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# Capacity Analysis of Lattice reduction Aided Detection in Massive-MIMO systems

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## Abstract

With the growing demand of future communication system, massive MIMO become a hot research topic. And Lattice reduction (LLR)-aided equalizers have been well researched for multi-input multi-output (MIMO) systems due to its bit-error-rate (BER) improvement over linear equalizers (LEs) and with relatively low complexity. This paper deal with the analysis of the channel capacity in massive MIMO system. Here, the performance of the LLR aided ZF and MMSE receives have been demonstrated along with the comparison between the linear receivers. The detail mathematical calculation for the improvement inn channel capacity due to LLR aided receiver over the linear receiver has been presented and verified through the simulated results.

## 1. Introduction

Over last few decades MIMO gained lots of attraction for its benefits for the wireless communication. Under present regime of large scale or massive MIMO, large number (greater than ten) of antennas are integrated in the system to boost the system performance. As in [1], signal processing techniques, coding and decoding algorithms for massive MIMO is presented. Till today lots of receiver algorithms have been proposed and implemented. Linear detection techniques such as linear zero forcing (ZF), minimum mean squared error (MMSE), successive interference cancellation (SIC), and ordered SIC (OSIC) presents a performance clearly inferior to the maximum likelihood (ML) detector [2]. At the same time. As in [3], the sphere decoding (SD) produces near optimal performance but its complexity becomes comparable to the ML detector. This puts a limitation on the SD and ML detector for the real time implementation.
In massive MIMO systems, receiver complexity is directly related to the number of antennas used. Therefore, practical implementation of massive MIMO is very challenging due to the complexity of the receiver. For large MIMO systems, several detection schemes have been proposed such as detection based on local search [4,5]; detection based on meta-heuristics [6,7]; detection via message passing on graphical models [8,9]; lattice reduction (LLR) aided detection [10,11]; and detection using Monte Carlo sampling [12]. However, for these schemes to achieve a near-ML performance with high orders of antennas.
This paper deal with the Lattice reduction aided receiver system for MIMO. The basic approach of this receiver is to improve the channel condition and the with the help of the low complex linear receiver, it produces near acceptable performance [13,14]. The new channel matrix is more orthogonal than the old one. Since LLR is used to improve channel conditions, it allows the use of simpler detection techniques. As a result, with very little addition of complexity, BER and the capacity of the system is able to achieve tremendous improvement. As in [15,16], it demonstrates the performance of the LLR-ZF and LLR-MMSE receiver. LLR-MMSE detector performs better than LLR-ZF detector in terms of the BER performance. And in [17], capacity analysis of LLR-ZF receiver has been demonstrated.
Therefore, in this paper, capacity of massive MIMO system has been presented with LLR-ZF and LLR-MMSE receiver system. And also, performance has been compared with the linear receiver system (ZF and MMSE).
The rest of the paper is organized as follows. Section II introduces the mathematical model for capacity comparison. Section III represents the simulated result with the different LLR aided receiver system. Section IV concludes the paper.

## 2. Mathematical Analysis

### *A. MIMO system*

In this paper, we consider a MIMO antenna system with $N_t$ transmitting antenna and $N_r$ receiving antennas. The received signal y can be computed by the equation as given below,

$$Y=Hx+ n \qquad \text{----- (1)}$$

Where the transmit signal is represented by x and n is the $N_r \times 1$ additive white Gaussian noise vector. The vector H represents the time varying flat fading channels for the wireless transmission. The fading behavior of the channel is governed by a Nakagami –m probability distribution function (pdf). Let γ represent the instantaneous SNR and for Nakagami fading channel it can be defined as which can be defined [17] by

$$P_{\gamma}(\gamma) = \frac{2}{\Gamma(m)}\left(\frac{m}{\bar{\gamma}}\right)^{m} \gamma^{m-1} \, exp\left(\frac{-m\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0 \qquad \text{----- (2)}$$

*B. Capacity gain due to f LLR-aided receiver:*

Main aim of this paper is to analyze the capacity improvement due to LLR aided MMSE receiver. Here authors have used complex LLR technique. Main goal of LLR aided system is convert the ill conditioned channel matrix H into a well-conditioned $\widetilde{\mathrm{H}}$ by exploiting orthogonalization. LLR algorithm produces the reduced channel matrix $\widetilde{\mathrm{H}} = \mathrm{HT}$, where T is unimodular matrix. Therefore, in case of a LLR aided MIMO system, the overall system can be represented as,

$$\mathrm{y} = \mathrm{HT}(\mathrm{T}^{-1}\mathrm{x}) + \mathrm{w} \qquad \text{---- (8)}$$

For ZF receiver, the channel capacity can be represented as

$$C_{ZF} = \log_2\left[1 + \frac{SNR}{(H^H H)^{-1}}\right] \qquad \text{-------(9)}$$

Similarly, for LLR aided ZF receiver, the channel capacity can be represented by

$$C_{ZF}^{LR} = \log_2\left[1 + \frac{SNR}{\left(\widetilde{H}^H \widetilde{H}\right)^{-1}}\right] \qquad \text{--------(10)}$$

Considering the high SNR approximation, the above two equations can be again simplified
as mentioned below,

$$C_{ZF} = \log_2[1 + SNR(H^H H)] \qquad \text{-----(11)}$$

$$= SNR(H^H H)\left[1 - \frac{SNR(H^H H)}{2} + o(SNR)^2 - - - - \infty\right] \qquad \text{----(12)}$$

Similarly, for LLR aided ZF receiver in high SNR condition,

$$C_{ZF}^{LR} = SNR.\left(\widetilde{H}^H \widetilde{H}\right)\left[1 - \frac{SNR.\left(\widetilde{H}^H \widetilde{H}\right)}{2} + o(SNR)^2 - - \infty\right] \text{ - (13)}$$

$$\text{Therefore, } \frac{C_{ZF}^{LR}}{C_{ZF}} = \frac{\left(\widetilde{H}^H \widetilde{H}\right)}{(H^H H)} \frac{\left[1 - \frac{SNR.\left(\widetilde{H}^H \widetilde{H}\right)}{2} + o(SNR)^2 - - - - - \infty\right]}{\left[1 - \frac{SNR(H^H H)}{2} + o(SNR)^2 - - - - - \infty\right]} \qquad \text{--(14)}$$

This represents the improvement in the capacity of the system due to LLR aided ZF receiver.

In a similar manner, the capacity improvement due to LLR aided MMSE receiver can be formulated.
The channel capacity for a conventional MMSE [12] system can be represented as,

$$C_{MMSE} = \log_2\left[\frac{1}{\left(I_{N_r} + SNR.H^H H\right)^{-1}}\right] \qquad \text{------- (15)}$$

The channel capacity of a LLR aided MMSE receiver can be represented as

$$C_{MMSE}^{LR} = \log_2\left[\frac{1}{\left(I_{N_r} + SNR.\widetilde{H}^H \widetilde{H}\right)^{-1}}\right] \qquad \text{-------- (16)}$$

At high SNR condition, the factor $\left(I_{N_r} + SNR.H^H H\right)^{-1}$ can be further expressed as

$$\left(I_{N_r} + SNR.H^H H\right)^{-1}$$
$$= (SNR.H^H H)^{-1} - I_{N_r}(SNR.H^H H)^{-2} + O(SNR^{-3})$$
$$= (SNR.H^H H)^{-1}\left[1 - I_{N_r}(SNR.H^H H)^{-1} + O(SNR^{-2})\right] \qquad \text{-------- (17)}$$

Therefore, at high SNR condition, the capacity difference between LLR-MMSE and MMSE can be expressed as,

$$C_{MMSE}^{LLR} - C_{MMSE}$$
$$= \log_2\left(SNRH^H H\right)$$
$$- \log_2\left[1 - I_{N_r}\left(SNRH^H H\right)^{-1} + O\left(SNR^{-2}\right)\right]$$
$$- \log_2\left(SNR.\widetilde{H}^H \widetilde{H}\right)$$
$$+ \log_2\left[1 - I_{N_r}\left(SNR.\widetilde{H}^H \widetilde{H}\right)^{-1} + o(SNR^{-2})\right] \qquad \text{------- (18)}$$

$$C_{MMSE}^{LLR} - C_{MMSE} = \log_2\left(\frac{\widetilde{H}^H \widetilde{H}}{H^H H}\right) + \log_2\left(\frac{1 - I_{N_r}(SNR..H^H H)^{-1} + o(SNR^{-2})}{\left[1 - I_{N_r}\left(SNR.\widetilde{H}^H \widetilde{H}\right)^{-1} + o(SNR^{-2})\right]}\right) \qquad \text{---- (19)}$$

Equation (19) represent the capacity improvement due to LLR-MMSE system.

## 3. Results

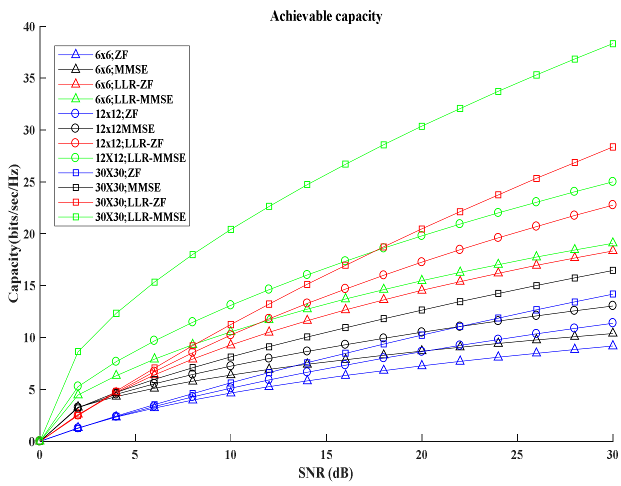


Fig.1. Capacity comparison with different antenna configurations.

In order to analyze the Large scale MIMO system performance, authors have used MATLAB software for the simulation purpose. Here spatial multiplexing MIMO with OFDM have been considered along with the un-correlated Nakagami-m channel. Authors have considered Nakagami-m channel because of its diverse fading nature. For the simulation purpose, power at the transmitter side is equally distributed and channel state information (CSI) is available at the receiver side only.

Above figure 1 shows the capacity comparison with the different numbers of antennas at the transmitter and receiver side. The main aim of this result is to figure out the improvement in channel capacity for Large Scale MIMO with lattice reduction aided receivers. As in figure, ZF receiver producing the worst performance whereas LLLR-MMSE receiver producing significant improvement.

Now if we consider, ZF receiver and SNR=30 dB then the channel capacities for 6x6,12x12 and 30x30 are 9.177 bits/sec/Hz, 11.39 bits/sec/Hz and 14.18 bits/sec/Hz respectively. At the SNR level the channel capacities for LLLR-MMSE receiver for 6x6,12x12 and 30x30 are 19.1 bits/sec/Hz, 25 bits/sec/Hz and 38.33 bits/sec/Hz respectively. Therefore, with the increase in number of antennas, LLLR-MMSE receiver improves the channel capacity in a much faster rate with respect to that of ZF receiver as presented in Table 1.

Table.1.
Capacity Improvement

| Receiver | Capacity Gain ($G_c$) w.r.to 6x6 MIMO system | |
|---|---|---|
| | Large Scale MIMO system | |
| ZF | 12x12; $G_c$ =1.24 | 30x30; $G_c$ =1.54 |
| LLLR-MMSE | 12x12; $G_c$ =1.3 | 30x30; $G_c$ =2.0 |

As indicated in figure 1, LLLR-MMSE receiver provide the best solution for capacity enhancement. Therefore, we are considering the same receiver for further analysis.

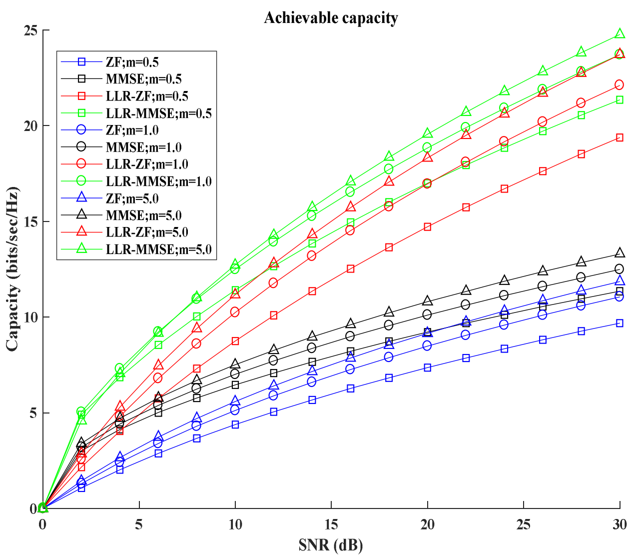


Fig.3. Capacity comparison of 10x10 MIMO with the variation in m.

As indicated in figure 3, Nakagami-m fading parameter m have significant effect on the system performance. And also, from this curve one can easily conclude that LLLR aided receiver is much more superior than the conventional linear receiver. At 30dBSNR condition, with m=5.0, MIMO with ZF receiver provides a capacity of 11.86 bits/sec/Hz whereas MIMO with LLLR-MMSE receiver provides a capacity of 24.76 bits/sec/Hz. At low SNR condition (like 2dB) with m=5, the channel capacities for ZF and LLLR-MMSE are 1.429 bits/sec/Hz and 4.581 bits/sec/Hz respectively. Its shows the tremendous improvement in capacity due to LLLR aided receiver.

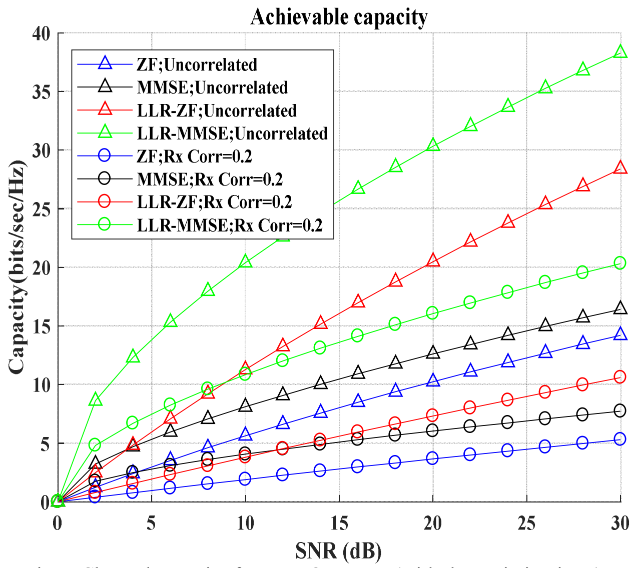


Fig.5. Channel capacity for MIMO system (with the variation in m).

Figure 5 shows the effect of the correlation over the channel capacity. Here, we have considered 30x30 MIMO system with Nakagami-m channel having m=1 and also, we have considered receiver side correlation. It proves that the performance of the system improves even in correlated environment by using lattice reduction aided receiver system.

## 4. Conclusion

In this paper, we have studied the capacity of LLR-aided receivers. We derived capacity gain of LLR-aided receiver system over the linear receiver system. And also, we derived the relation to show the effect of the channel correlation over the receiver system. From the simulated results one can conclude that massive MIMO system boost the channel capacity quite significantly and the LLLR-MMSE receiver provide better performance in comparison to other receiver system. Finally the mathematical analysis have been endorsed by the simulated results.

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