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Multifrequency electromagnetic wave propagation in a dielectric slab with Kerr nonlinearity: perturbative and nonperturbative guided waves

V.Yu. Kurseeva[(1)], D.V. Valovik[(1)]
(1) Department of Mathematics and Supercomputing, Penza State University, 40, Krasnaya street, Penza, Russia

Abstract

The paper focuses on a particular problem of nonlinear multifrequency electromagnetic wave propagation that is called problem P. The problem P describes propagation of a finite sum of n monochromatic TE waves guide by a dielectric layer having infinitely conducted walls. The permittivity of the dielectric is described by the Kerr law. The multifrequency guided wave is thus characterised by n different frequencies and n propagation constants. The physical problem is reduced to a nonlinear multiparameter eigenvalue problem. It is shown that there are nonlinear guided waves with and without linear counterparts.

1 General statement of the problem

Let $\Sigma = \{(x,y,z) \in \mathbb{R}^3 : 0 \leqslant x \leqslant h, (y,z) \in \mathbb{R}^2\}$ be a layer placed in $\mathbb{R}^3$ and filled with nonlinear dielectric. The permittivity ε of the dielectric will be described below; the permeability μ of the dielectric is a positive constant. The layer has infinitely conducting walls at $x = 0$ and $x = h$.

In accordance with [1], introduce the multifrequency field

$$\mathbf{E}_\omega = \sum_{j=1}^{n} \mathbf{E}_j e^{-i\omega_j t}, \quad \mathbf{H}_\omega = \sum_{j=1}^{n} \mathbf{H}_j e^{-i\omega_j t}, \tag{1}$$

where $\mathbf{E}_j = \mathbf{E}_j^+ + i\mathbf{E}_j^-$ and $\mathbf{H}_j = \mathbf{H}_j^+ + i\mathbf{H}_j^-$ are the complex amplitudes [2]. The real (physical) field $\tilde{\mathbf{E}}_\omega$, $\tilde{\mathbf{H}}_\omega$ has the form $\tilde{\mathbf{E}}_\omega(x,y,z,t) = \mathrm{Re}\,\mathbf{E}_\omega$, $\tilde{\mathbf{H}}_\omega(x,y,z,t) = \mathrm{Re}\,\mathbf{H}_\omega$. Frequencies ω_j are different but there can be restrictions related to a particular nonlinear law chosen for ε [1, 3, 4].

We assume that the permittivity ε is a diagonal (3×3)-tensor that depends on the field by the Kerr law, that is,

$$\varepsilon(\tilde{\mathbf{E}}_\omega) \equiv \begin{pmatrix} \varepsilon_x + f_x & 0 & 0 \\ 0 & \varepsilon_y + f_y & 0 \\ 0 & 0 & \varepsilon_x + f_x \end{pmatrix}, \tag{2}$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are real positive constants and

$$f_r \equiv \sum_{j=1}^{n} \left(\beta_{x,j,r} |(\mathbf{E}_j, \mathbf{e}_x)|^2 + \beta_{y,j,r} |(\mathbf{E}_j, \mathbf{e}_y)|^2 + \beta_{z,j,r} |(\mathbf{E}_j, \mathbf{e}_z)|^2 \right);$$

here β_{r,j,r_1} are real constants, $(\cdot\,,\cdot)$ is the euclidian scalar product, $\mathbf{e}_r$ is a unit vector in r-direction, $r, r_1 \in \{x,y,z\}$.

The permittivity in the form (2) is not as general as possible of course.Nevertheless, such a permittivity is in agreement with some real situations [2, 4–13] and is sufficient to study various types of waves, for example, TE, TM, and, so called, coupled TE-TE and TE-TM waves in the Kerr case.

Well, substituting fields (1) into Maxwell's equations, one derives that the complex amplitudes $\mathbf{E}_k$, $\mathbf{H}_k$ satisfy the following (coupled) equations

$$\begin{cases} \mathrm{rot} \sum\limits_{j=1}^{n} \mathbf{H}_j e^{-i\omega_j t} = -i\varepsilon \sum\limits_{j=1}^{n} \omega_j \mathbf{E}_j e^{-i\omega_j t}, \\ \mathrm{rot} \sum\limits_{j=1}^{n} \mathbf{E}_j e^{-i\omega_j t} = i\mu \sum\limits_{j=1}^{n} \omega_j \mathbf{H}_j e^{-i\omega_j t}. \end{cases}$$

The operator rot is linear and thus the latter gives

$$\begin{cases} \sum\limits_{j=1}^{n} e^{-i\omega_j t}\, \mathrm{rot}\, \mathbf{H}_j = -i\varepsilon \sum\limits_{j=1}^{n} \omega_j \mathbf{E}_j e^{-i\omega_j t}, \\ \sum\limits_{j=1}^{n} e^{-i\omega_j t}\, \mathrm{rot}\, \mathbf{E}_j = i\mu \sum\limits_{j=1}^{n} \omega_j \mathbf{H}_j e^{-i\omega_j t}. \end{cases}$$

Since the derived system must be fulfilled for all t, then one arrives at the following system of n (coupled) systems

$$\begin{cases} \mathrm{rot}\, \mathbf{H}_j = -i\varepsilon \omega_j \mathbf{E}_j, \\ \mathrm{rot}\, \mathbf{E}_j = i\mu \omega_j \mathbf{H}_j, \quad \text{where } j = \overline{1,n}. \end{cases} \tag{3}$$

Thus the complex amplitudes $\mathbf{E}_j$, $\mathbf{H}_j$ satisfy equations (3), tangential components of the electric fields $\mathbf{E}_j$ vanish at the interfaces $x = 0$, $x = h$. An additional condition is also needed; for example, one can fix value of the field at one of the boundaries, see second formulas in (7) and (10).

2 Multifrequency guided waves of TE type

Let us consider a particular configuration of the filed (1) that results in the problem studied in sections 3–4. Let an integer index j' be such that $1 \leqslant j' \leqslant n$. We consider the fields $\mathbf{E}_j, \mathbf{H}_j$ to be of the form

$$\begin{aligned} &\mathbf{E}_j = (0, e_y^{(j)}, 0)^\top e^{i\gamma_j z}, \quad \mathbf{H}_j = (h_x^{(j)}, 0, h_z^{(j)})^\top e^{i\gamma_j z}, \\ &\mathbf{E}_j = (0, 0, e_z^{(j)})^\top e^{i\gamma_j y}, \quad \mathbf{H}_j = (h_x^{(j)}, h_y^{(j)}, 0)^\top e^{i\gamma_j y} \end{aligned} \tag{4}$$

for $1 \leqslant j \leqslant j'$ and $j' \leqslant j \leqslant n$ in the former and letter lines of (4), respectively; here components $e_y^{(j)}$, $e_z^{(j)}$, $h_x^{(j)}$, $h_y^{(j)}$,

$h_z^{(j)}$ depend on spatial variable x only (of course, these quantities, as solutions to Maxwell's equations, also depend on other parameters of the problem) and γ_j are unknown real constants. In other words, we consider a sum of transverse-electric fields propagating in directions Oz and Oy, respectively.

Substituting fields (4) into equations (3), taking into account (2), and using the notation $u_j := e_y^{(j)}$ for $j = \overline{1, j'}$ and $u_j := e_z^{(j)}$ for $j = \overline{j', n}$, after some algebra one arrives at the following system

$$\begin{cases} u_1'' = -(\varepsilon_{1,1} - \mu\omega_1^2\gamma_1^2)u_1 - \left(\beta_{1,1}u_1^2 + \ldots + \beta_{1,n}u_n^2\right)u_1, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ u_{j'}'' = -(\varepsilon_{1,j'} - \mu\omega_{j'}^2\gamma_{j'}^2)u_{j'} - \left(\beta_{1,1}u_1^2 + \ldots + \beta_{1,n}u_n^2\right)u_{j'}, \\ u_{j'+1}'' = -(\varepsilon_{2,j'+1} - \mu\omega_{j'+1}^2\gamma_{j'+1}^2)u_{j'+1} - \\ \qquad - \left(\beta_{2,1}u_1^2 + \ldots + \beta_{2,n}u_n^2\right)u_{j'+1}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ u_n'' = -(\varepsilon_{2,n} - \mu\omega_n^2\gamma_n^2)u_n - \left(\beta_{2,1}u_1^2 + \ldots + \beta_{2,n}u_n^2\right)u_n, \end{cases} \tag{5}$$

where $\varepsilon_{1,j} = \varepsilon_y\mu\omega_j$ for $j = \overline{1, j'}$, $\varepsilon_{2,j} = \varepsilon_z\mu\omega_j$ for $j = \overline{j', n}$, $\beta_{1,j} = \mu\omega_j^2\beta_{y,j,y}$ for $j = \overline{1, j'}$, $\beta_{1,j} = \mu\omega_j^2\beta_{z,j,y}$ for $j = \overline{j'+1, n}$, $\beta_{2,j} = \mu\omega_j^2\beta_{y,j,z}$ for $j = \overline{1, j'}$, $\beta_{2,j} = \mu\omega_j^2\beta_{z,j,z}$ for $j = \overline{j'+1, n}$.

Tangential electric field components vanish at perfectly conducting walls [14]. In this case $e_y^{(j)}$ and $e_z^{(j)}$ are tangential components. Thus $u_j|_{x=0} = u_j|_{x=h} = 0$ for $j = \overline{1, n}$. We also fix values of u_j' at the boundary $x = 0$. The conditions listed in this section result in conditions (7), (8) if $n \geqslant 2$ and conditions (10), (11) if $n = 1$.

Field (4) propagates in Σ only for special values of γ_j. These values are called *propagation constants* (PCs). Thus, the main problem is to determine PCs. From the mathematical standpoint, the above formulated problem is a nonlinear multiparameter eigenvalue problem for system (5) with the above listed boundary conditions. The eigentuples (or eigenvalues in the one-parameter case) are PCs.

Nonlinear laws that are used in the waveguiding nonlinear optics have small factors; these factors can be considered as small parameters (this is true for the Kerr nonlinearity) [3, 4, 15]. This allows one to apply a perturbation method based on linear problems and prove existence of solutions to the nonlinear problem that are close to solutions of the used linear ones (see, for example, [16, 17]).

In linear eigenvalue problems eigenfunctions are determined up to a constant factor; the eigenvalues are uniquely determined [18]. For nonlinear eigenvalue problems (when equations depend nonlinearly on the searched for functions), the same boundary conditions are not enough.

As is known, linear electromagnetic wave propagation problems in a plane layer have discrete sets of PCs (see, for example, [14, 19]). If one generalises a linear problem to the nonlinear situation, then it is natural to formulate the nonlinear problem in such a way that solutions to the nonlinear problem have linear counterparts at least for 'small' nonlinearities. Thus, the necessity of an additional conditions is clear.

As an additional condition, one can fix (or prescribe) value of the field components (or their derivatives) at one of the boundaries, for example at $x = 0$. Fixing norms (in an appropriate function space) of $e_y^{(j)}$ and $e_z^{(j)}$, one gets another variant of the additional condition. We should stress however that in an open waveguide, a natural additional condition is the former one; moreover, the latter condition is not suitable from the physical point of view, see also [1].

3 Nonlinear eigenvalue problems

Below integer indexes i, j vary from 1 to $n \geqslant 2$ and often we do not indicate this explicitly.

We introduce $2n$ positive constants a_i, b_i and n^2 nonnegative constants α_{ij} as well as n real parameters λ_i. In addition, we consider n-tuple λ and $(n \times n)$-tuples α, α', and 0.

The tuple λ consists of n parameters λ_i and can be considered as an n-dimensional vector $\lambda = (\lambda_1, \ldots, \lambda_n)$. The tuple α consists of n^2 parameters α_{ij} and can be considered as a $(n \times n)$-matrix. The tuple α' consists of n^2 parameters α_{ij}, where $\alpha_{ii} > 0$ and α_{ij} for $i \neq j$ are zeros; the tuple 0 can be considered as a zero $(n \times n)$-matrix.

We define sets $\Lambda_i = [0, \lambda_i^*)$, where λ_i^* are positive sufficiently big constants. The choice of λ_i^* will be clear from theorems 3 and 4. It is assumed that $\alpha_{ij} \in \mathrm{A}_{ij}$, where $\mathrm{A}_{ij} = (0, \alpha_{ij}^*)$. In this notation α_{ii}^* are arbitrary but fixed positive constants and α_{ij}^* for $i \neq j$ are positive constants that depend on α_{ii}^* and λ_i^*. The parameters α_{ij}^* for $i \neq j$, in general, are sufficiently small, see theorem 4.

Below we use the notation $\prod_l \mathrm{C}_l$ as well as $\mathrm{C}_1 \times \ldots \times \mathrm{C}_k$ to define a (finite) Cartesian product of sets C_l. We define the following Cartesian products $\Lambda = \prod_i \Lambda_i$, $\mathbf{A} = \prod_{i,j} \mathrm{A}_{ij}$. The notation $\lambda \in \Lambda$ and $\alpha \in \mathbf{A}$ mean that $\lambda_i \in \Lambda_i$ and $\alpha_{ij} \in \mathrm{A}_{ij}$, respectively. We denote the interval $(0, h)$ and segment $[0, h]$ by I and $\bar{\mathrm{I}}$, respectively.

Now let us consider the system of n coupled equations

$$\begin{cases} u_1'' = -(a_1 - \lambda_1)u_1 - \left(\alpha_{11}u_1^2 + \ldots + \alpha_{1n}u_n^2\right)u_1, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ u_n'' = -(a_n - \lambda_n)u_n - \left(\alpha_{n1}u_1^2 + \ldots + \alpha_{nn}u_n^2\right)u_n, \end{cases} \tag{6}$$

where the prime marks denote differentiation with respect to x; here it is assumed that $(x, \lambda, \alpha) \in \bar{\mathrm{I}} \times \mathbb{R}^n \times \mathbf{A}$. Solutions to system (6) are denoted by u_i, $u_i(x)$, or $u_i(x; \lambda, \alpha)$.

The problem $P = P(\alpha)$ consists in finding n-tuples λ for which there exist solutions $u_1 \equiv u_1(x;\lambda,\alpha),\ldots,u_n \equiv u_n(x;\lambda,\alpha)$ to system (6) that satisfy boundary conditions

$$u_i(0;\lambda,\alpha) = 0, \quad u_i'(0;\lambda,\alpha) = b_i, \tag{7}$$

$$u_i(h;\lambda,\alpha) = 0, \tag{8}$$

and such that $u_1,\ldots,u_n \in C^2(\bar{\mathrm{I}})$.

The correspondence between equations (5) and (6) is clear. In fact, system (6) is more general than (5).

If $\alpha \to \alpha'$, that is, $\alpha_{ij} \to +0$ for $i \neq j$, then the problem $P(\alpha)$ degenerates into the problem $P(\alpha')$. As is seen from system (6) and formulas (7)–(8), the problem $P(\alpha')$ consists of n independent nonlinear problems. These problems are denoted by P_i.

In order to formulate the problems P_i rigorously let us consider the equation

$$v_i'' = -(a_i - \lambda_i)v_i - \alpha_{ii}v_i^3, \tag{9}$$

where the prime marks denote differentiation with respect to x; here it is assumed that $(x,\lambda_i,\alpha_{ii}) \in \bar{\mathrm{I}} \times \mathbb{R} \times \mathbb{R}_+$, $\mathbb{R}_+ = (0,+\infty)$. Solutions to equation (9) are denoted by v_i, $v_i(x)$, or $v_i(x;\lambda_i,\alpha_{ii})$.

Every problem P_i consists in finding values λ_i for which there exist solutions $v_i \equiv v_i(x;\lambda_i)$ to equation (9) that satisfy boundary conditions

$$v_i(0;\lambda_i,\alpha_{ii}) = 0, \quad v_i'(0;\lambda_i,\alpha_{ii}) = b_i, \tag{10}$$

$$v_i(h;\lambda_i,\alpha_{ii}) = 0, \tag{11}$$

and such that $v_i \in C^2(\bar{\mathrm{I}})$.

Since γ_j in (4) are real, then $\mu\omega_j^2\gamma_j^2$ are positive and for this reason electromagnetic applications require only positive λ_j in the problems $P(\alpha)$ and P_j. If $n=1$ and, therefore, $j'=0$ or $j'=1$, then one comes to one of the problems P_j. If all $\beta_{1,j}$ and $\beta_{2,j}$ are zeros, then one arrives at n linear problems that arise when one needs to determine linear guided TE waves propagating in the layer Σ with linear permittivity. These linear problems are equivalent to the problems P_j^0 formulated in section 3.

4 Results

Below we use additional notation for the eigentuples and eigenvalues. Eigentuples λ of the problem $P(\alpha)$ will be denoted by $\bar{\lambda}_{k_1\ldots k_n} = (\bar{\lambda}_{1,k_1},\ldots,\bar{\lambda}_{n,k_n})$, where $k_1,\ldots,k_n$ are nonnegative integer indexes. Eigenvalues λ_i of the problems P_i and P_i^0 will be denoted by $\widehat{\lambda}_{i,k_i'}$ and $\widetilde{\lambda}_{i,k_i'}$, respectively, where k_i' are nonnegative integer indexes. It is assumed that eigenvalues $\widehat{\lambda}_{i,k_i'}$, $\widetilde{\lambda}_{i,k_i'}$ are arranged in the descending and ascending.

Since the problems P_i^0 are easily solved, then we immediately start with the following fact.

Statement 1 *For any $h \geqslant h_{\min} = \frac{\pi}{\sqrt{a_i}} > 0$ the problem P_i^0 has a finite number (not less than 1) of simple (positive) eigenvalues $0 \leqslant \widetilde{\lambda}_{i,1} < \ldots < \widetilde{\lambda}_{i,k} < a_i$; if $a_i = 0$, then the problem P_i^0 does not have positive solutions.*

Let us consider the problem P_i. We consider functions $\theta_i = v_i^2$, $\mu_i = v_i'/v_i$, where $v_i \equiv v_i(x;\lambda_i,\alpha_{ii})$ is a solution to the Cauchy problem for equation (9) with initial data (10). By virtue of (9), functions $\theta_i(x)$ and $\mu_i(x)$ satisfy the following system

$$\begin{cases} \theta_i' = 2\theta_i\mu_i, \\ \mu_i' = -(\mu_i^2 + a_i - \lambda_i + \alpha_{ii}\theta_i). \end{cases} \tag{12}$$

Taking into account (10), the first integral of system (12) has the form

$$\tfrac{1}{2}\alpha_{ii}\theta_i^2 + (\mu_i^2 + a_i - \lambda_i)\theta_i = b_i^2. \tag{13}$$

Let $T_i(\lambda_i) = \int\limits_{-\infty}^{+\infty} \frac{ds}{s^2 + a_i - \lambda_i + \alpha_{ii}\theta_i(s)}$, where $\theta_i(s)$ is defined from (13) with $\mu_i = s$.

Using the IDEM, we obtain

Statement 2 (of equivalence) *The value $\widehat{\lambda}_i$ is a solution to the problem P_i if and only if there exists an integer $m_i' = \widehat{m}_i \geqslant 0$ such that $\lambda_i = \widehat{\lambda}_i$ is a solution to the DE*

$$(m_i' + 1)T_i(\lambda_i) = h \tag{14}$$

for $m_i' = \widehat{m}_i$; the corresponding eigenfunction $v_i \equiv v_i(x;\widehat{\lambda}_i,\alpha_{ii})$ has $\widehat{m}_i$ (simple) zeros $x_{i,r}' \in \mathrm{I}$, where $x_{i,r}' = rT_i(\widehat{\lambda}_i) = \frac{rh}{\widehat{m}_i + 1}$, $r = \overline{1,\widehat{m}_i}$.

Analyzing dispersion equation (14), we get

Theorem 3 *There exist an integer $m_i' \geqslant 0$ such that for every integer $m \geqslant m_i'$ equation (14) has at least one (positive) solution $\widehat{\lambda}_i = \widehat{\lambda}_{i,m}$ with $\widehat{\lambda}_{i,m} \to +\infty$ as $m \to +\infty$ and, therefore, the problem P_i has infinitely many (positive) eigenvalues $\widehat{\lambda}_{i,m}$ with an accumulation point at infinity. Furthermore,*

1) *there is a constant $\lambda_i' > a_i$ such that all eigenvalues $\widehat{\lambda}_{i,m} \in [0,a_i) \cup (\lambda_i',+\infty)$ are simple eigenvlaues;*

2) *if the problem P_i^0 has p (positive) solutions $\widetilde{\lambda}_{i,0} < \widetilde{\lambda}_{i,1} < \ldots < \widetilde{\lambda}_{i,p-1}$, then there exists a constant $\alpha_{ii}'' > 0$ such that for any (positive) $\alpha_{ii} = \alpha_{ii}' < \alpha_{ii}''$ it is true that*

$$\widehat{\lambda}_{i,m} \in [0,a_i) \text{ and } \lim_{\alpha_{ii}' \to +0} \widehat{\lambda}_{i,m} = \widetilde{\lambda}_{i,m} \text{ for } m = \overline{0,p-1},$$

where $\widehat{\lambda}_{i,0},\ldots,\widehat{\lambda}_{i,p-1}$ are first p solutions to the problem P_i with $\alpha_{ii} = \alpha_{ii}'$;

3) *if* $m \geqslant p$, *then* $\widehat{\lambda}_{i,m}$ *has no linear counterpart and* $\lim_{\alpha_{ii}\to+0}\widehat{\lambda}_{i,m} = +\infty$;

4) $\max_{x\in(0,h)}|v_i(x;\widehat{\lambda}_{i,m},\alpha_{ii})| = O(s_m^{1/2})$ *as* $m\to\infty$, *where* $s_m = \widehat{\lambda}_{i,m}$.

Using problems P_i as nonperturbed and applying the IDEM, we obtain the main result of this paper.

Theorem 4 *Let every problem* P_i *have* m_i *simple eigenvalues* $\widehat{\lambda}_{i,1},\ldots,\widehat{\lambda}_{i,m_i} \in [0,a_i)\cup(\lambda_i',\lambda_i^*)\subset\Lambda_i$, *respectively. Then there exist positive constants* α_{ij}^* *for* $i\neq j$ *such that for any* $0<\alpha_{ij}<\alpha_{ij}^*$ $(i\neq j)$ *the problem* $P(\alpha)$ *has at least* $m_1\times\ldots\times m_n$ *eigentuples* $\bar{\lambda}_{k_1,k_2,\ldots,k_n} = (\bar{\lambda}_{1,k_1},\bar{\lambda}_{2,k_2},\ldots,\bar{\lambda}_{n,k_n})$, *where* $k_i=\overline{1,m_i}$; *furthermore, every* $\bar{\lambda}_{k_1,k_2,\ldots,k_n}$ *belongs to a neighbourhood of the point* $(\widehat{\lambda}_{1,k_1},\widehat{\lambda}_{2,k_2},\ldots,\widehat{\lambda}_{n,k_n})$.

Since values λ_i^* in Λ_i can be chosen as big as necessary, then theorem 4 states existence of eigentuples of the problem $P(\alpha)$ that, in particular, belong to the domain where there are no solutions to the problems P_i^0. This result predicts existence of a novel type of nonlinear guided waves.

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