Cross-correlation index and multiple-access performance of spreading codes for a wireless system

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***Abstract*— Cross-correlation index is a measure of multi-user interference combating a spread-spectrum signal. Its magnitude (relative to auto-correlation index of a reference user) is a critical parameter affecting the system performance. In this work, a mathematical model is presented for cross-correlation index of spreading codes, and thereafter the system loading capacity and bit-error-rate performance. The model was tested for different sets of Gold codes. Results show that 63-chip Gold codes have the capacity to support a maximum of about four users, above which bit-error-rate increases rapidly, ultimately resulting in emergence of error floor. The point at which the cross-correlation index equals to auto-correlation index marks the turning point around which the system performance revolves. The outcome was found to be in close agreement with results of simulations obtained for the system performance.**

Keywords*— wireless communication, channel coding, spread-spectrum system, cross-correlation index, multi-user interference, loading capacity, Gold codes, bit-error-rate, error floor.

I. INTRODUCTION

Spread spectrum communication is a tried and tested technique developed originally for military applications but later became a significant worldwide technique in the larger society for commercial and civilian applications. It has been an important communication technique in wireless telephony as well as satellite navigation. The popularity of spread spectrum technique in mobile telephony has been diminishing in recent times, but it remains the primary modality used for radio communication in the global positioning system (GPS). The technique is also used in other areas of application [1-3] including tomography imaging [4], digital terrestrial television [5], anti-jam underwater communication [6] and in encryption of medical images [7]. There exist other advanced hybrids of spread-spectrum systems involving some channel enhancement techniques like multicarrier transmission (or orthogonal frequency-division multiplexing – OFDM), multiple-input multiple-output (MIMO) antenna system, and different variants of space-time coding, but these are not the focus of this paper.

Performance of any spread-spectrum system is affected by properties of spreading codes. It is well known that for any particular code, there is a maximum number of users that would be tolerated and that this number would be well below nominal full load. However, the proportion of the full code set that are actually available for use before the onset of system saturation is not well known. This work seeks to address this problem. In this work, the author presents a simple but effective means for determining performance limits of spreading codes (or sequences). The model was tested for different sets of Gold codes, and the outcome was found to agree with results of software simulations for the system.

This work involves the development of a concept of cross-correlation index and its use for the determination of the system loading capacity. The outcome shows that cross-correlation index is very effective for determining upper limit of simultaneous users that a spread-spectrum system can support in practice with respect to its spreading sequences. If cross-correlation index is much less than auto-correlation index, then there is little interference, and the user signal survives. On the other hand, if cross-correlation index is much greater than auto-correlation index, then there is significant interference, and the user signal becomes swamped.

This paper is part of a larger study. Related work have been presented in part in conferences [8-14] and published in journal papers [15-17]. The rest of this paper is organized as follows. Mathematical model for the system is developed in Section II. Research methods used for the work are outlined in Section III. Results are presented in Section IV. Finally, this paper concludes with a summary in Section V.

II. SYSTEM MODEL

Consider a direct-sequence spread-spectrum system (DSSS). Spread spectrum signal transmitted by a user k can be expressed as

$$s_k(t) = Ac_k(t)b_k(t)\cos(\omega_c t + \theta_k)\,, \tag{1}$$

where $b_k(t)$ is the user binary data, $c_k(t)$ is the spreading code and ω_c is the carrier frequency. The spreading code $c_k(t)$ for the user can be denoted as

$$c_k(t) = \sum_{i=1}^{N} c_k^i P_c(t - iT_c), \qquad c_k^i \in \{-1, +1\}, \tag{2}$$

where N is length of the code, and P_c is a rectangular pulse having a duration T_c. Let the wireless communication channel be represented by multiple paths having a real positive gain β_l, propagation delay τ_l and phase shift γ_l, where l is the path index. The channel impulse response $h_k(t)$ for L independent paths can be modelled as

$$h_k(t) = \sum_{l=1}^{L} \beta_{kl} e^{j\gamma_{kl}} \delta(t - \tau_{kl})\,. \tag{3}$$

At the receiving end, the received signal $r_k(t)$ for the user is obtained by convolving $s_k(t)$ with $h_k(t)$:

$$r_k(t) = \int_{-\infty}^{\infty} s_k(\tau) h_k(t - \tau) d\tau \tag{4}$$

Substituting the expressions for $s_k(t)$ and $h_k(t)$ into this integral, and using relevant properties of the Dirac delta function $\delta(t)$ gives (5). For a multi-user system comprising K users, the received signal $r(t)$ is a linear superposition of the signals for the users, and is given by (6). Let user-1 be the reference user. Assuming coherent demodulation, the receiver output $z(m)$ for

m^{th} bit during the bit duration T_b of the user is given by (7), where n(t) is receiver noise. Substituting for $r(t)$ gives (8). Define k = 1 for the reference user. Using this in (8) gives (9), where z_{11} represents the desired signal for the reference user, z_{12} is interference term, and z_{13} is noise term.

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$$r_k(t) = \sum_{l=1}^{L} A\beta_{kl} e^{j\gamma_{kl}} c_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \cos(\omega_c t - \theta_{kl}) \tag{5}$$

$$r(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} A\beta_{kl} e^{j\gamma_{kl}} c_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \cos(\omega_c t - \theta_{kl}) \tag{6}$$

$$z_1(m) = \int_{mT_b}^{(m+1)T_b} \big(r(t) + n(t)\big) c_1(\mathrm{t}-\tau_{kl}) \cos(\omega_c t - \theta_{kl})\, dt \tag{7}$$

$$z_1(m) = \int_{mT_b}^{(m+1)T_b} \left\{\sum_{k=1}^{K}\sum_{l=1}^{L} A\beta_{kl} e^{j\gamma_{kl}} c_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \cos(\omega_c t - \theta_{kl})\right\} c_1(\mathrm{t}-\tau_{kl}) \cos(\omega_c t - \theta_{kl}) dt + \int_{mT_b}^{(m+1)T_b} n(t)\, c_1(\mathrm{t}-\tau_{kl}) \cos(\omega_c t - \theta_{kl})\, dt \tag{8}$$

$$\begin{aligned} z_1(m) = & \int_{mT_b}^{(m+1)T_b} \left\{\sum_{l=1}^{L} A\beta_{1l} e^{j\gamma_{1l}} c_1(t-\tau_{1l}) b_1(t-\tau_{1l}) \cos(\omega_c t - \theta_{1l})\right\} c_1(\mathrm{t}-\tau_{1l}) \cos(\omega_c t - \theta_{1l}) dt \\ & + \int_{mT_b}^{(m+1)T_b} \left\{\sum_{k=2}^{K}\sum_{l=1}^{L} A\beta_{kl} e^{j\gamma_{kl}} c_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \cos(\omega_c t - \theta_{kl})\right\} c_1(\mathrm{t}-\tau_{1l}) \cos(\omega_c t - \theta_{kl}) dt \\ & + \int_{mT_b}^{(m+1)T_b} n(t) c_1(\mathrm{t}-\tau_{kl}) \cos(\omega_c t - \theta_{kl})\, dt = z_{11} + z_{12} + z_{13} \end{aligned} \tag{9}$$

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Consider the mathematical expression for z_{11} of (9). In this expression, signal recovery at the receiver involves matrix multiplication of code sequences, such that the term $c_1^2(t-\tau_{1l})$ represents the inner product of the first user-assigned code c_1(t) with itself. Now, the user's code is a sequence of chip elements (+1,-1). Using this, we can show that $c_1^2(t-\tau_{1l})$ is equal to the length N of the code. Also, let ω_c be chosen such that $f_c = \frac{n}{T_b}$, $n \in \boldsymbol{Z}$. By substituting these, we can show that the expression for z_{11} reduces to:

$$z_{11} = \frac{AN}{2}\sum_{l=1}^{L} \beta_{1l} e^{j\gamma_{1l}} \left\{\int_{mT_b}^{(m+1)T_b} b_1(t-\tau_{1l})\, dt\right\} \tag{10}$$

If $b_1(t-\tau_{1l}) = 1$ in the period T_b, then

$$z_{11} = \frac{AN}{2}\sum_{l=1}^{L} \beta_{1l} e^{j\gamma_{1l}}\, T_b \tag{11}$$

For a Gaussian channel, number of paths L = 1. Consequently, there is no multipath fading. Therefore, $\beta_{1l} = 1$ and $\gamma_{1l} = 0$. Hence, we have

$$z_{11} = \frac{ANT_b}{2}. \tag{12}$$

Similarly, we can show that the interference term z_{12} reduces to:

$$z_{12} = \pm\frac{A}{2}\sum_{k=2}^{K}\int_{mT_b}^{(m+1)T_b} d_{1k} dt\,, \tag{13}$$

where $d_{1k} = c_1(\mathrm{t}-\tau_{1l}) c_k(t-\tau_{kl})$. Consider the noise term z_{13}:

$$z_{13} = \int_{mT_b}^{(m+1)T_b} n(t) c_1(\mathrm{t}-\tau_{kl}) \cos(\omega_c t - \theta_{kl})\, dt \tag{14}$$

In this equation, it can be seen that the spreading code $c_1(\mathrm{t}-\tau_{kl})$ multiplies the receiver noise n(t). Effect of this is to spread out the noise. Hence the noise is suppressed greatly. Bit-error-rate (BER) or bit-error-probability P_e can now be calculated using:

$$P_e = Q\left(\sqrt{2\frac{E_b}{N_o + I_o}}\right) = Q\left(\sqrt{2\frac{|z_{11}|^2}{N_o + |z_{12}|^2}}\right)$$

$$= Q\left(\sqrt{2\frac{\left|\frac{ANT_b}{2}\right|^2}{N_o + \left|\frac{A}{2}\sum_{k=2}^{K}\int_{mT_b}^{(m+1)T_b} d_{1k}\, dt\right|^2}}\right)$$

$$= Q\left(\sqrt{2\frac{\left|\frac{ANT_b}{2}\right|^2}{N_o + \left|\frac{A}{2}\sum_{k=2}^{K}\int_{mT_b}^{(m+1)T_b} d_{1k}\, dt\right|^2}}\right) \quad (15)$$

where I_o is interference term. This expression shows that BER depends on code length N. Since the q-function Q is a monotonically decreasing function, it implies that as N increases, BER decreases. That is to say, longer codes give better BER. The equation also shows that BER depends on the cross-correlation d_{1k}. This implies that BER for a multi-user system is affected by the cross-correlation. Define cross-correlation index D_{1k} as:

$$D_{1k} = \pm\frac{A}{2N}\sum_{k=2}^{K}\int_{mT_b}^{(m+1)T_b} d_{1k}\, dt \quad (16)$$

Gold codes are a type of pseudo-noise (PN) sequences derived from combination of certain pairs of m-sequences called *preferred sequences*, implemented using linear feedback shift registers. A set of Gold code sequences consists of $2^n - 1$ sequences, each one with a period of $2^n - 1$. A Gold code has a period $N = 2^n - 1$, where n is the length of the shift register. Gold codes exhibit triple-valued cross-correlation function [18-20] with values {-1, -t(n), t(n)-2}by , where

$$t(n) = \begin{cases} 2^{(n+1)/2} + 1, & n \text{ odd} \\ 2^{(n+2)/2} + 1, & n \text{ even} \end{cases} \quad (17)$$

III. Methodology

Graphs of cross-correlation index for the codes were generated by direct software implementation of the mathematical model that was developed in the previous section. The model was tested using a set of 63-chip Gold codes. The outcome of this was compared with results of simulations. For the software simulations, various sets of Gold codes were generated from appropriate combinations of preferred pairs of m-sequences, using linear feedback shift registers (Table 1). The software simulations were carried out for the transmission of typically about one million random QPSK symbols for uncoded as well as coded data transmission, through a Gaussian channel of zero mean and unit variance Gaussian noise. Following signal recovery, original transmitted data was compared with recovered data for the determination of system (BER) for the various sets of codes.

TABLE I. Generator Polynomials for the Gold Codes

n	$P_i^n(x)$	*Generator polynomial	N
6	$P_1^6(x)$	$x^6 + x^5 + 1$	63
	$P_2^6(x)$	$x^6 + x^5 + x^4 + x + 1$	
8	$P_1^8(x)$	$x^8 + x^7 + x^6 + x + 1$	255
	$P_2^8(x)$	$x^8 + x^7 + x^5 + x^3 + 1$	
10	$P_1^{10}(x)$	$x^{10} + x^7 + 1$	1023
	$P_2^{10}(x)$	$x^{10} + x^9 + x^8 + x^5 + 1$	
12	$P_1^{12}(x)$	$x^{12} + x^{11} + x^{10} + x^4 + 1$	4095
	$P_2^{12}(x)$	$x^{12} + x^{11} + x^{10} + x^2 + 1$	

*$P_1^n(x)$ and $P_2^n(x)$ are the generator polynomials of the preferred pair used for obtaining corresponding set of Gold codes of degree n.

IV. Results and Discussion

Cross-correlation index for spreading codes is an important parameter affecting the system performance. Figure 1 shows the graph of cross-correlation index D_{1k} for a set of 63-chip Gold codes. This graph was generated by direct implementation of (16) at zero lag. For the purpose of comparison, the value of auto-correlation index D_{11} was also obtained and plotted. Both quantities were normalised by code length. This figure shows that D_{1k} increases linearly with number of users, reaching a peak value of 15 at maximum loading (63 users) for the code. Figure 1 also shows that $D_{1k} = D_{11}$ for four users. Therefore, the system is expected to saturate when number of users approaches four. These predictions agree with results obtained for the system performance. This is explained as follows.

Consider a family of BER curves for a DSSS system for a family of Gold codes. If there is no system saturation, horizontal spacing between adjacent (neighbouring) performance curves for the various code lengths are expected to remain the same. Figure 2(a) and (b) illustrate this for all the code lengths under consideration. Now, if any of the codes undergoes system saturation, it will deviate from this normal behaviour, and will drift away from the other characteristic curves. On Figure 2(c), this anomalous behaviour can be observed for code $N = 63$. The performance curve for this code can be seen to drift away gradually from the others. This is an evidence of system saturation.

It is interesting to note that this figure (i.e. Figure 2(c)) represents the performance for four users. Thus we see that the anomalous behaviour agrees exactly with what the mathematical model (cross-correlation index) predicts for this code. The model predicts that system saturation should set in after four users. This situation becomes more obvious when number of users increases to five and above. This can be seen on Figures 2(d) to (f). These figures show that when number of users become increasingly higher than 4, system performance degrades rapidly, ultimately resulting in emergence of error floor. Going back to Figure 1, we see that peak value of D_{1k} is 15. This implies that at that point, cross-correlation index is 15 times larger than autocorrelation index. This means that multi-user interference is 15 times larger than the desired signal. Therefore the signal becomes swamped, and level of error floor worsens. This shows that the cross-correlation index is an accurate tool for predicting the system loading capacity.

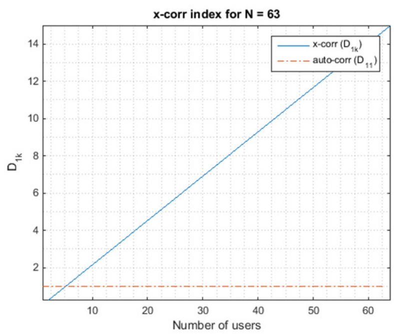

Figure 1. Cross-correlation index for $N = 63$

V. Conclusion

This paper presented a mathematical model for cross-correlation index of spreading codes. The model was tested for a set of 63-chip Gold codes and was found to agree with results of simulations for the system performance. The results confirmed that the cross-correlation index is an accurate tool for predicting level of multiple-access interference experienced by a spread-spectrum signal. It is also useful for the estimation of the system loading capacity, as well as the onset of system saturation and emergence of error floor. Further results shall be presented in future writings.

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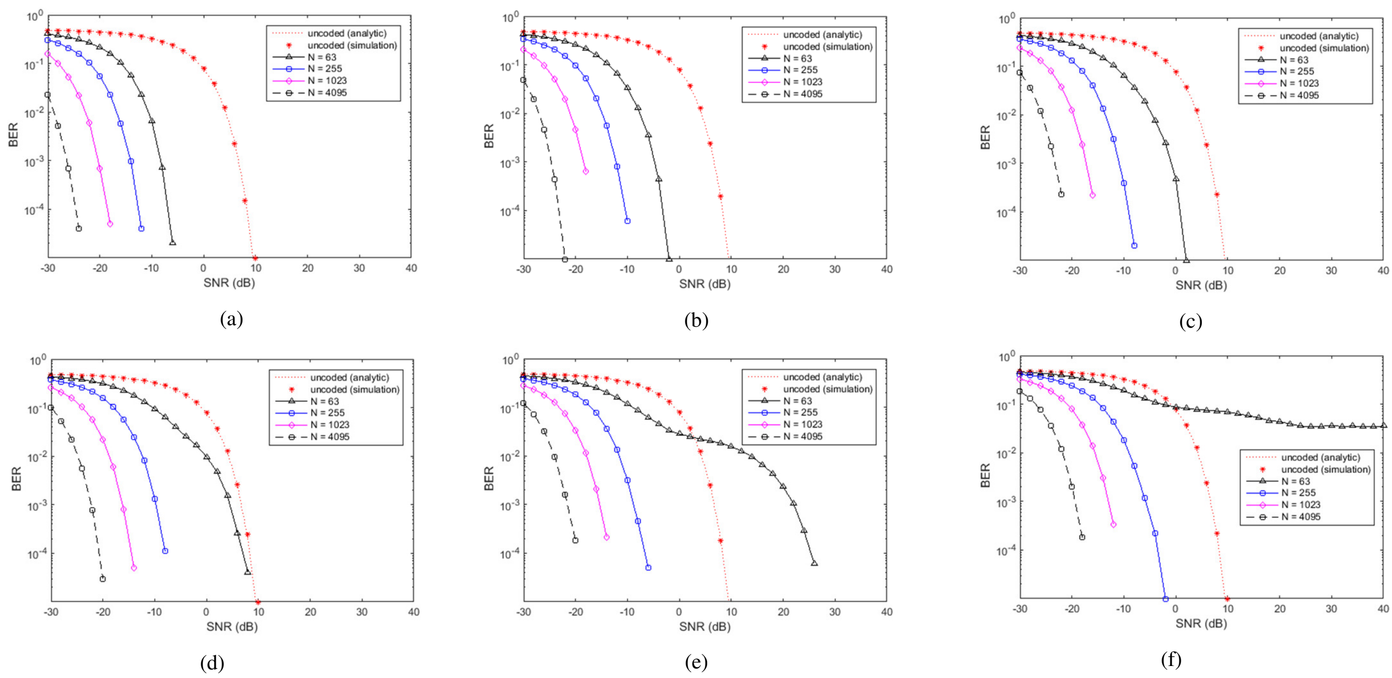

Figure 2. Performance for (a) two, (b) three, (c) four, (d) five, (e) six and (f) ten users.