown; for extremely fine spatial discretization where the traditional FETD fails, this mixed FETD method remains stable, highly accurate and efficient.