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Electromagnetic Wave Excitation by a Nonsymmetric Antenna Located on the Surface of an Open Gyrotropic Cylindrical Waveguide

Oleg M. Ostafiychuk*, Vasiliy A. Es'kin, and Alexander V. Kudrin
University of Nizhny Novgorod, 23 Gagarin Ave., Nizhny Novgorod 603950, Russia

Abstract

Excitation of electromagnetic waves by distributed given sources located on the surface of an open cylindrical waveguide filled with a gyrotropic medium is studied. It is assumed that the waveguide is placed in free space and aligned with an external static magnetic field. The considered sources have nonsymmetric electric-current distributions. The radiation resistances of the sources are numerically calculated using an approach based on an expansion of the total field in terms of the discrete- and continuous-spectrum waves.

1 Introduction

In the past decades, there has been a great deal of interest in the problem of excitation and propagation of guided electromagnetic waves in the presence of cylindrical irregularities with gyrotropic properties. In the case of a magnetoplasma, such an interest is explained by an important role that such waves play in various processes in the Earth's ionosphere [1] and the laboratory experiments with helicon antennas [2, 3] and helicon plasma sources which can be used in high-energy particle accelerators [4]. An overwhelming majority of theoretical works deal with the cases where electromagnetic waves in such systems are excited by an axisymmetric source in the form of a loop antenna. Moreover, in most works, consideration is limited to the excitation of only the discrete-spectrum waves (eigenmodes). For arbitrary sources, an exhaustive theoretical investigation was made in [5] for a cylindrical guiding structure surrounded by a general anisotropic medium. However, the radiation from the source having the nonsymmetric distribution of the current and operating in an open gyrotropic waveguide, which is surrounded by an isotropic medium, has not yet been studied in sufficient detail. Therefore, this type of sources, which is close to helicon antennas, requires more detailed analysis.

This work employs the theoretical approach developed for estimating the characteristics of the helicon-type sources. The energy characteristics of the antenna are obtained by solving the problem of wave excitation by the antenna current. The analysis is performed using the full-wave approach presented in [6].

2 Formulation of the Problem

We consider an infinitely long cylindrical waveguide of radius a which is filled with a gyrotropic medium and located in free space. In our case, the gyrotropic medium is a cold collisionless magnetoplasma. It is assumed that the axis of the waveguide is parallel to an external static magnetic field $\mathbf{B}_0$ and coincides with the z axis of a cylindrical coordinate system (ρ,ϕ,z). The field is excited by a source placed on the surface of the cylinder with the current density distribution written, with the $\exp(i\omega t)$ time dependence dropped, as

$$\mathbf{J}(\mathbf{r}) = (\boldsymbol{\phi}_0 j_\phi + \mathbf{z}_0 j_z)\delta(\rho - a) \times \exp\left(-im\phi - ik_0 p_0 z\right), \quad |z| < d, \tag{1}$$

where d is the half-length of the antenna. The electric current components are assumed to be related to the total-current amplitude I_0 as $|I_0|^2 = |I_\phi|^2 + |I_z|^2$, where $I_\phi = 2d j_\phi$ and $I_z = 2\pi a j_z$. The quantities m and p_0 in Eq. (1) define the nonunifom current distribution along the azimuthal and longitudinal coordinates, respectively, and $k_0 = \omega/c$ is the free-space wave number (c is the speed of light in free space). The cold collisionless magnetoplasma is described by the dielectric permittivity tensor

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}, \tag{2}$$

where

$$\begin{aligned} \varepsilon &= 1 + \frac{\omega_{\rm p}^2}{\omega_{\rm H}^2 - \omega^2} + \frac{\Omega_{\rm p}^2}{\Omega_{\rm H}^2 - \omega^2}, \\ g &= -\frac{\omega_{\rm p}^2\omega_{\rm H}}{\omega_{\rm H}^2 - \omega^2} + \frac{\Omega_{\rm p}^2\Omega_{\rm H}}{\Omega_{\rm H}^2 - \omega^2}, \\ \eta &= 1 - \frac{\omega_{\rm p}^2}{\omega^2} - \frac{\Omega_{\rm p}^2}{\omega^2}. \end{aligned} \tag{3}$$

Here, $\omega_{\rm p}$ and $\Omega_{\rm p}$ are the electron and ion plasma frequencies, and $\omega_{\rm H}$ and $\Omega_{\rm H}$ are the gyrofrequencies of the corresponding particles, respectively. It should be noted that the Gaussian system of units is used throughout the work, except for the numerical results presented in the figures in what follows.

3 The Source-Excited Field in the Presence of an Open Cylindrical Waveguide

The field excited by the source with current density (1) in the source-free regions $|z| > d$ can be represented as an expansion in terms of the discrete- and continuous-spectrum waves as follows:

$$\begin{bmatrix}\mathbf{E}(\mathbf{r})\\ \mathbf{H}(\mathbf{r})\end{bmatrix} = \sum_n a_{s,m,n}\begin{bmatrix}\mathbf{E}_{s,m,n}(\mathbf{r})\\ \mathbf{H}_{s,m,n}(\mathbf{r})\end{bmatrix} + \sum_{\alpha=1}^{2}\int_0^\infty a_{s,m,\alpha}(q)\begin{bmatrix}\mathbf{E}_{s,m,\alpha}(\mathbf{r},q)\\ \mathbf{H}_{s,m,\alpha}(\mathbf{r},q)\end{bmatrix} dq. \quad (4)$$

Here,

$$\begin{bmatrix}\mathbf{E}_{s,m,n}(\mathbf{r})\\ \mathbf{H}_{s,m,n}(\mathbf{r})\end{bmatrix} = \begin{bmatrix}\mathbf{E}_{s,m,n}(\rho)\\ \mathbf{H}_{s,m,n}(\rho)\end{bmatrix}\exp(-im\phi - ik_0 p_{s,m,n}z),$$

$$\begin{bmatrix}\mathbf{E}_{s,m,\alpha}(\mathbf{r},q)\\ \mathbf{H}_{s,m,\alpha}(\mathbf{r},q)\end{bmatrix} = \begin{bmatrix}\mathbf{E}_{s,m,\alpha}(\rho,q)\\ \mathbf{H}_{s,m,\alpha}(\rho,q)\end{bmatrix}\exp(-im\phi - ik_0 p_s(q)z),$$

where $\mathbf{E}_{s,m,n}(\rho)$, $\mathbf{B}_{s,m,n}(\rho)$ and $\mathbf{E}_{s,m,\alpha}(\rho,q)$, $\mathbf{B}_{s,m,\alpha}(\rho,q)$ are vector functions describing the radial distributions of the fields of the discrete- and continuous-spectrum waves, respectively; m is the azimuthal index of the modes; the subscript s obeys the relations $s = +$ for $z > d$ and $s = -$ for $z < -d$; n is the radial index of eigenmodes (discrete-spectrum waves) with the longitudinal wave numbers $p_{s,m,n}$ ($p_{+,m,n} = -p_{-,m,n}$); the subscript α corresponds to two kinds of the continuous-spectrum waves for which the function $p_s(q) = (1-q^2)^{1/2}$ is the longitudinal wave number in free space, and q is the transverse wave number in the outer region ($\rho > a$) normalized to k_0. It is assumed that the fields are regular on the z axis and satisfy the boundedness conditions at infinity. Detailed expressions for the field components of the discrete- and continuous-spectrum waves can be found in [6]. It is taken into account in Eq. (4) that only the modes with the azimuthal index m are excited by source (1). The quantities $a_{s,m,n}$ and $a_{s,m,\alpha}(q)$ can be obtained using the well-known technique developed for finding the excitation coefficients of the modes of open waveguides [7], and are given by the formulas

$$\begin{aligned}a_{s,m,n} &= \frac{1}{N_{s,m,n}}\int \mathbf{J}(\mathbf{r})\cdot\mathbf{E}^{(\mathrm{T})}_{-s,-m,n}(\mathbf{r})d\mathbf{r}\\ &= \left[j_\phi E^{(\mathrm{T})}_{\phi;-s,-m,n}(a) + j_z E^{(\mathrm{T})}_{z;-s,-m,n}(a)\right]\\ &\quad\times\frac{4\pi a}{N_{s,m,n}}\frac{\sin[k_0(p_0 - p_{s,m,n})d]}{k_0(p_0 - p_{s,m,n})},\\ a_{s,m,\alpha}(q) &= \frac{1}{N_{s,m,\alpha}(q)}\int \mathbf{J}(\mathbf{r})\cdot\mathbf{E}^{(\mathrm{T})}_{-s,-m,\alpha}(\mathbf{r},q)d\mathbf{r}\\ &= \left[j_\phi E^{(\mathrm{T})}_{\phi;-s,-m,\alpha}(a,q) + j_z E^{(\mathrm{T})}_{z;-s,-m,\alpha}(a,q)\right]\\ &\quad\times\frac{4\pi a}{N_{s,m,\alpha}(q)}\frac{\sin[k_0(p_0 - p_s(q))d]}{k_0(p_0 - p_s(q))},\end{aligned} \quad (5)$$

where integration is performed over the region occupied by current (1), the superscript (T) denotes fields taken in an auxiliary medium described by the transposed dielectric tensor $\hat{\varepsilon}^{\mathrm{T}}$, $N_{s,m,n}$ and $N_{s,m,\alpha}(q)$ are the normalization quantities which are defined by the orthogonality relations presented in [6].

It can also be established that the fields with the azimuthal index m in Eq. (4) satisfy the following power orthogonality relations in the case of a lossless medium:

$$\begin{aligned}&\int_0^\infty \Big[\mathbf{E}_{s,m,n}(\mathbf{r})\times\mathbf{H}^*_{\tilde{s},m,\tilde{n}}(\mathbf{r}) + \mathbf{E}^*_{\tilde{s},m,\tilde{n}}(\mathbf{r})\times\mathbf{H}_{s,m,n}(\mathbf{r})\Big]\cdot\mathbf{z}_0\rho d\rho = \frac{8}{c}\mathscr{P}_{s,m,n}\delta_{s,\tilde{s}}\delta_{n,\tilde{n}},\\ &\int_0^\infty \Big[\mathbf{E}_{s,m,\alpha}(\mathbf{r},q)\times\mathbf{H}^*_{\tilde{s},m,\tilde{\alpha}}(\mathbf{r},\tilde{q}) + \mathbf{E}^*_{\tilde{s},m,\tilde{\alpha}}(\mathbf{r},\tilde{q})\times\mathbf{H}_{s,m,\alpha}(\mathbf{r},q)\Big]\cdot\mathbf{z}_0\rho d\rho = \frac{8}{c}\mathscr{P}_{s,m,\alpha}(q)\delta(q-\tilde{q})\delta_{s,\tilde{s}}\delta_{\alpha,\tilde{\alpha}},\\ &\int_0^\infty \Big[\mathbf{E}_{s,m,\alpha}(\mathbf{r},q)\times\mathbf{H}^*_{\tilde{s},m,\tilde{n}}(\mathbf{r}) + \mathbf{E}^*_{\tilde{s},m,\tilde{n}}(\mathbf{r})\times\mathbf{H}_{s,m,\alpha}(\mathbf{r},q)\Big]\cdot\mathbf{z}_0\rho d\rho = 0.\end{aligned} \quad (6)$$

Here, $\delta_{\alpha,\beta}$ is the Kronecker delta, $\delta(q)$ is the Dirac function, and the asterisk denotes complex conjugation. Note that $\mathscr{P}_{+,m,n} = -\mathscr{P}_{-,m,n}$ and $\mathscr{P}_{+,m,\alpha}(q) = -\mathscr{P}_{-,m,\alpha}(q)$. With allowance for this fact and relations (6), the total power radiated from the source is represented as

$$P_\Sigma = \sum_{s=\pm} P_s \,\mathrm{sgn}\, s, \quad (7)$$

where

$$P_s = \sum_n |a_{s,m,n}|^2\mathscr{P}_{s,m,n} + \sum_{\alpha=1}^{2}\int_0^1 |a_{s,m,\alpha}(q)|^2\mathscr{P}_{s,m,\alpha}(q)dq. \quad (8)$$

Note that the terms under the summation sign in Eq. (7) are the powers radiated to the positive ($s = +$) or negative ($s = -$) direction of the z axis. Introducing the total radiation resistance $R_\Sigma = 2P_\Sigma/|I_0|^2$ of the source, we obtain

$$R_\Sigma = R_{\mathrm{mod}} + R_{\mathrm{cs}}, \quad (9)$$

where

$$R_{\mathrm{mod}} = \sum_n R_n, \qquad R_{\mathrm{cs}} = \int_0^1 \mathscr{R}_{\mathrm{cs}}(q)dq, \quad (10)$$

$$R_n = \frac{2}{|I_0|^2}\left(|a_{+,m,n}|^2 + |a_{-,m,n}|^2\right)\mathscr{P}_{+,m,n},$$

$$\mathscr{R}_{\mathrm{cs}}(q) = \frac{2}{|I_0|^2}\sum_{\alpha=1}^{2}\left(|a_{+,m,\alpha}(q)|^2 + |a_{-,m,\alpha}(q)|^2\right)\mathscr{P}_{+,m,\alpha}(q).$$

Here, R_n and R_{cs} are the partial radiation resistances which correspond to the powers going to the nth eigenmode and the continuous-spectrum waves, respectively. Integration in Eq. (10) is performed over the q values for which the function $p_s(q)$ is purely real. The quantity $\mathscr{R}_{cs}(q)$ has the meaning of spatial-spectrum distribution of the partial radiation resistance for the continuous-spectrum waves.

4 Numerical Results

4.1 Radiation Resistances in the Resonant Frequency Ranges of a Magnetoplasma

The partial radiation resistances were calculated for the source with parameters typical of helicon antennas which are often used in the laboratory experiments [2–4]. In expression (1) for the current density, we put $a = 2.5\,\text{cm}$, $d = 4a$, $p_0 = 0$, and $m = 1$. First, let the source frequency ω belong to the resonant interval $\omega_{\rm LH} < \omega < \omega_{\rm H}$ of the whistler range, where $\omega_{\rm LH} = (\omega_{\rm H}\Omega_{\rm H})^{1/2}$ is the lower-hybrid frequency. Figure 1(a) shows the partial radiation resistances R_n for the individual eigenmodes with the radial indices n at $\omega/\omega_{\rm H} = 2.5 \cdot 10^{-2}$. In this case, the waveguide supports an infinite number of the propagated eigenmodes which are excited by source (1) due to a resonance character of the plasma. The most efficient excitation is observed for the modes with $p_n < P = (\varepsilon - g)^{1/2} = 44.48$, for which both the helicon and quasielectrostatic parts of the mode fields have the volume structure. The value of P is indicated by the vertical dashed line in Fig. 1(a). In the considered case, the radiation resistance for the eigenmodes amounts to $R_{\rm mod} \simeq 0.52\,\Omega$. Moreover, it was established that for the given length-to-radius ratio of the source, an increase in the axial component j_z of the current density with respect to its azimuthal component j_ϕ leads to redistribution of the radiated power to the higher-order modes with $p_n > P$. It is worth noting that the value of $R_{\rm mod}$ also increases in this case. The distribution of the quantity $\mathscr{R}_{cs}$ over the spatial spectrum of the excited waves is illustrated by Fig. 1(b) at the same frequency. The radiation resistance corresponding to the continuous-spectrum waves is equal to $R_{cs} = 1.1 \cdot 10^{-6}\,\Omega$. It is evident that the inequality $R_\Sigma \simeq R_{\rm mod} \gg R_{cs}$ holds with a sufficient margin. This means that almost all the radiated power goes to the eigenmodes in the considered frequency range. Analogous dependences for R_n and $\mathscr{R}_{cs}$ are shown in Fig. 2 in the case where the frequency of the excited field belongs to the range $\omega_{\rm p} < \omega < \omega_{\rm UH}$, where $\omega_{\rm UH} = (\omega_{\rm H}^2 + \omega_{\rm p}^2)^{1/2}$ is the upper-hybrid frequency. Here, the radiation resistance for the eigenmodes amounts to $R_{\rm mod} \simeq 0.075\,\Omega$. However, in contrast to the previous case, the total radiation resistance of the antenna is mainly determined by the continuous-spectrum waves, for which $R_{cs} = 62.3\,\Omega$.

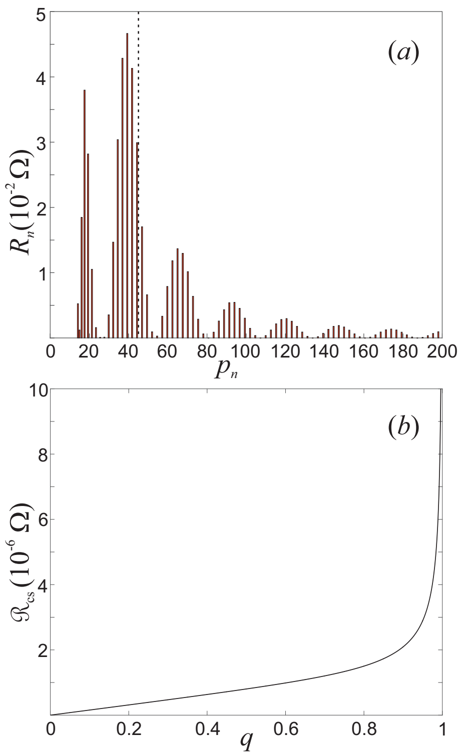

Figure 1. (a) Partial radiation resistances R_n for the individual eigenmodes with the radial indices n and the longitudinal wave numbers p_n and (b) the distribution of $\mathscr{R}_{cs}$ over q for $\omega/\omega_{\rm H} = 2.5 \times 10^{-2}$, $\omega_{\rm p}/\omega_{\rm H} = 6.95$, $\omega_{\rm LH}/\omega_{\rm H} = 3.7 \times 10^{-3}$, and $j_z = 0.25 j_\phi$.

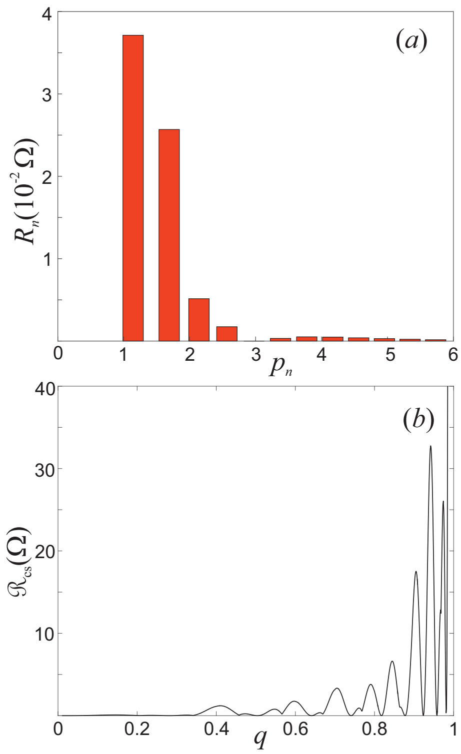

Figure 2. The same as in Fig.1 but for $\omega/\omega_{\rm UH} = 0.995$.

4.2 Radiation Resistances of Short and Long Antenna

The radiation resistances for antennas with the purely longitudinal ($R^{(z)}_{\rm mod}$) and purely azimuthal ($R^{(\phi)}_{\rm mod}$) components of the current density (1) as functions of the parameter d/a are presented in Fig. 3(a) for a wide range of d. It can be seen that for a long antenna (compared with its radius), the radiation resistance is mainly determined by the longitudinal component of the electric current density. On the contrary, a relatively large value of $R^{(\phi)}_{\mathrm{mod}}$ is observed for small values of d (short antenna). Figure 3(b) shows the ratios of the

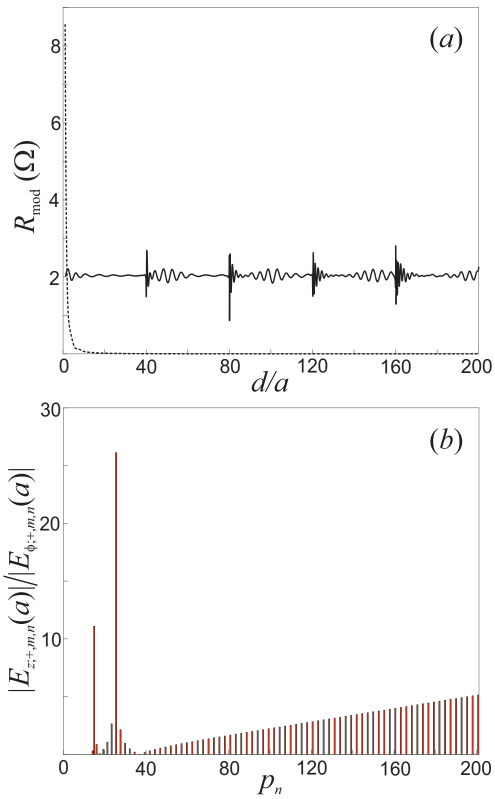

Figure 3. (a) Radiation resistances $R^{(z)}_{\mathrm{mod}}$ (solid line) and $R^{(\phi)}_{\mathrm{mod}}$ (dashed line) as functions of the parameter d/a. (b) Ratios of the absolute values of the tangential electric-field components for the individual eigenmodes at the surface of the waveguide. The same parameters as in Fig.1.

longitudinal and azimuthal electric-field components on the surface of the cylinder for individual modes with the radial indices n and the longitudinal wave numbers p_n. The radiation resistance of the antennas with various electric current distributions can be tuned by choosing the appropriate value of $p_0 = \tilde{p}$ in Eq. (1). For example, if we put $p_0 = 200.7$ ($\tilde{p} \simeq p_{75}$), the radiation resistance $R^{(\phi)}_{\mathrm{mod}} = 4.82\,\Omega$, while $R^{(z)}_{\mathrm{mod}} = 12.32\ \Omega$ for the sources with $d = a$. On the contrary, it is seen in Fig.3 (a) that for the same length of the antenna and $p_0 = 0$, the inverse inequality $R^{(\phi)}_{\mathrm{mod}} \gg R^{(z)}_{\mathrm{mod}}$ takes place.

5 Conclusions

In this work, the radiation characteristics of the nonsymmetric antenna placed on the surface of an open cylindrical gyrotropic waveguide have been studied. The radiation resistances of such a source have numerically been calculated for two characteristic frequencies. It is shown that in the low-hybrid frequency range, almost all the radiated power goes to the guided waves (eigenmodes), in contrast to the case of the upper-hybrid range where the radiation resistance of the antenna is determined by the contribution of the unguided (continuous-spectrum) waves. It has been demonstrated that the values of the radiation resistances of the antennas with the purely azimuthal and purely longitudinal electric-current components can significantly be varied, depending on the longitudinal current-density distribution.

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