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Output SNR Analysis of Phased MIMO Radar

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Abstract

Exploitation of coherency gain and diversity gain to improve the MIMO system performance is a hot research topic. This paper deals with the performance analysis of phased MIMO radar by utilizing the above said gains. This paper also includes the analysis of the trade-off between the coherency and spatial diversity gain in terms of the signal to noise (SNR) and performance of the phased MIMO system has been compared with the MIMO Radar system in term of the output SNR and probability of detection.

1. Introduction

Researchers are continuously exploring and engaging themself to improve the radar performance [1-2] through advance hardware and signal processing techniques. The advanced multiple antenna signal processing algorithm provide additional degrees of freedom and it offers tremendous boost in radar performance improvement [2]. The performance of a radar is very much influenced by target scintillations [3-4]. With the increase in demand of more sophisticated radar, tar-get localization and characterization become more and more demanding aspect. And it has become even more complex in presence of rich scattering environment [4]. With the change in target aspect angle, the scintillations can be more than 10 dB [2, 5]. Therefore the way out to reduce this effect is to enhance the receiver signal processing gain. And that can be achieved by exploiting different kind of diversity technique.

The diversity technique can be utilized by exploiting multiple antenna configuration. Mainly, it is categorized into MIMO radar [6-8], Phased array radar and hybrid phased-MIMO radar [1, 9]. Phased array radar deal with the and in MIMO radar system, it to send multiple orthogonal signal from each antenna elements of the radar [10-13]. Generally this approach is known as waveform diversity and MIMO radar exploits the diversity gain.

In MIMO radar, as the transmitted waveforms are uncorrelated, this helps to capture the spatial characteristic of a target (radar cross section (RCS)) [14-15]. Whereas in phased array radar, it utilize the coherence in signal. As an advance-ment of radar signal processing, researcher are trying to exploit both the benefits. This leads to the development of phased-MIMO system.

In this paper, the output SNR corresponding to MIMO and phased-MIMO radar have been compared. The mathematical expression have been derived based on the low and high SNR approximation.

The rest section of the paper is furnished as follows. Section II deals with the mathematical formulation of phased-MIMO radar. Section III an-alyzes the performance of phased-array radar, MIMO radar and phased-MIMO radar. While Section IV represents the simulated results to compare the system performances. Section V provides the conclusion remarks.

2. Mathematical Model

Let us consider the radar detection problem at delay τ as follows [3-4],

H_0 : Fall Detection

H_1 : Target Detected

Based on the Neyman-Pearson sense, the optimal detector likelihood ratio test (LRT) can be given as [16]

$$T = \log\frac{f(r(t)|H_1)}{f(r(t)|H_0)} \mathop{\gtrless}_{H_0}^{H_1} \delta_{th} \qquad (1)$$

Where, $f(r(t)|H_0)$ and $f(r(t)|H_1)$ are the probability density function of the observation corresponding to the radar detection hypotheses and δ_{th} is a threshold, which is govern by the probability of false alarm.

For the analysis purpose, in this paper we have considered M and N number of transmitter and receiver antennas.

Let x be the output of the matched filter banks and x is a complex random variable with zero-mean. And the elements of the correlation matrix are $\sigma_n^2 I_{M_t N_r}$ and $(\left(\frac{E}{M_t}\right) + \sigma_n^2) I_{M_t N_r}$ under the radar detection hypothesis. Where E be the total transmitted energy and σ_n^2 is the noise level per receive element.

2.1. Discussion Output SNR

As in [4], the detector's output SNR, β is defined as given below,

$$\beta = \frac{|E(T|H_0) - E(T|H_1)|^2}{\frac{1}{2}[Var(T|H_0) + Var(T|H_1)]} \qquad (2)$$

2.1.1. MIMO Radar

For MIMO radar, $(T/H_0) = NM(\sigma_n)^2 and\ E(T/H_1) = MN\left(\sigma_n{}^2 + \frac{E}{M}\right) = MN\sigma_n{}^2 + EN$. And also $Var(T/H_0) = MN(\sigma_n{}^2)^2 = NM\sigma_n{}^4$ and

$$Var(T/H_1) = MN\left\{\left(\sigma_n{}^2 + \frac{E}{M}\right)^2\right\} = MN\left(\sigma_n{}^4 + \frac{E^2}{M^2} + 2.\sigma_n{}^2\frac{E}{M}\right). \qquad (3)$$

Therefore, using equation (2), the output SNR level can be calculated as mentioned below,

$$\beta_{MIMO} = \frac{|E(T/H_0) - E(T/H_1)|^2}{\frac{1}{2}[Var(T/H_0) + Var(T/H_1)]} = \frac{(EN)^2}{\frac{1}{2}\left[2MN\sigma_n{}^4 + \frac{E^2}{M^2} + 2\frac{\sigma_n{}^2E}{M}\right]} = \frac{\rho^2 N}{M\left(1 + \frac{\rho^2}{2M^2} + \frac{\rho}{M}\right)} \qquad (4)$$

Where, the SNR is denoted byρ, as the ratio between the total transmitted energy and the noise level per receive element and can be define as $\rho = E/\sigma_n^2$

2.1.2. Phased MIMO Radar

To analyze the phased MIMO radar following parameters are considered, M_{coh} represents the number of elements per sub-array at the transmitter side. N_{coh} is the number of elements per sub-array at the receiver side.M_{div} and N_{div} represent the number of Sub-Array at the transmitter and receiver side respectively.

In case of phased-MIMO radar, $E(T/H_0) = \sigma_n{}^2 N_{coh} M_{div} N_{div}$ and $E(T/H_1) = \left(\sigma_n{}^2 N_{coh} + \frac{EN_{coh}{}^2 M_{coh}}{M_{div}}\right) M_{coh}$ .And $Var(T/H_0) = M_{div} N_{div} (\sigma_n{}^2 N_{coh})^2$ and

$$Var(T/H_1) = M_{div} N_{div}\left(\sigma_n{}^2 N_{coh} + \frac{EN_{coh}{}^2 M_{coh}}{M_{div}}\right)^2 \qquad (5)$$

Therefore, the output SNR level can be calculated as,

$$\beta_{phased-MIMO} = \frac{\left(\frac{EN_{coh}{}^2 M_{coh}}{M_{div}} . M_{div} N_{div}\right)^2}{\frac{1}{2} M_{div} N_{div}\left[\sigma_n{}^4 N_{coh}{}^2 + \sigma_n{}^4 N_{coh}{}^2 + \frac{EN_{coh}{}^2 M_{coh}{}^2}{M_{div}{}^2} + 2\frac{\sigma_n{}^2 N_{coh}{}^3 E}{M_{div}}\right]}$$

$$= \frac{E^2 N_{coh}{}^2 M_{coh}{}^2 N_{div} / \sigma_n{}^4}{M_{div}\left[1 + \frac{N_{coh}{}^2 M_{coh}{}^2}{2M_{div}{}^2}\frac{E^2}{\sigma_n{}^4} + \frac{E}{\sigma_n{}^2}\frac{N_{coh} M_{coh}}{M_{div}}\right]}$$

$$\beta_{phased-MIMO} = \frac{\rho^2 N_{coh}{}^2 M_{coh}{}^2 N_{div}}{M_{div}\left[1 + \frac{N_{coh}{}^2 M_{coh}{}^2}{2M_{div}{}^2}\rho^2 + \frac{N_{coh} M_{coh}}{M_{div}}\rho\right]} \qquad (6)$$

2.2. Low SNR approximation

In high SNR regime, equations (4) and (6) can be expressed as

$$\beta_{MIMO}|_{low} = \frac{N/M}{\left(\frac{1}{2\rho^2} + \frac{1}{\rho M}\right)} \qquad (7)$$

$$\beta_{phased-MIMO}\Big|_{low} = \frac{N_{coh}{}^2 M_{coh}{}^2 N_{div}/M_{div}}{\left[\frac{1}{\rho^2} + \frac{N_{coh} M_{coh}}{\rho M_{div}}\right]} \qquad (8)$$

$$\frac{\beta_{MIMO}|_{low}}{\beta_{phased-MIMO}|_{low}} = \frac{N/M}{\left(\frac{1}{2\rho^2} + \frac{1}{\rho M}\right)} \times \frac{\left[\frac{1}{\rho^2} + \frac{N_{coh} M_{coh}}{\rho M_{div}}\right]}{N_{coh}{}^2 M_{coh}{}^2 N_{div}/M_{div}} \qquad (9)$$

Therefore, using (7) and (8) equation (9) can be written as

$$\frac{\beta_{MIMO}|_{low}}{\beta_{phased-MIMO}|_{low}} = \frac{\rho N_{coh} M_{coh} + M_{div}}{(\rho + M_{coh} M_{div}) N_{coh} M_{coh}{}^2} \qquad (10)$$

For the simplicity, we have considered a symmetrical configuration i.e,

$$N_{coh} = N_{div} = M_{coh} = M_{div} = h$$

Therefore,

$$\frac{\beta_{MIMO}|_{low}}{\beta_{phased-MIMO}|_{low}} = \frac{\rho h + 1}{(\rho + h^2) h^2} = k(say) \qquad (11)$$

Therefore, as indicated in (11), at low SNR condition, Phased-MIMO radar performs better than the MIMO radar system.

2.3. High SNR approximation

In high SNR regime, equations 4 and 6 can be expressed as

$$\beta_{MIMO}|_{high} = \frac{N/M}{\left(\frac{1}{2M^2} + \frac{1}{\rho M}\right)} \qquad (12)$$

$$\beta_{phased-MIMO}\Big|_{high} = \frac{N_{div}/M_{div}}{\left[\frac{1}{2M_{div}{}^2} + \frac{1}{\rho M_{div} N_{coh} M_{coh}}\right]} \qquad (13)$$

$$\frac{\beta_{MIMO}|_{high}}{\beta_{phased-MIMO}|_{high}} = \frac{N/M}{\left(\frac{1}{2M^2} + \frac{1}{\rho M}\right)} \times \frac{\frac{1}{2M_{div}{}^2} + \frac{1}{\rho M_{div} N_{coh} M_{coh}}}{N_{div}/M_{div}}$$

(14)

Let us consider that the total number of antennas are same for all the systems,

$$N = N_{coh} N_{div} \qquad (15)$$

$$M = M_{coh} M_{div} \qquad (16)$$

Therefore, equation 6 can be written as

$$\frac{\beta_{MIMO}|_{high}}{\beta_{phased-MIMO}|_{high}} = \frac{\rho N_{coh} M_{coh} + 2M_{div}}{(\rho + 2M_{coh} M_{div})} \qquad (17)$$

For the simplicity, we have considered a symmetrical configuration i.e,

$N_{coh} = N_{div} = M_{coh} = M_{div} = h$

Therefore,

$$\frac{\beta_{MIMO}|_{high}}{\beta_{phased-MIMO}|_{high}} = \frac{\rho h^2 + 2h}{(\rho + 2h^2)} \qquad (18)$$

Therefore, as indicated in (18), at high SNR condition, MIMO radar performs better than the Phased-MIMO radar system.

3. Results

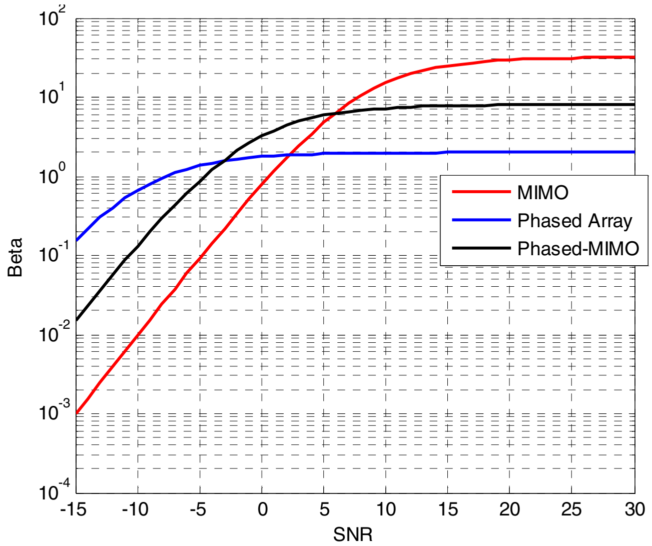

Figure 1. Detector Output SNR variation for different radar system.

Fig. 1 represents the performance comparison phased array radar ($M_t = 4, N_r = 4$), MIMO radar ($M_t = 4, N_r = 4$) and phased-MIMO ($M_{coh} = 2, N_{coh} = 2, M_{div} = 2, N_{div} = 2$). As in figure, at low SNR region phased array radar outperform the other radar system whereas at high SNR region MIMO radar is superior that other counter parts. But phased-MIMO radar perform moderately. Therefore, one can claim that phased-MIMO radar is a best solution to eliminate the problems associated with MIMO and phased array radar.

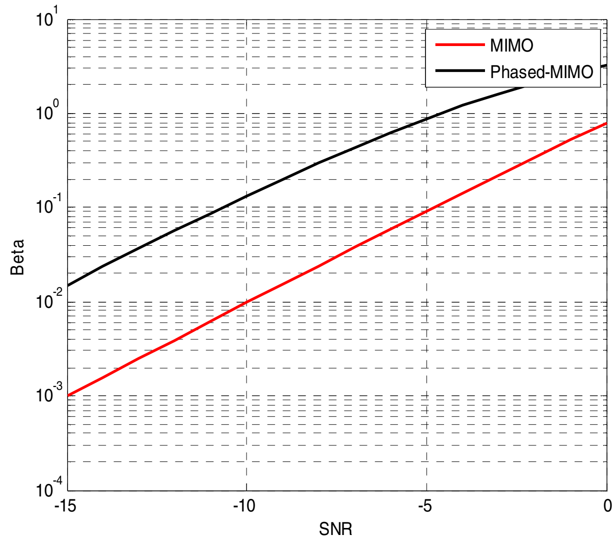

Figure 2. Detector Output SNR variation for MIMO and Phased-MIMO radar in low SNR Condition.

As depicted in above figure, in low SNR condition, there is almost 6dB SNR improvement in Phased-MIMO radar in comparison to MIMO radar. And this performance gap will increase with the increase in number of antennas.

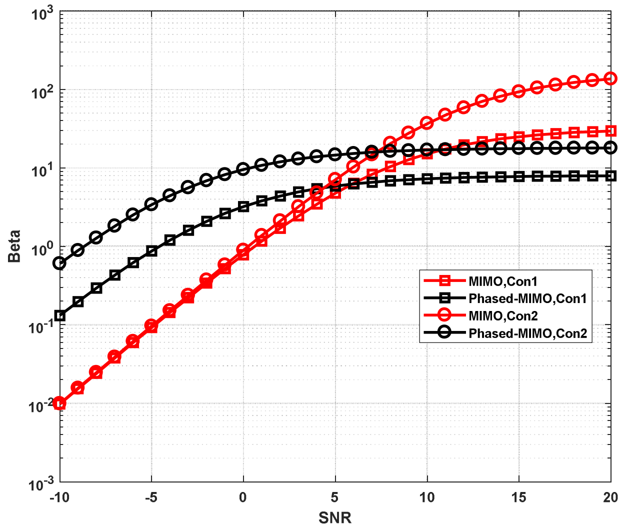

Figure 3. Output SNR Comparison between MIMO and Phased-MIMO Radar.

Above figure shows the performance comparison between the MIMO and Phased MIMO Radar in two different configurations. In Configuration 1 (Con1), antenna distribution for MIMO radar ($M_t = 4, N_r = 4$) and phased-MIMO ($M_{coh} = 2, N_{coh} = 2, M_{div} = 2, N_{div} = 2$). In Configuration 1 (Con2), antenna distribution for MIMO radar ($M_t = 9, N_r = 9$) and phased-MIMO ($M_{coh} = 3, N_{coh} = 3, M_{div} = 3, N_{div} = 3$). It is clearly from the figure that in low SNR region, increase in number of antennas for MIMO system does not improve the system performance. Whereas in high SNR region, impact of the increase in number of antennas is clearly visible and is outperform the phased-MIMO system. But for phased-MIMO system, system performance improves in all SNR condition.

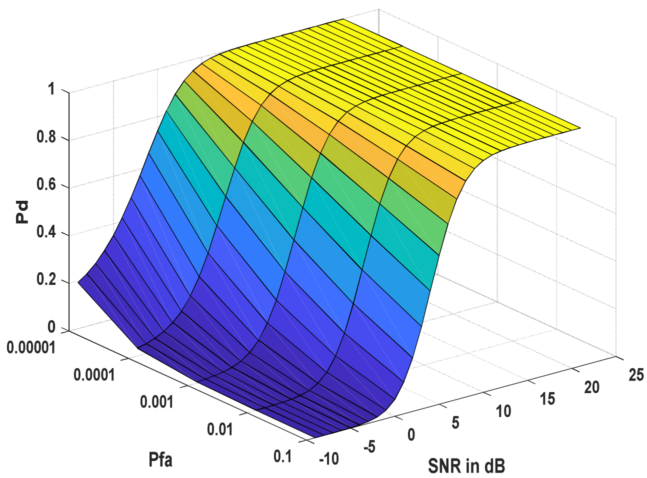

Figure 4. Variation Pd with the variation in SNR and Pfa.

Figure 4 depicts the performance of a phased MIMO ($M_{coh} = 2, N_{coh} = 2, M_{div} = 2, N_{div} = 2$) radar system. Here, Pd is observed with the variation in SNR (--10 dB to 25 dB) and Pfa (10^{-5} to 0.1). For a fixed SNR level, curve between Pd and Pfa represent the receiver operating curves (ROC) for the phased-MIMO system under the known noise condition.

4. Conclusion

This paper represents the analysis of different radar system in terms of output SNR and probability of detection. It reflects the tread-off between the diversity gain and coherence signal processing gain of multi antenna system. It proves the fact that at low SNR region coherence signal processing is more advantageous that the diversity. Whereas in high SNR region diversity gain produce significant boosting in system performance. Therefore, the hybrid approach, phased-MIMO improve the system performance because of its exploitation of the both diversity and coherence processing gain.

5. References

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