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The Use of the Predictive Posterior of the 3-Parameter Log-Normal Distribution for the Quantification of Measurement Uncertainty in EMC

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Abstract

In some fields of metrology, such as impulse measurements, the measurement model function, on which the quantification of uncertainty is based, may be highly non-linear. Further, physical constraints determine corresponding constraints on the statistical model, that mandate the use of skewed probability density functions (PDFs). Hence a numerical analysis based on Monte Carlo random sampling is necessary and the standard set of available PDFs (such as the one proposed by the Supplement 1 to the Guide to the expression of Uncertainty in Measurement, the GUMS1) is to be enlarged to include asymmetric and flexible (i.e. adaptable to the problem at hand) PDFs. An application of these concepts to the measurement of the rise-time of an impulse is presented where the use of the 3-parameter log-normal PDF is suggested.

1. Introduction

Calculation of measurement uncertainty of impulse parameters (peak, rise-time, duration, …) entails the use of non-linear measurement model functions. Examples of these non-linear models can be found in the annexes on measurement uncertainty of the IEC 61000-4-X series on immunity to impulses, e.g. IEC 61000-4-4 [1]. The Supplement 1 of the Guide to the expression of Uncertainty in Measurement (GUMS1, [2]) specifically provides a procedure for the quantification of measurement uncertainty when non-linearity in the model function cannot be neglected. Such procedure is based on the propagation of the probability density functions (PDFs) of the input quantities through the measurement model in order to obtain the PDF of the output quantity (the measurand).

In addition to model non-linearity, quantification of measurement uncertainty of impulse parameters is further complicated by the fact that the relevant quantities are subject to physical constraints (e.g. the quantity is, due to its nature, larger than a minimum value). Unfortunately the set of the available PDFs that the GUMS1 provides for quantification of measurement uncertainty is restricted to symmetric PDFs or asymmetric but very specialized PDFs (such as the exponential, whose use is recommended for a quantity of which a best estimate but no other information is available, or the gamma PDF whose use is recommended when the counting of objects is involved).

An additional limitation, already pointed out in [3, 4], is that, in some applications, the measurand is not characterized by an essentially unique value then the Student's t PDF shifted by the mean and scaled by the standard deviation of the mean, the only sampling PDF available in the GUMS1, is not the appropriate choice. This circumstance applies to impulse measurements since high-voltage and high-current impulses are generally less repeatable than the measurement systems used to observe them.

In the next two sections these ideas are illustrated through an example. In particular in section 2 the predictive posterior of a generalization of the lognormal PDF (the 3-parameter lognormal PDF) is derived. An application to the measurement of the rise-time of an impulse is described in section 3.

2. The predictive posterior of the 3-Parameter Lognormal PDF

The 3-parameter lognormal PDF has the following mathematical expression

$$f(x;\mu,\sigma,a) = \frac{1}{\sqrt{2\pi}\sigma(x-a)}\exp\left\{-\frac{[\ln(x-a)-\mu]^2}{2\sigma^2}\right\} \qquad (1)$$

where $x > a$, $-\infty < \mu < \infty$ and $\sigma > 0$. Note that (1) corresponds to the normal distribution of $\ln(x-a)$. Suppose that a set of n observations $x_1, x_2, \ldots, x_n$ of the quantity x is available, where $x_i > a$ for $i = 1, 2, \ldots, n$. If a is known then the predictive posterior PDF $f(\hat{x}|a)$ of a future observation $\ln(\hat{x}-a)$ is a Student's t PDF with $n-1$ degrees of freedom shifted by

$$m(a) = \frac{1}{n}\sum_{i=1}^{n}\ln(x_i - a) \qquad (2)$$

and scaled by

$$s(a) = \sqrt{\frac{1+n}{n(n-1)}\sum_{i=1}^{n}[\ln(x_i-a)-m(a)]^2}. \qquad (3)$$

If a is unknown and $f(a)$ is its marginal PDF then the joint PDF $f(\hat{x}, a)$ is obtained as

$$f(\hat{x}, a) = f(\hat{x}|a)f(a). \tag{4}$$

Note that (4) is valid if $x_i > a$ for any possible value of a and $i = 1, 2, \ldots, n$. Hence the right tail of $f(a)$ cannot extend to infinity or, equivalently, a value a^+ shall exist such that $f(a) = 0$ for any $a > a^+$.

3. Application

Assume that the rise-time of an impulse (e.g. the current of the electrostatic discharge) generated by a test generator is measured for the purpose of calibration of the test generator. The result of the calibration is an interval encompassing 95 % of the possible values of the rise time. Measurement is carried out with a specified measuring system (e.g. a system composed by a current to voltage transducer, an attenuator, a cable and a real-time sampling oscilloscope). The measuring system has an upper bandwidth limit and therefore the rise-time of its step response shall be $t_{MS} > 0$. From information about the bandwidth of the measuring system (calibration and/or specification of the voltage transducer, cable and oscilloscope) it is concluded that t_{MS} cannot get values outside the interval $[t_{MS}^-, t_{MS}^+]$.
The rise time of n impulses generated by the test generator are recorded thus obtaining the observations $t_{o1}, t_{o2}, \ldots, t_{on}$. The measuring system, due to its limited bandwidth, distorts the waveform of the impulse and, in particular, the measured (observed) rise-time is larger than the actual (true) one. This effect can be accurately estimated by the following equation

$$t_o^2 = t_{MS}^2 + t_r^2 \tag{5}$$

where t_o is the observed rise-time and t_r is the true rise-time. The assumptions that have to be satisfied for the validity of (5) and the inherent limits of error are analyzed in [5]. From (5)

$$t_r = \sqrt{t_o^2 - t_{MS}^2}. \tag{6}$$

(5) implies that $t_o \geq t_{MS}$ hence a skewed PDF shall be assigned to t_o. Indeed, if a symmetric PDF is assigned to t_o then there will be some chance that $t_o < t_{MS}$ and this is not physically plausible. The predictive posterior of the 3-parameter lognormal PDF is assigned to t_o. To summarize, the information available to derive an interval of the possible values of the true rise-time consists in the n observations $t_{o1}, t_{o2}, \ldots, t_{on}$ and in the interval $[t_{MS}^-, t_{MS}^+]$ of the rise-time of the step response of the measuring system. It is assumed that no other influences may contribute to significantly modify this state of knowledge about t_r (e.g. inaccuracy of the time scale of the oscilloscope, non-linearity of the vertical scale, mismatch …). If this would not be the case then other contributions to measurement uncertainty would appear in the measurement model function and a corresponding PDF would be assigned to each of them.
A numerical example is here provided. $n = 5$ observations are recorded whose values are $t_{o1} = 850$ ps, $t_{o2} = 900$, $t_{o3} = 920$ ps, $t_{o4} = 870$ ps, $t_{o5} = 930$ ps. The nominal bandwidth of the measuring system is 1 GHz and the limits of the possible values of the rise-time of its step response are estimated as $t_{MS}^- = 290$ ps and $t_{MS}^+ = 430$ ps. A uniform PDF is assigned to t_{MS} within these limits. A 3-parameter lognormal PDF is assigned to t_o and the corresponding posterior is derived as in (4), where $\hat{x} = \hat{t_o}$ and $a = t_{MS}$.

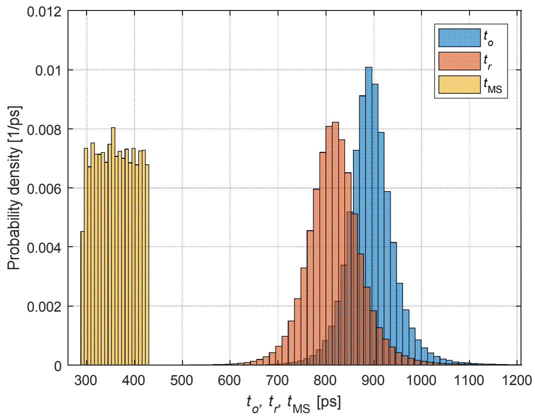

Figure 1. Probability density functions for the observed values of the rise time ($\boldsymbol{t_o}$), the rise time of the step response of the measuring system ($\boldsymbol{t_{MS}}$) and the true rise time ($\boldsymbol{t_r}$).

The histogram plot of the uniform PDF of t_{MS} is shown in Figure 1 on the left side (yellow), while the histogram plots of t_r (brown) and t_o (blue) are visible on the right side. Note again that the PDF of t_o which is shown in Figure 1 is the marginal PDF obtained as (see (4))

$$f(\hat{t_o}) = \int_{t_{MS}^-}^{t_{MS}^+} f(\hat{t_o}, t_{MS}) f(t_{MS}) dt_{MS}. \tag{6}$$

The coverage interval corresponding to a coverage probability of 95 % is numerically obtained as $(800, 1000)$ ps for t_o and $(707, 942)$ ps for t_r.

4. Conclusion

The Supplement 1 to the GUM provides a sophisticated tool for quantification of measurement uncertainty. As any sophisticated tool its use requires particular care for a proper implementation in specific cases, such measurement of the parameters of impulses. In particular, the provided standard set of PDFs requires an extension to include statistical models applicable to the case where the measurand cannot be characterized by an essentially unique true value and to quantities whose state of knowledge is inherently skewed toward positive or negative values.

5. References

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