



## Discontinuities in Multilayer Waveguides to Model 2-D Photonic Crystal Structures

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### Abstract

This paper presents an alternative, quasi-analytic method for evaluating the resonances of the 2-D photonic crystals (PCs) pertinent to Photonic Crystal Surface Emitting Lasers (PCSELs), based on viewing the 2-D periodic structure as discontinuities in a (periodic) multilayer waveguide. This method is easy to implement, computationally fast and can readily also compute structures of small number of periodicities.

### 1. Introduction

The study of wave propagation in periodic media has been of considerable interest for well over a century, [1], [2], [3]; wave propagation in solids liquids and gases have been investigated over the years. Electromagnetic wave propagation in periodic structures at microwave frequencies have led to the development of filters and microwave sources such as Traveling-Wave Tubes [4]. Very early use of periodic structures pertinent to optical wavelengths led to the development of gratings and are extensively used in monochromators and spectrometers. At the quantum mechanical level, the analysis of electron (Schrödinger) waves in crystalline (periodic) media led to the development of the theory of band structure in crystalline solids. In the area of optics, the advent of the laser, semiconductor lasers in particular, and the rapid development of integrated optics there has been a massive resurgence of interest and activity in periodic structures at optical wavelengths. Integrated gratings for use as optical filters, couplers and for use in the development of semiconductor lasers such as Distributed Bragg Reflector (DBR), Distributed Feed Back (DFB) Vertical Cavity Surface Emitting Lasers (VCSEL) have been widespread since the 1980 - but they have been structures with periodicity in one direction.

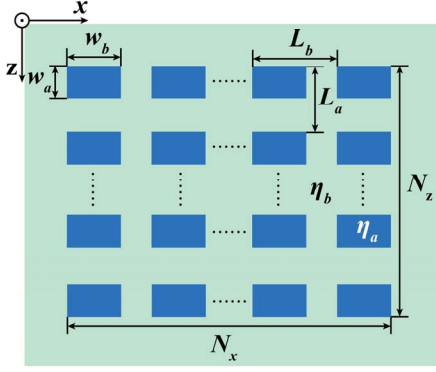
More recently, however, optical waves in structures with periodicities in two and three dimensions have received intense attention. These structures are referred to as Photonic Crystals (PCs), [3], [5] – in analogy with electron waves in Crystalline solids. As can be expected the analysis of the characteristics of two and three dimensional (2-D & 3-D) photonic periodic structures (PCs) is considerably more difficult than analysing the more traditional periodic structure such as Bragg gratings.

PCs have been developed and used extensively for various photonic applications, [6], [7]. As an almost natural extension to the use in earlier years of 1-D periodic (Bragg) structures in the development of semiconductor lasers (DBR, DFB, VCSEL), significant attention has been directed in recent years to using, in particular, 2-D PCs to attain a large area (for high power) semiconductor laser that emits vertically in a single mode (single lasing wavelength) - referred to as Photonic Crystal Surface Emitting Laser, PCSEL, [8].

As with the modelling of any laser, a primary requirement for the modelling of PCSELs is the evaluation of the optical field resonances that identify the lasing mode. The analysis of 2-D periodic structures is considerably more difficult than evaluating the characteristics of (1-D) Bragg gratings. Predominantly three different methods are used to analyse 2-D PCs; they are Plane Wave Expansion (PWE), [9], Coupled Mode Theory, [10], and the purely numerical, Finite Difference Time Domain (FDTD) [11].

As the name implies, the PWE is based on describing the field as an expansion in a complete set of Plane Waves with the media periodicity is imposed by a Floquet-Bloch form which, however, is strictly suitable for an infinitely extending periodic structure. Thus, the often-used devices with very finite extent periodicities cannot be reliably modelled by this method. This shortcoming is avoided by using the CMT but the mathematical complexities are quite daunting and takes very considerable effort to implement. The FDTD is a powerful numerical method and can handle very general conditions but the computation times can be very long indeed; besides, the purely numerical method can analyse specified structures but does not provide any 'physical feel' and hence not convenient for the synthesis, design of devices.

Thus, there is very much a need to have another technique to solve for fields in 2-D PCs which is relatively simple, easy to implement and requires modest computer resources and time while retaining a 'physical feel'. Such an alternative quasi-analytic method is presented here that is based on considering the 2-D PCs as discontinuities in (periodic) multilayer waveguides.

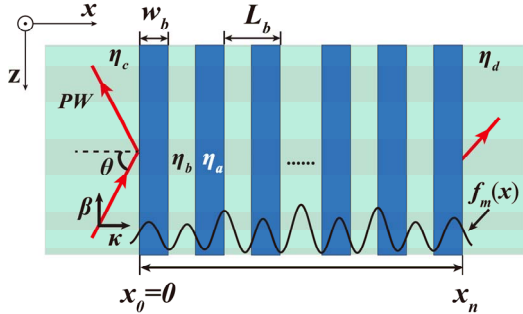


**Fig. 1** Schematic of PC structure.  $N_x$  and  $N_z$  are number of periods along  $x$  and  $z$  respectively.

## 2. Model Description

A rectangular co-ordinate system  $(x, y, z)$  is used throughout consistent with the rectangular device geometry. Fig. [1] illustrate a schematic representation of the planar periodic structure in the  $x$ - $z$  plane with a 2-D refractive index distribution  $\eta(x, z)$ , and the excitation is such that any non-zero field component,  $\hat{F}(x, y, z) = \hat{F}(x, z)$  [12], i.e.,  $\partial/\partial y \approx 0$  is applicable. Any nonzero field component  $\hat{F}_{m,n}(x, z)$  for the resonance modes ( $m, n$ ; integers) satisfied the acceptably approximate wave equation:

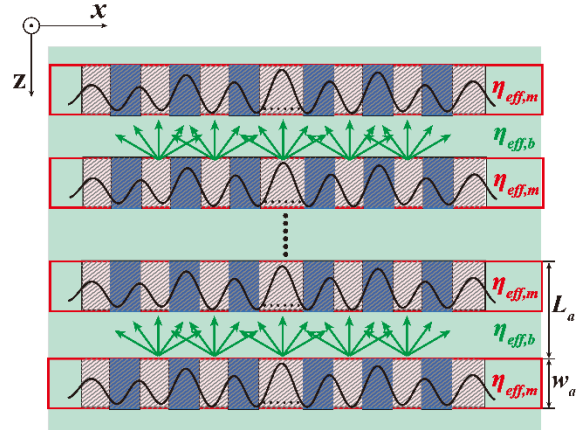
$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_0^2 \eta^2(x, z) \right] \hat{F}(x, z) = 0 \quad (1)$$



**Fig. 2** Model description: lateral modes are computed for a relevant range of wavelengths.

To obtain  $\hat{F}_{m,n}(x, z)$ , the procedure begins by first considering index uniformity along  $z$  direction. The structure is then become a piecewise-constant multilayer structure as shown in Fig. [2]; the eigenmode of such structure,  $f_m(x)$  with the corresponding mode indices  $\eta_{eff,m}$  are evaluated over a pertinent range of wavelength using the transfer matrix method [13] as an excitation problem rather than as an eigenvalue problem [14]. The excitation angles corresponding to transmission resonances provide the  $\beta_m$  and corresponding eigenfunction  $f_m(x)$ .

Then, referring to Fig. [3], discontinuities along the  $z$ -axis are introduced in to the (waveguide) structure, Fig. [2] to ‘create’ the 2-D PC structure corresponding to Fig. [1] and this is then analysed as a waveguide discontinuity problem. In general, waveguide discontinuities excite all the modes of the structure. But in the present context of dimensions and magnitude of discontinuities, multimode generation at discontinuities [15] is approximated by self-mode conditions so that a simple Modal Index Analysis (MIA) at discontinuities is used. However, importantly, consideration of diffraction of modes in to the homogenous region ( $\eta_b$ ) at the discontinuities provides a modified (effective) index for the homogenous regions. Thus, the refractive indices in segmented regions (as shown in Fig. [3] with green arrows) are further modified accordingly into  $\eta_{eff,b}$  using in-plane diffraction based on the dominant plane wave component [16].



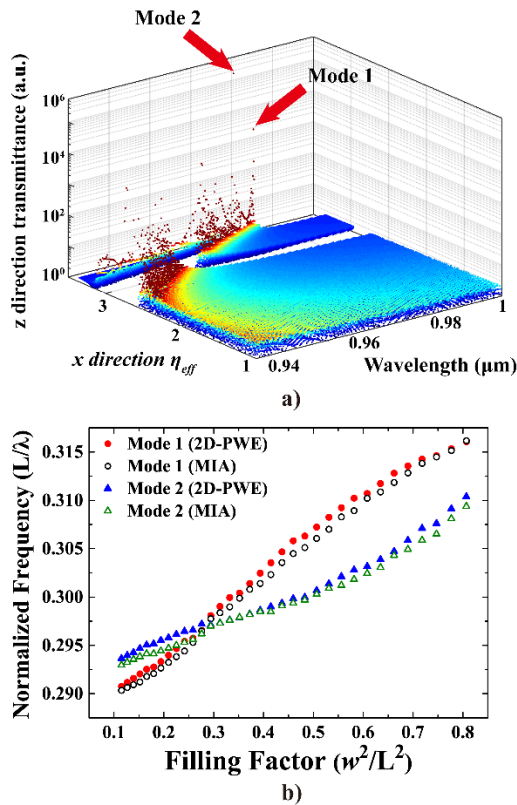
**Fig. 3** The shaded area with dark blue region representing the multilayer waveguide in Fig. [2] is replaced by a homogeneous medium of effective modal index, resulting in a 1-D effective periodic grating along  $z$ .

The transfer matrix method is then used to obtain the resonances  $g_{n,m}[z; f_m(x)]$  of such effective periodic structure (Fig. [3]) composited of unit cell  $\{\eta_{eff,m}, \eta_{eff,b}\}$ , which in effect, includes reasonably well the characteristics of the original 2-D periodic media (Fig. [1]). Hence the final 2-D field distribution at resonances are  $\hat{F}_{m,n}(x, z) = f_m(x)g_{n,m}[z; f_m(x)]$ .

## 3. Results and Discussions

To obtain the 2-D resonance, the resonance wavelength is searched over an appropriate range and the final result (resonances),  $\hat{F}_{m,n}(x, z)$ , of such structure are represented by  $(\eta_{eff,m}, \lambda_0)$  pairs, referring to Fig. [4a]. Optical gain is included as a convenience for easier identifying band edge resonance (corresponds to lasing mode) as illustrated in Fig. [4a]. Band edge modes obtained by this method match

closely with those calculated using PWE over a large range of filling factor (Fig. [4b]).



**Fig. 4 a)** 2-D resonances of PC calculated using MIA. (band edge resonances: Mode 1 and Mode 2). **4)** Band edge resonances calculated with varying filling factor.

In plane field distribution using MIA matches with the PWE and far field pattern obtained matches well with the experimental results. The finite size effect and the consistency of the model will be presented in the conference.

## 4. Conclusions

In this work, a novel method for solving resonance in 2-D PCs using mode index analysis is presented. The method is based on evaluating the eigenfunction and eigenstate of the periodic multilayer structure. Full use of transfer matrix makes the computation process modest and fast. It is shown that the method is versatile and yields very reliable results. In view of above, the MIA method is promising in more comprehensive device modelling such as spatial and temporal variation in optical gain and other PC configurations.

The ‘excitation’ method used here to obtain eigen-mode solutions have raised issues about the various categories of modes, such as bound (confined), leaky and radiating modes that can occur in multilayer (finite and infinitely) periodic waveguides; these interesting issues will be discussed in the conference presentation.

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