



## Study On The Physical Aspect Of Singularity Expansion Method

Nandan Bhattacharyya\* <sup>(1)</sup>, Jawad Y. Siddiqui <sup>(2)</sup>, and Y.M.M. Antar <sup>(3)</sup>

(1) RCC Institute of Information Technology, Kolkata, India

(2) Institute of Radio Physics and Electronics, University of Calcutta

(3) Royal Military College of Canada, Kingston, Canada

### Abstract

This paper deals with the physical aspects of singularities present in the radar echo. Two different objects namely a cylinder and a wire has been analysed in this study. Matrix pencil method has been applied to the late time response to extract the poles. The extracted poles have been found to match with the predicted poles – thereby validating the physical reasoning. The first order extracted pole from the received late time response of an object is found to yield useful circumferential information of the object.

### 1. Introduction

Radar echo signals can be expressed as a sum of damped sinusoids. These oscillation frequencies can be found by applying singularity expansion method on the late time portion of the received echo signal. As these oscillation frequencies are intrinsic to the object geometry, they provide a means for object identification [1,2].

Purpose of this study is to find the physical reasons behind the singularities in the late time response and to match them with poles extracted using matrix pencil method. Knowledge of the physical reason could lead us to have an idea about the object dimension.

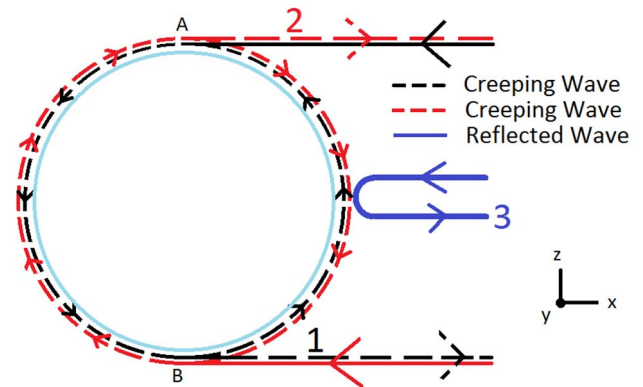
EM scattering response of 2 different objects namely a cylinder and a wire have been found using HOBBIES [4] software. The frequency domain data has been transformed to time domain by IFFT. Then matrix pencil method [3] of singularity expansion has been applied to get the singularities in the time domain data. These singularities have been compared with theoretical singular values.

### 2. CASE A: A CYLINDER

#### 2.1 Theory:

A Hollow metallic cylinder having diameter  $D = 0.15$  m and height 1 mm with the cylinder axis towards  $y$ -direction as shown in Figure 1 is being illuminated by uniform plane wave from the  $x$  direction.

Wave incident on the cylinder in between right side of point A and B will be reflected back to the source. This has been shown using ray 3 (blue solid line) in Fig. 1.



**Figure 1.** Reflected and creeping wave component in Cylindrical object echo

Wave incident at point A traverses the cylinder circular circumference in anti-clockwise direction as shown by ray 1 (black dashed line). This surface creeping wave will again meet the incident signal at point A. These two signal's phase matches, if the frequency of the incident signal is such that the circumference is equal to wavelength ( $\lambda$ ). This corresponds to the first resonant frequency of the creeping wave. Part of this creeping wave is scattered back towards the source.

Same arguments hold for wave incident at point B and has been shown by ray 2 (red dashed line). These two components of creeping wave meeting at point A and B produces standing wave patterns.

The first resonant frequency,  
 $f = c / \lambda = c / \text{circumferential length} = c / \pi * D = 0.64$  GHz, where  $c$  is the velocity of light.

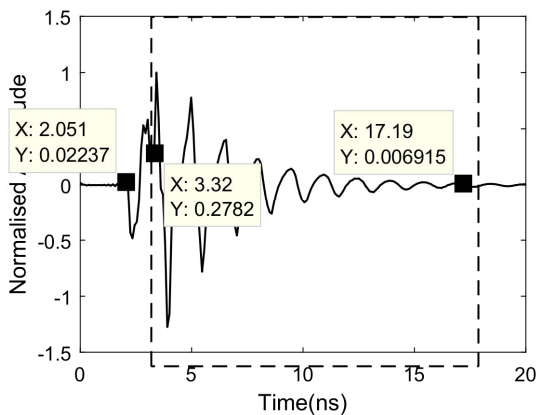
The part of the standing creeping wave scattered back towards the source reaches the source at different time instants than the directly reflected component does. The difference in path length between the creeping wave and reflected wave component is  $2 * \text{radius} + \text{half of the circumference} = D + \pi * D / 2$ . Thus the received time

domain signal could be divided in two parts namely early time effect which is due to the directly reflected component and late time effect which is due to the creeping wave component.

Clearly in this case, the late time response should start  $[D+\pi*D/2]/c = 1.285$  ns after the early time response.

## 2.2. Simulation Results - poles extracted using matrix pencil method:

The simulated received signal obtained using HOBBIES is shown in Figure 2. Late time response starting after 1.285 ns has been shown inside dashed rectangle.



**Figure 2.** Early time and late time response in the received signal from Cylindrical object

The first order pole extracted from this late time response using Matrix Pencil method is, frequency = 0.66 GHz, Damping coefficient = - 0.29

## 3. CASE B : A WIRE

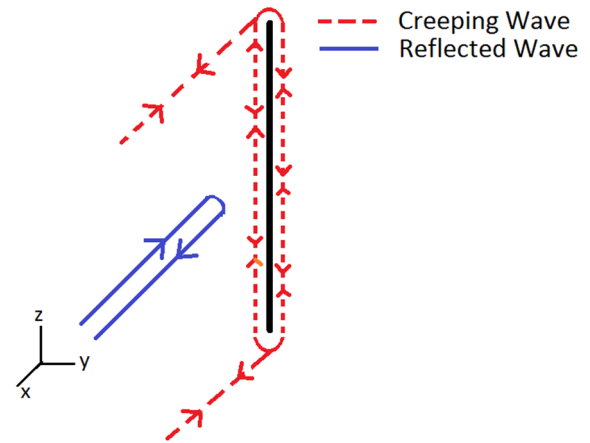
### 3.1 Theory:

A Wire having a length  $D = 0.1$  m, radius 0.1 mm oriented towards  $z$  axis as shown in Figure 3 is being illuminated by uniform plane wave from the  $x$  direction. As evident from Figure 3, the creeping wave resonance condition is:

$$\text{resonant frequency, } f = c / \lambda$$

$$= c / \text{round trip distance} = c / (2*D) = 1.5 \text{ GHz}$$

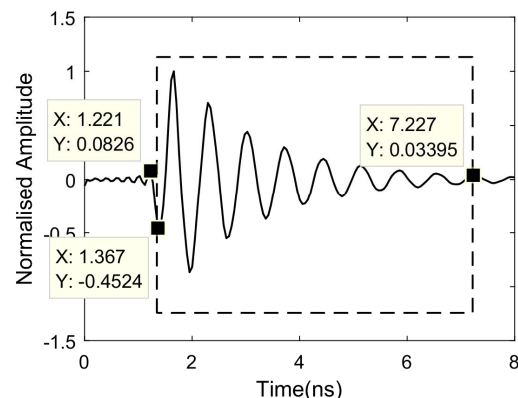
Also the path difference between the reflected and scattered creeping wave from the edges is  $D/2$ . Therefore late time response should start  $D/2c = 0.166$  ns after the early time response.



**Figure 3.** Reflected and creeping wave component in Wire object echo

## 3.2. Simulation Results - poles extracted using matrix pencil method:

The simulated received signal obtained using HOBBIES is shown in Figure 4. Late time response starting after 0.166 ns has been shown inside dashed rectangle.



**Figure 4.** Early time and late time response in the received signal from Wire object

The first order pole extracted from this late time response using MP method is, frequency = 1.4 GHz, Damping coefficient = - 0.58.

## 4. References

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