URSI AP-RASC 2019, New Delhi, India, 09 - 15 March 2019

Statistical Modelling of GNSS Multipath Error Using Triple-Frequency Linear Combination

V. Sirisha [(1)], V. Satya Srinivas* [(1)], and P Rajini[(2)]
(1) Department of ECE, Geethanjali College of Engineering and Technology, Cheeryal(V), TS, India,
http://www.geethanjaliinstitutions.com/engineering/
(2) Department of Mathematics and Statistics, Bhavan's Vivekananda College, Sainikpuri, Secundrabad, TS India,
http://www.bhavansvc.org/

Abstract

Multipath is considered as the major debilitating factor affecting the accuracy of global navigation satellite system (GNSS) and can lead to position error of 10 meters. Therefore, multipath characterization and modelling is indispensable. Now multipath error can be precisely estimated using triple frequency linear combination of GNSS signals. In this paper the triple frequency linear combination of code measurements of GPS (L1/L2C/L5) and Galileo (E1/E5a/E5b) signals are considered to precisely estimate the multipath and statistical model the error distribution. For multipath free environment the data with residual multipath error, does not follow any distribution.

1. Introduction

The shadowing of the signal from obstructions, foliage etc., and signal reflections due to terrain, buildings, vehicles etc., cause multipath error. The combination of multipath and shadowing is more detrimental in the context of multi-GNSS positioning. Multipath is considered as systematic as well as random error depending upon the type of application. The calibration of multipath remained as unsolved problem even after efforts by many investigators. Multipath introduces errors in both code phase and carrier phase measurements and subsequently in Position, Velocity and Time estimation. To reduce multipath effects various counter measures are deployed. These approaches include hardware (Multipath Estimating Delay Lock Loop (MEDLL) technique, Multiple Signal Classification (MUSIC) technique with multiple antennas etc.,), software (filtering techniques like RLS, MLS etc.,) and hybrid (combination of both hardware and software) [1]. Altogether, these methods have their own advantages and limitations and can be found in open literature [2]. The new receivers today available in market are capable of Tracking signals of multi-GNSS systems. Therefore, the receiver should be capable of processing the multi-frequency signals of these systems in complex environment, while adopting suitable models for various errors of GNSS link-budget. Further, in the development of software-based receiver and simulators for GNSS applications, the algorithms for multipath characterization for various environments will improve the commercial value of the receivers for various applications. Therefore, deep understanding of multipath characteristics is essential.

In the present study the linear combination of code measurements of GPS and Galileo signals are considered to precisely estimate the multipath at the station (GCET). As triple frequency approach found to be promising for precise estimation of multipath at a location, the three frequencies signals of GPS (L1/L2/L5) and Galileo (E1/E2/E5) are used. The following distributions namely Weibull, Gamma, Normal Beta and uniform ones are tested with the experimental data.

2. Multipath estimation: triple frequency linear combination

Direct and indirect signals received at the Global Positioning System (GPS) receiver have relative phase offsets and the phase differences, which are proportional to the differences of the path lengths. Multipath error can be estimated by using linear combinations of code and carrier phase measurements. The code phase and carrier phase multipath using triple frequency GPS measurements is given as [3],

$$M_{P_{125}} = \lambda_5^2 (P1 - P2) + \lambda_2^2 (P5 - P1) + \lambda_1^2 (P2 - P5) \quad (1)$$

$$M_{\phi_{125}} = \lambda_5^2 (\phi 1 - \phi 2) + \lambda_2^2 (\phi 5 - \phi 1) + \lambda_1^2 (\phi 2 - \phi 5) \quad (2)$$

Eq.(1) and (2), shows triple frequency linear model for multipath estimation from code and carier phase observations pertaining to three frequency signals respectively. The indexing of 1, 2, 5 in above equations corresponds to three frequencies, in case of GPS (U.S.A) L1 (1575.42 MHz), L2 (1227.60 MHz) and L5 (1176.54 MHz), for Galileo (Europe) E1 (1575.42 MHz), E5a (1176.45 MHz) and E5b (1207.14 MHz). λ1,2,5 denotes wavelengths. This linear combination completely removes ionospheric error and other measurement errors as well and gives absolute estimate of multipath.

3. Distributions and statistical modelling

To characterize the behavior of a random variable PDFs can be used. Multipath effect is also random and thus can be described by using PDFs. In order to understand which distribution, the experimental data follows, the following distributions are considered namely Weibull, Gamma, Normal Beta and uniform etc. The standard mathematical expressions to describe these density functions is expressed as [4],

$$P(x)=N_x/N_h \quad (3)$$

Where,
N: multipath data values, $\{x_n\}$ n=1,2,3………N.
H: narrow interval width (bin width) centered at x,
N_x: data values that fall within the range $x \pm h/2$
The histogram determine the shape of the probability density curve and it depends on h. Different values of h gives different features of data. In literature many methods are suggested to select h [5][6]. One approach is both histogram and density function can be superimposed on the same plot for easy comparison. Further, to represent tail and center probability distribution of multipath, one parameter and multiparameter PDFs are considered.

4. Methodology

In this paper the linear combination of code-phase measurements of GPS and Galileo signals are considered to precisely estimate the multipath at the station. A triple frequency approach using data of three frequencies signals available from GPS and Galileo is used. To model the multipath, parameters such as elevation angle of signal, azimuth, satellite orbit and signal nose ratio can be analyzed. At present, elevation specific analysis of measurements dividing into bins of for range of elevations from 10° to 30°, 30° to 60° and 60° to 90° has been carried out. As the idea is to investigate the statistical distribution of GNSS multipath error, taking into consideration different elevation angle ranges. The focus of work in this context is mainly applied using the empirical model based on the experimental measurements. A common method of studying distribution function is to use all potential functions such as Weibull, uniform, Rayleigh, Rician, log-normal etc., with similar trend to fit the statistical histogram. The chisquare and/or Anderson darling test result is used to check the fitting consistency of each function.

5. Experimental Setup and Data

A multi-frequency GNSS receiver of Make: Septentrio, NV (Model: PolaRxs pro) capable of tracking GPS, GlONASS, Galileo and SBAS (WAAS, GAGAN, EGNOS) satellite signals was setup at Geethanjali College of Engineering and Technology (GCET), Hyderabad. The experimental setup is depicted in Fig.1. The antenna was mounted on the terrace for better view of SVs and the mask angle is set to 10°. The data with sampling interval of 15 s is used for the analysis.

Figure 1. Septentrio PolaRxs-Pro GNSS receiver and antenna setup at GCET, Hyderabad

6. Results and Discussion

A typical day data corresponding to 27th September 2018 for dual system (GPS+Glileo) is used for the multipath characterization. The sampling interval of data is 15 seconds. With a typical day data, various distributions are tested to fit the multipath obtained at GNSS site i.e., GCET. The distributions considered are Weibull, Gamma, Normal, Beta, uniform, exponential, lognormal and cauchy distributions.

Table 1 Distribution parameters of multipath data for elevation range 10° to 30°

Distribution	Parameter values	Table value	P-value
Weibull	Scale = 0.023 Shape = 1.73179	D = 0.10624	2.20E-16
Normal	Mean = 0.02214 Sd = 0.01629	D = 0.13688	2.20E-16
Gamma	Rate = 52.4238 Shape = 1.170402	D = 0.10333	2.20E-16
Beta	shape1 = 1.15269 Shape2 = 50.5463	D = 0.99718	2.20E-16
Uniform	Min = 0.000009315 Max = 0.11146	D = 0.50635	2.20E-16
Exponential	Rate = 45.1595	D = 0.10494	2.20E-16
Log Normal	Mean = -4.28849 Sd = 1.213464	D = 0.11318	2.20E-16
Cauchy	Location = 0.014861 Scale = 0.011153	D = 0.20503	2.20E-16

The experimental data obtained of our multi-frequency GNSS receiver is a test case with mostly LOS signal in a benign environment and can be considered a multipath free condition. Anderson-Darling test of goodness-of-fit is used to test best distribution fit to multipath data from triple frequency measurements of the GPS and Galileo. Table 1, 2 and 3 depicts the descriptive statistics of distribution parameters and P values for 3 bins of elevation ranges considered, to decide error distribution.
The distribution parameters estimated by using Maximum Likely hood method. Using Kolmogrov Smirnov test the goodness of fit of particular distribution to the data has to

identified based on p-values. But as can be observed from the tables 1-3, for all distributions P-Value is very small and is almost near to zero. That means P value is <0.05, statistically. Therefore, the Null Hypothesis is rejected. This confirms that none of the distribution out of eight considered is suitable to fit the data.

Table 2 Distribution parameters of multipath data for elevation range 30° to 60°

Distribution	Parameter values	Table value	P-value
Weibull	Scale = 0.023828 Shape = 1.21483	D = 0.14241	2.20E-16
Normal	Mean = 0.02246 Sd = 0.016408	D = 0.15579	2.20E-16
Gamma	Rate = 55.107715 Shape = 1.237919	D = 0.14506	2.20E-16
Beta	shape1 = 1.22366 Shape2 = 53.42168	D = 0.9976	2.20E-16
Uniform	Min = 0.00000943 Max = 0.05572218	D = 0.20811	2.20E-16
Exponential	Rate = 44.517614	D = 0.14451	2.20E-16
Log Normal	Mean = -4.251157 Sd = 1.176418	D = 0.14661	2.20E-16
Cauchy	Location = 0.01590 Scale = 0.012435	D = 0.21127	2.20E-16

Table 3 Distribution parameters of multipath data for elevation range 60° to 90°

Distribution	Parameter values	Table value	P-value
Weibull	Scale = 0.02059 Shape = 1.1123	D = 0.13663	2.20E-16
Normal	Mean = 0.01983 Sd = 0.016344	D = 0.21707	2.20E-16
Gamma	Rate = 57.40749 Shape = 1.138479	D = 0.1356	2.20E-16
Beta	shape1 = 1.12433 Shape2 = 55.674046	D = 0.99717	2.20E-16
Uniform	Min = 0.000007 Max = 0.051750	D = 0.30772	2.20E-16
Exponential	Rate = 50.422520	D = 0.13028	2.20E-16
Log Normal	Mean = -4.420109 Sd = 1.1951806	D = 0.12428	2.20E-16
Cauchy	Location = 0.01019 Scale = 0.007211	D = 0.23716	2.20E-16

7. Conclusions

Eight prominent distributions namely Weibull, Gamma, Normal, Beta, uniform, exponential, lognormal and cauchy distributions are checked with the multipath data. The observations processed are under multipath free environment, thus the data is not following any distribution. This is evident from the p values. However, efforts are in progress to raise the data in multipath environment, to extend the analysis.

8. Acknowledgements

This work was supported by the Department of Science and Technology, New Delhi. Vide sanction letter No:SR/FTP/ES-156/2014, dated:4th September 2015.

9. References

1. John W. Betz, “Engineering Satellite Based Navigation and Training”, John Wiley & Sons, Inc, 2016.
2. K. Yedukondalu, A. D. Sarma, and V. Satya Srinivas, “Estimation And Mitigation of PSs Multipath Interference Using Adaptive Filtering”, PIER–M, Vol.21, 133-148, 2011.
3. Yedukondalu Kamamtham and Satya Srinivas V (2017), “YAnalysis and Estimation of Multipath Interference using Dual and Triple Frequency GNSS Signals”, IEEE AEMC Bangalore.
4. Dawid W. Scott, “ On optimal and data-based histograms”, Biometrika (1979),66, 3.pp. 605-610, doi:10.1093/ biomet/ 66.3.605.
5. Athanasious populios., S. Unnikrishna pillo.s., “Probability, Random Variables and Stochastic process”,Mc-Grawhill, 4/e, 2002.
6. Jay L. Devore., “Probability and Statistics for Engineering and the Sciences”, 8/e, Cengage Learing, 2009.