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Propagation of TM waves in a shielded plane waveguide with anisotropic Kerr nonlinearity

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Abstract

An eigenvalue problem for Maxwell's equations with anisotropic cubic nonlinearity is studied. The problem describes propagation of transverse magnetic waves in a dielectric layer with perfectly conducted walls. Layer is filled with (nonlinear) anisotropic Kerr medium. It is shown that the anisotropy coefficients differently and crucially influences the propagation process. It is proved that in some cases (even for small values of the nonlinearity coefficients), the nonlinear problem has infinitely many nonperturbative solutions, whereas the corresponding linear problem always has a finite number of solutions.

1 Statement of the problem

Let ε_x, ε_z, γ, h be positive and α, β be nonnegative real parameters. Consider the system of equations

$$\begin{cases} -Z'' + \gamma X' = (\varepsilon_z + \beta X^2 + \alpha Z^2)Z, \\ -Z' + \gamma X = \gamma^{-1}(\varepsilon_x + \alpha X^2 + \beta Z^2)X, \end{cases} \tag{1}$$

where $X \equiv X(x;\gamma)$, $Z \equiv Z(x;\gamma)$ and $x \in [0,h]$.

The problem Q is to find positive numbers $\gamma = \widehat{\gamma}$ such that there exist functions $X \equiv X(x;\widehat{\gamma})$, $Z \equiv Z(x;\widehat{\gamma})$ satisfying (1) and boundary conditions

$$Z(0;\widehat{\gamma}) = 0, \quad X(0;\widehat{\gamma}) = X_0 > 0, \tag{2}$$

$$Z(h;\widehat{\gamma}) = 0, \tag{3}$$

where X_0 is a constant. The choice of this constant is described below, see section 2.2.

It is also assumed that

$$X \in C^1[0,h], \quad Z \in C^2[0,h]. \tag{4}$$

Definition 1 *A positive number $\gamma = \widehat{\gamma} > 0$ such that there exist nontrivial functions $X \equiv X(x;\widehat{\gamma})$, $Z \equiv Z(x;\widehat{\gamma})$ satisfying (1) and (2)–(4) is called an eigenvalue of the problem Q; the corresponding functions $X(x;\widehat{\gamma}), Z(x;\widehat{\gamma})$ are called eigenfunctions (or eigenmodes) of the problem Q.*

Remark 2 *If a three-tuple (X,Z,γ) is a solution to system (1), then the three-tuples $(-X,Z,-\gamma)$ and $(X,-Z,-\gamma)$ are also solutions to (1); for this reason it is enough to study the case of positive $\gamma > 0$, $X_0 > 0$.*

If $\alpha = \beta = 0$, one gets the linear problem, which is denoted by Q_0. The eigenvalues of this problem are denoted by $\widetilde{\gamma}$.

It is easy to check that the problem Q_0 has a finite number of positive eigenvalues $\gamma = \widetilde{\gamma}$, where $\widetilde{\gamma}^2 \in (0, \varepsilon_x)$. Indeed, solving (1) with $\alpha = \beta = 0$ and using boundary conditions (2), (3), one obtains the dispersion equation

$$\sin h\sqrt{\varepsilon_x^{-1}\varepsilon_z(\varepsilon_x - \gamma^2)} = 0, \tag{5}$$

which determines the eigenvalues. Solving (5), one gets

$$\widetilde{\gamma} = \sqrt{\varepsilon_x - \varepsilon n^2},$$

where $\varepsilon = \varepsilon_x \varepsilon_z^{-1} h^{-2}\pi^2$ and $n = 1, 2, \ldots$ until the radicand is positive.

Problem Q describes the propagation of a monochromatic TM wave $(\mathbf{E},\mathbf{H})e^{-i\omega t}$ in a plain waveguide

$$\Sigma = \{(x,y,z) : 0 \leqslant x \leqslant h, (y,z) \in \mathbb{R}^2\}$$

with perfectly conducted walls, where

$$\mathbf{E} = \left(\mathrm{E}_x(x), 0, \mathrm{E}_z(x)\right)^\top e^{i\gamma z}, \quad \mathbf{H} = \left(0, \mathrm{H}_y(x), 0\right)^\top e^{i\gamma z} \tag{6}$$

are the complex amplitudes [1], γ is an unknown real spectral parameter, ω is the circular frequency and $(\,\cdot\,)^\top$ is the transposition operation. The field (6) is called TM-polarized [2].

The permittivity ε of the waveguide is described by the (3×3)-tensor

$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}, \tag{7}$$

where

$$\varepsilon_{xx} = \varepsilon_x + a|\mathrm{E}_x|^2 + b|\mathrm{E}_z|^2, \quad \varepsilon_{zz} = \varepsilon_z + b|\mathrm{E}_x|^2 + a|\mathrm{E}_z|^2,$$

and ε_x, ε_z, a, b are real positivie constants [3]; the entry $*$ does not affect the propagation process.

The fields (6) satisfy Maxwell's equations

$$\begin{cases} \operatorname{rot}\mathbf{H} = -i\omega\varepsilon\mathbf{E}, \\ \operatorname{rot}\mathbf{E} = i\omega\mu\mathbf{H}, \end{cases} \tag{8}$$

where $\mu > 0$ is the permeability of free space [3].

Substituting fields (6) into equations (8), one finds

$$\begin{cases} i\gamma \mathrm{E}_x(x) - \mathrm{E}_z'(x) = i\omega\mu \mathrm{H}_y(x), \\ \mathrm{H}_y'(x) = -i\omega\varepsilon_{zz}\mathrm{E}_z(x), \\ i\gamma \mathrm{H}_y(x) = i\omega\varepsilon_{xx}\mathrm{E}_x(x). \end{cases}$$

Expressing $\mathrm{H}_y(x)$ from the first equation of the latter system, differentiating it, and then using the second and third equations of the system, one arrives at the system (1), where $X := i\mathrm{E}_x$, $Z := \mathrm{E}_z$ and $\varepsilon_x := \omega^2\mu\varepsilon_x$, $\varepsilon_z := \omega^2\mu\varepsilon_z$, $\alpha := \omega^2\mu a$, $\beta := \omega^2\mu b$.

It is known that tangential components of the electric field (in our case it is the z-th component) vanish on the perfectly conducted walls [2]. We assume that the x-th component of the electric field has a fixed value at the point $x = 0$. These physical conditions lead to boundary conditions (2)–(3). Natural requirement of the smoothness of the field for $x \in [0, h]$ leads to condition (4).

The fields (6) propagate in the layer Σ only for special values of γ; in electromagnetics this values are called propagation constants (PCs) [1–3]. From the mathematical standpoint the PCs are eigenvalues of the problem Q, see definition 1.

For an open plane waveguide, the problem Q has a long history. Such a problem, for the first time, was mentioned in the beginning of 1970's in [1] for the case of scalar nonlinear permittivity $\varepsilon = \varepsilon_l + a|\mathbf{E}|^2$ in the layer. From that time it stayed unsolved in spite of many attempts, see [3–6] and the bibliography therein. In [5] many special cases are considered, numerical results are presented in [3, 5, 7, 8]. For the scalar case, an essential advance has been achieved in [9]. Up to now for an open waveguide, the anisotropic case remains unsolved.

The permittivity of the form (7) is simplified in different ways by some researchers. Roughly speaking, it is assumed that either one of the entries of ε in formula (7) is constant and the other one depends on the field (or one of its components) [10–12] or both entries depend only on one component of the electric field [13, 14]. Both approaches are not correct of course; it is just an opportunity to integrate Maxwell's equations exactly and then derive the dispersion equation (DE) in an explicit form (in both cases, equations (8) are integrated in elliptic functions). However, in this case, the DE derived via complicated special functions is not easy to handle.

2 Preperatory results

2.1 First integral

Differentiating the second equation in (1), one finds

$$-Z'' + \gamma X' = \tfrac{2}{\gamma}(\alpha XX' + \beta ZZ')X + \tfrac{1}{\gamma}\left(\varepsilon_x + \alpha X^2 + \beta Z^2\right)X'.$$

Using the found relation, system (1) is reduced to the following one

$$\begin{cases} \dfrac{dX}{dx} = \dfrac{2\beta(\varepsilon_x - \gamma^2 + \alpha X^2 + \beta Z^2)}{\gamma(\varepsilon_x + 3\alpha X^2 + \beta Z^2)} X^2 Z + \\ \qquad + \dfrac{\gamma(\varepsilon_z + \beta X^2 + \alpha Z^2)}{\varepsilon_x + 3\alpha X^2 + \beta Z^2} Z, \\ \dfrac{dZ}{dx} = -\tfrac{1}{\gamma}\left(\varepsilon_x - \gamma^2 + \alpha X^2 + \beta Z^2\right)X. \end{cases} \quad (9)$$

System (9) has the first integral, which can be written in the form

$$\begin{aligned} &\left(\varepsilon_x + \alpha X^2 + \beta Z^2\right)^2 X^2 + \\ &\quad + \gamma^2(\varepsilon_x X^2 + \varepsilon_z Z^2) - 2\gamma^2\left(\varepsilon_x + \alpha X^2 + \beta Z^2\right)X^2 + \\ &\qquad + \tfrac{1}{2}\gamma^2(\alpha X^4 + 2\beta X^2 Z^2 + \alpha Z^4) \equiv C \end{aligned} \quad (10)$$

where C is a constant.

2.2 Additional condition

Calculating (10) at $x = 0$ and using (2), one obtains

$$(\varepsilon_l - \gamma^2 + \alpha X_0^2)^2 X_0^2 + \gamma^2(\varepsilon_l - \gamma^2)X_0^2 + \tfrac{1}{2}\alpha\gamma^2 X_0^4 = C, \quad (11)$$

where $X_0 = X(0)$.

Let $A > 0$ be a given constant. The value X_0 mentioned in (2) is determined from the equation

$$C(X_0^2) = A, \quad (12)$$

where $C(X_0^2)$ is the left-hand side of (11).

Statement 3 *Let $\alpha > 0$ and $\beta \geqslant 0$, then for any $A > 0$ and $\gamma > 0$ equation (12) has a unique (positive) solution X_0^2, such that for sufficiently big γ it is true that*

$$X_0^2 = \frac{3}{2\alpha}\gamma^2 + O(1). \quad (13)$$

Statement 4 *Let $\alpha = 0$ and $\beta \geqslant 0$, then for any $A > 0$ and $\gamma \in (0, \sqrt{\varepsilon_x})$ equation (12) has a unique (positive) solution*

$$X_0^2 = \frac{A}{\varepsilon_x(\varepsilon_x - \gamma^2)}.$$

2.3 New variables

Introduce new variables $\bar\tau$ and $\bar\eta$

$$\bar\tau = \frac{\varepsilon_x + \alpha X^2 + \beta Z^2}{\gamma^2}, \quad \bar\eta = \frac{\bar\tau X}{Z}, \quad (14)$$

where $\bar\tau \equiv \bar\tau(x;\gamma), \bar\eta \equiv \bar\eta(x;\gamma)$; in addition, we will use the notation

$$\bar\varepsilon_x = \varepsilon_x\gamma^{-2},\ \bar\varepsilon_z = \varepsilon_z\gamma^{-2},\ \bar A = A\gamma^{-6}.$$

Equations (9) take the form

$$\begin{cases} \frac{d\bar{\tau}}{dx} & = V(\bar{\tau}, \bar{\eta}; \gamma), \\ \frac{d\bar{\eta}}{dx} & = W(\bar{\tau}, \bar{\eta}; \gamma), \end{cases} \tag{15}$$

where

$$V(\bar{\tau},\bar{\eta};\gamma) \equiv \gamma \frac{2\bar{\eta}\bar{\tau}(\bar{\tau}-\bar{\varepsilon}_x)}{(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)} \cdot \frac{\alpha\bar{\varepsilon}_z(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)+\alpha(\bar{\tau}-\bar{\varepsilon}_x)(\beta\bar{\eta}^2+\alpha\bar{\tau}^2)}{\bar{\tau}(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)+2\alpha\bar{\eta}^2(\bar{\tau}-\bar{\varepsilon}_x)} - \gamma \frac{2\bar{\eta}\bar{\tau}(\bar{\tau}-\bar{\varepsilon}_x)}{(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)} \cdot \frac{\beta\bar{\tau}(\bar{\tau}-1)(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)}{\bar{\tau}(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)+2\alpha\bar{\eta}^2(\bar{\tau}-\bar{\varepsilon}_x)},$$

$$W(\bar{\tau},\bar{\eta};\gamma) \equiv \gamma \frac{\left(\bar{\varepsilon}_z\bar{\tau}+\bar{\eta}^2(\bar{\tau}-1)\right)(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)+\bar{\tau}(\bar{\tau}-\bar{\varepsilon}_x)(\beta\bar{\eta}^2+\alpha\bar{\tau}^2)}{\bar{\tau}(\alpha\bar{\eta}^2+\beta\bar{\tau}^2)}.$$

Integral (10) takes the form

$$\bar{F}(\bar{\tau},\bar{\eta};\gamma) \equiv \bar{a}_1\bar{\eta}^4 + 2\bar{a}_2\bar{\tau}^2\bar{\eta}^2 + \bar{a}_3\bar{\tau}^4 = 0, \tag{16}$$

where $\bar{a}_i \equiv \bar{a}_i(\bar{\tau};\gamma)$ and

$$\bar{a}_1(\bar{\tau};\gamma) \equiv \alpha\bar{\tau}(\bar{\tau}-\bar{\varepsilon}_x)(2\bar{\tau}-3) + \alpha\bar{\varepsilon}_x(\bar{\tau}-\bar{\varepsilon}_x) - \alpha^2\bar{A},$$
$$\bar{a}_2(\bar{\tau};\gamma) \equiv \beta\bar{\tau}(\bar{\tau}-\bar{\varepsilon}_x)(\bar{\tau}-1) + \alpha\bar{\varepsilon}_z(\bar{\tau}-\bar{\varepsilon}_x) - \alpha\beta\bar{A},$$
$$\bar{a}_3(\bar{\tau};\gamma) \equiv \alpha(\bar{\tau}-\bar{\varepsilon}_x)^2 + 2\beta\bar{\varepsilon}_z(\bar{\tau}-\bar{\varepsilon}_x) - \beta^2\bar{A}.$$

In spite of the fact that formulas (15)–(16) are derived under condition $\alpha,\beta > 0$, one can check, however, that the cases $\alpha > 0, \beta = 0$ and $\alpha = 0, \beta > 0$ are also covered by these formulas.

Let us define the set Γ, where $\Gamma := (0, +\infty)$ for $\alpha > 0, \beta \geqslant 0$ and $\Gamma := (0, \sqrt{\bar{\varepsilon}_x})$ for $\alpha = 0, \beta > 0$.

The following result takes place.

Statement 5 *First integral* (16) *defines a unique function* $\bar{\tau} \equiv \bar{\tau}(\bar{\eta};\gamma)$, *that depends continuously on* $(\bar{\eta},\gamma) \in \mathbb{R}\times\Gamma$ *and satisfies the estimate* $\bar{\varepsilon}_x < \bar{\tau}(\bar{\eta},\gamma) < \bar{\tau}_0$, *where* $\bar{\tau}_0$ *is bounded.*

3 Main results

Below, eigenvalues of the problem Q will be denoted $\widehat{\gamma}$ as well as $\widehat{\gamma}_i$. The notation $\widehat{\gamma}_i$ means that the eigenvalues are ordered in the ascending order.

In addition, we will use the notation

$$T(\gamma) := \int_{-\infty}^{+\infty} \frac{ds}{W(\bar{\tau}, s; \gamma)},$$

where function $\bar{\tau} \equiv \bar{\tau}(s;\gamma)$ is defined by (16) for $\bar{\eta} = s$.

It can be proved that DE of the problem Q has the form

$$(n+1)T(\gamma) = h, \tag{17}$$

where $n = 0, 1, \ldots$ [15].

The following result takes place.

Theorem 6 (of equivalence) *For any fixed nonnegative* α,β *with* $\alpha+\beta > 0$ *problem* Q *is equivalent to DE* (17) *in the sense that the value* $\widehat{\gamma}(>0)$ *is an eigenvalue of the problem* Q *if and only if there is an integer* $\widehat{n} \geqslant 0$ *such that* $\gamma = \widehat{\gamma}$ *is a solution to* (17) *for* $n = \widehat{n}$ *and* $\widehat{\gamma} \in \Gamma$.

Relation (17) is a family (but not a system!) of equations for different n.

Remark 7 *DE* (17) *is valid even for* $\alpha = \beta = 0$. *In this case* $\gamma \in (0, \sqrt{\bar{\varepsilon}_x})$. *In order to get well-defined integrands in* (17), *one should set* $\alpha = \beta$ *and after this set* $\alpha = 0$. *Since* $\bar{\tau}$ *becomes* $\bar{\varepsilon}_x$ *(constant), then one does not need first integral* (16).

Two corollaries result from theorem 6.

Corollary 8 *Let* $\gamma = \widehat{\gamma}$ *be a solution to* (17) *with* $n = \widehat{n}$ *and* $X(x;\widehat{\gamma})$, $Z(x;\widehat{\gamma})$ *be the eigenfunctions, then* $Z(x;\widehat{\gamma})$ *has exactly* $\widehat{n}+2$ *(simple) zeroes* $x_i = iT(\widehat{\gamma})$, $i = \overline{0, \widehat{n}+1}$.

Corollary 9 *Let* $\gamma = \widehat{\gamma}$ *be a solution to* (17) *with* $n = \widehat{n}$, *then* $T(\widehat{\gamma}) = \frac{h}{\widehat{n}+1}$.

The periodicity result is given by

Theorem 10 *If the eigenfunction* $Z(x;\widehat{\gamma})$, *where* $\widehat{\gamma}$ *is an eigenvalue, has more than one zero for* $x \in (0,h)$, *then* $Z(x;\widehat{\gamma})$ *and* $X(x;\widehat{\gamma})$ *are periodic with the period* $\Theta = 2T(\widehat{\gamma})$.

Note: theorem 10 takes place for nonnegative α,β, see also remark 7.

Properties of the function $T(\gamma)$ play a crucial role. For this reason the solvability result is premised on the following

Statement 11 *If* $\alpha > 0, \beta \geqslant 0$, *then* $T(\gamma)$ *is positive and depends continuously on* γ *for* $\gamma \in \Gamma$; *for big* γ *it is true that*

$$T(\gamma) = \delta\frac{\ln\gamma}{\gamma} + O(\gamma^{-1}),$$

where $\delta = 2\alpha^{-\frac{1}{2}}\left((3\alpha^2+\beta^2)^{\frac{1}{2}}+\beta\right)^{\frac{1}{2}}$.

Statement 12 *If* $\alpha = 0, \beta > 0$, *then* $T(\gamma)$ *is positive and depends continuously on* γ *for* $\gamma \in \Gamma$; *in addition*

$$\lim_{\gamma \to \sqrt{\bar{\varepsilon}_x}-0} T(\gamma) = +\infty.$$

Results of statements 11–12 are main ingredients to prove solvability of the problem Q. Such results are given by theorems 13 and 15.

Theorem 13 *If $\alpha > 0, \beta \geqslant 0$, then the problem Q has infinetely many eigenvalues $\widehat{\gamma}_i$ with accumulation point at infinity. In addition, the following assertions take place*:

(i) *If there are p eigenvalues $\widetilde{\gamma}_i$ $(i = \overline{1,p})$ in the problem Q_0, then there exist $\alpha_0 > 0, \beta_0 \geqslant 0$ such that for any positive $\alpha = \alpha' < \alpha_0$ and nonnegative $\beta = \beta' \leqslant \beta_0'$ it is true that $\widehat{\gamma}_i \in (0, \sqrt{\varepsilon_x})$ and*

$$\lim_{\alpha' \to +0} \lim_{\beta' \to +0} \widehat{\gamma}_i = \lim_{\beta' \to +0} \lim_{\alpha' \to +0} \widehat{\gamma}_i = \widetilde{\gamma}_i \ \ (i = \overline{1,p}),$$

where $\widehat{\gamma}_1 < \ldots < \widehat{\gamma}_p$ are first p solutions to the problem Q with $\alpha = \alpha', \beta = \beta'$;

(ii) *If $X \equiv X(x; \widehat{\gamma}_i)$ and $Z \equiv Z(x; \widehat{\gamma}_i)$ are eigenfunctions for a particular $\widehat{\gamma}_i$, then*

$$\max_{x \in [0,h]} X^2 = \frac{3}{2\alpha} \widehat{\gamma}_i^2 + O(1), \ \ \max_{x \in [0,h]} Z^2 = \frac{1}{\alpha+\beta} \widehat{\gamma}_i^2 + O(1)$$

as $\widehat{\gamma}_i \to \infty$.

Note: if $\beta = 0$, then $\lim_{\beta' \to +0}$ is dropped.

Corollary 14 *Infinitely many eigenvalues $\widehat{\gamma}_i$ of the problem Q for $\alpha > 0$, $\beta \geqslant 0$ do not have linear counterparts.*

Theorem 15 *If $\alpha = 0$, $\beta > 0$, then the problem Q has a finite number* (*possibly no one*) *of eigenvalues $\widehat{\gamma}_i \in \Gamma$. In addition, if there are p eigenvalues $\widetilde{\gamma}_1 < \widetilde{\gamma}_2 < \ldots < \widetilde{\gamma}_p$ in the problem Q_0, then there exists $\beta_0 > 0$ such that for any positive $\beta = \beta' < \beta_0$ and $\alpha = 0$ the problem Q has at least p eigenvalues $\widehat{\gamma}_i$ and it is true that*

$$\widehat{\gamma}_i \in (0, \sqrt{\varepsilon_x}) \quad \text{and} \quad \lim_{\beta' \to +0} \widehat{\gamma}_i = \widetilde{\gamma}_i, \quad i = \overline{1,p},$$

where $\widehat{\gamma}_1, \ldots, \widehat{\gamma}_p$ are first p solutions to the problem Q with $\alpha = 0$, $\beta = \beta'$.

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