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Ion-acoustic shock waves in ion-beam plasma with non-Maxwellian electrons

N. S. Saini*[(1)], Nimardeep Kaur[(1)] and Kuldeep Singh[(1)]
(1) Department of Physics, Guru Nanak Dev University, Amritsar-143005, India

Abstract

The nonlinear propagation of ion-acoustic shock waves has been investigated in an unmagnetized ion-beam plasma with electrons featuring non-Maxwellian hybrid distribution. The reductive perturbation method has been employed to derive the Korteweg-de Vries Burgers (KdVB) equation and by employing the tangent hyperbolic method, the travelling wave solution has been acquired. It is observed that the superthermality of electrons, number density and temperature of positive ion beam, kinematic viscosity etc. crucially modify the propagation properties of IA shock structures. The findings of the present study may be useful in understanding the insight into the physics of nonlinear structures in the polar cap region.

1 Introduction

A number of investigations have been reported on the study of nonlinear propagation of ion acoustic (IA) waves in magnetized as well as unmagnetized plasmas under different physical situations in different kinds of laboratory, space and astrophysical environments. There is the existence of solitary waves in a nonlinear dispersive media due to the balance between the dispersion and nonlinearity. However, a medium supports the existence of shock waves which possesses dispersive and significant dissipative properties, The study of shock wave propagation plays a prominent role in order to understand the underlying physics of acceleration of charged particles in various laboratory and astrophysical phenomena. Vladimirov and Yu [1] derived the KdVB equation to study the ion acoustic shock waves in collisional plasma, where nonlinear effects originates from thermal forces and inter-particles heat exchange. It was highlighted that the derived KdVB equation cannot be transformed to KdV equation due to fixed normalization. Sahu and Roychoudhury [2] derived the non-planar KdVB equation to determine the characteristics of ion acoustic shock waves in collisional plasma, having non-isothermal electrons, and found that in the limits of small of values of time coordinate τ the considered plasma is conducive for the propagation of KdV soliton as well as Burger shock.

The presence of energetic charged particles e.g., ion, electron or positron beam in any given plasma system significantly modifies the characteristics of various nonlinear structures. Shah et al. [3] investigated the effect of positron beam on the propagation characteristics of ion-acoustic shock waves. It was found that both the amplitude and steepness of the ion-acoustic shock waves increases with the enhancement in the spectral index of the superthermal electrons, and concentration of positron beam. Kaur et al. [4] investigated the characteristics of ion acoustic Gardner solitons in an unmagnetized plasma composed of a positive warm ion fluid, two temperature superthermal electrons embedded by a positive ion beam. It was found that the different physical parameters have profound effect on the various characteristics of nonlinear electrostatic excitations. The omnipresence of non-Maxwellian distributed particles suprathermal tails has been confirmed by various satellite missions in many space end astrophysical plasma environments. The non-Maxwellian hybrid distribution involving two commonly adopted distribution functions for describing the plasmas are the kappa distribution, characterized by the κ parameter and the Cairns distribution, characterized by the nonthermal parameter α [5, 6]. During the last few years, some studies with non-Maxwellian hybrid distribution have been reported [7, 8]. Singh et al. [8] studied head-on collision among dust acoustic (DA) multi-solitons in a dusty plasma with ions featuring non-Maxwellian hybrid distribution under the effect of the polarization force. It was found that the presence of the non-Maxwellian ions leads to the significant modification in polarization force.

To the best of our knowledge, the study of IA shock waves in an unmagnetized plasma comprising of electrons following non-Maxwellian hybrid distribution penetrated by a positive ion beam (He^+ beam) has not been reported so far. Motivated by the observations of DE-1 satellite, which has shown the existence of ion beam and its penetration into different plasma environments, it is imperative to investigate the influence of density, temperature and velocity of ion beam, superthermality of electrons, and kinematic viscosity on the characteristics of IA shock waves. The layout of manuscript is organised as: In section 2, the fluid equations are illustrated. The derivation of KdVB equation is presented in section 3. The numerical analysis is discussed in section 4. Conclusions are highlighted in section 5.

2 Fluid equations

A collisionless, unmagnetized plasma consisting of positive ion fluid, positive ion beam and electrons obeying non-Maxwellian hybrid-distribution is considered to study the characteristics of IA shock waves. The number density of electrons is given as [8]

$$n_e = n_{e0}(1 + h_1(e/k_BT_e)\phi + h_2(e/k_BT_e)^2\phi^2) \qquad (1)$$

where

$$h_1 = \begin{cases} 1-\Lambda, & \text{Cairns distribution.} \\ \frac{\kappa_e-1/2}{\kappa_e-3/2}, & \text{Kappa distribution.} \end{cases} \qquad (2)$$

$$h_2 = \begin{cases} 1/2, & \text{Cairns distribution.} \\ \frac{\kappa_e^2-1/4}{(\kappa_e-3/2)^2}, & \text{Kappa distribution.} \end{cases} \qquad (3)$$

Here, $\Delta = 4\alpha/(1+3\alpha)$ and α is a parameter that determines the nonthermal effects in phase space of Cairns distribution. At equilibrium, the charge neutrality condition is given by $\mu_e = 1 + \mu_b$, where $\mu_e = \frac{n_{e0}}{n_{i0}}$ and $\mu_b = \frac{n_{b0}}{n_{i0}}$. The dynamics of IA shock waves are characterized by the following set of fluid equations in normalized form

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0, \qquad (4)$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} + 3\sigma_i n_i \frac{\partial n_i}{\partial x} = -\frac{\partial \phi}{\partial x} + \eta\frac{\partial^2 u_i}{\partial x^2}, \qquad (5)$$

$$\frac{\partial n_b}{\partial t} + \frac{\partial (n_b u_b)}{\partial x} = 0, \qquad (6)$$

$$\frac{\partial u_b}{\partial t} + u_b\frac{\partial u_b}{\partial x} + 3\sigma_b\alpha_1 n_b \frac{\partial n_b}{\partial x} = -\alpha_1\frac{\partial \phi}{\partial x}, \qquad (7)$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1-n_i) + (1-n_b)\mu_b + \mu_e(h_1\phi + h_2\phi_2) \qquad (8)$$

$\alpha_1 = \frac{m_i}{m_b}$ is the ratio of the masses of the positive ions to the ion beam, n_j ($j = i$ and b) indicates the number density of positive ions and ion beam, normalized by their equilibrium values n_{j0}. Also, we have included $\sigma_i = \frac{T_i}{T_e}$ and $\sigma_b = \frac{T_b}{T_e}$ where T_e, T_i and T_b are the temperatures of electrons, ions, and ion beam, respectively. The ion fluid speed u_i and ion beam speed u_b are normalized by the ion sound speed $C_i = (K_BT_e/m_i)^{1/2}$, the electrostatic potential ϕ by K_BT_e/e, the space (x) and time (t) coordinates are normalized by Debye length, $\lambda_{Di} = (K_BT_e/4\pi n_{i0}e^2)^{1/2}$ and the inverse of plasma frequency, $\omega_{pi}^{-1} = (m_i/4\pi n_{i0}e^2)^{1/2}$, respectively.

3 Derivation of KdVB equation

The stretching coordinates used to derive the KdV Burgers equation are $\xi = \varepsilon^{1/2}(x - v_{ph}t)$, and $\tau = \varepsilon^{3/2}t$, where ε is a small parameter $(0 < \varepsilon < 1)$ measuring the weakness of the dispersion, and v_{ph} is the phase velocity. The expansion of dependent variables is described as

$$\begin{pmatrix} n_i \\ n_b \\ u_i \\ u_b \\ \phi \\ \rho \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ u_{b0} \\ 0 \\ 0 \end{pmatrix} + \sum_{l=1}^{\infty} \varepsilon^l \begin{pmatrix} n_{il} \\ n_{bl} \\ u_{il} \\ u_{bl} \\ \phi_l \\ \rho_l \end{pmatrix} \qquad (9)$$

Using the stretching coordinates and expansion of dependent variables in Eqs. (4)-(8), evolution equations are determined after equating the coefficients of different powers of ε. On comparing the coefficients of terms with lowest power of ε, we get the following first order evolution equations:

$$u_{i1} = \frac{v_{ph}}{(v_{ph}^2 - 3\sigma_i)}\phi_1, n_{i1} = \frac{1}{(v_{ph}^2 - 3\sigma_i)}\phi_1, \qquad (10)$$

$$u_{b1} = \frac{\alpha_1(v_{ph} - u_{b0})}{(v_{ph} - u_{b0})^2 - 3\sigma_b\alpha_1}\phi_1, \qquad (11)$$

and

$$n_{b1} = \frac{\alpha_1}{(v_{ph} - u_{b0})^2 - 3\sigma_b\alpha_1}\phi_1. \qquad (12)$$

Using Eqs. (10) and (12) in Eq. (8), we get the following linear dispersion relation for IASWs

$$\frac{1}{(v_{ph}^2 - 3\sigma_i)} + \frac{\alpha_1\mu_b}{(v_{ph} - u_{b0})^2 - 3\sigma_b\alpha_1} - \mu_e h_1 = 0 \qquad (13)$$

which is a quadratic in v_{ph}^2 showing that four distinct modes can propagate in the given plasma system corresponding to the four different roots obtained from Eq. (13) provided that $v_{ph} \neq 3\sigma_i$ and $v_{ph} \neq u_{b0}$. In this present investigation, we are considering only one root corresponding to which IA shock structures have been studied. The next order of ε yields the following second order evolution equations,

$$-v_{ph}\frac{\partial n_{i2}}{\partial \xi} + \frac{\partial n_{i1}}{\partial \tau} + \frac{\partial u_{i2}}{\partial \xi} + \frac{\partial (n_{i1}u_{i1})}{\partial \xi} = 0, \qquad (14)$$

$$-v_{ph}\frac{\partial u_{i2}}{\partial \xi} + \frac{\partial u_{i1}}{\partial \tau} + u_{i1}\frac{\partial u_{i1}}{\partial \xi} + 3\sigma_i\frac{\partial n_{i1}}{\partial \xi} + 3\sigma_i n_{i1}\frac{\partial n_{i1}}{\partial \xi} = -\frac{\partial \phi_2}{\partial \xi} - \eta_{i0}\frac{\partial^2 u_{i1}}{\partial \xi^2}, \qquad (15)$$

$$-v_{ph}\frac{\partial n_{b2}}{\partial \xi} + \frac{\partial n_{b1}}{\partial \tau} + \frac{\partial u_{b2}}{\partial \xi} + u_{b0}\frac{\partial n_{b2}}{\partial \xi} + \frac{\partial (n_{b1}u_{b1})}{\partial \xi} = 0, \qquad (16)$$

$$-v_{ph}\frac{\partial u_{b2}}{\partial \xi} + \frac{\partial u_{b1}}{\partial \tau} + u_{b0}\frac{\partial u_{b2}}{\partial \xi} + u_{b1}\frac{\partial u_{b1}}{\partial \xi} + 3\sigma_b\alpha_1\frac{\partial n_{b2}}{\partial \xi} + 3\sigma_b\alpha_1 n_{b1}\frac{\partial n_{b1}}{\partial \xi} = -\alpha_1\frac{\partial \phi_2}{\partial \xi}, \qquad (17)$$

and

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \mu_e(h_1\phi_2 + h_2\phi_1^2) - n_{i2} - n_{b2}\mu_b. \qquad (18)$$

Eliminating the second order quantities from Eqs. (14)-(18) and making the use of first order Eqs. (10)-(12), we obtain the following nonlinear KdVB equation

$$\frac{\partial \phi}{\partial \tau} + A\phi\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} - C\frac{\partial^2 \phi}{\partial \xi^2} = 0, \qquad (19)$$

here $\phi_1 \equiv \phi$ for mathematical simplicity and the nonlinear coefficient A, dispersion coefficient B and dissipation coefficient C are given as $A = \gamma_1 B$, $B = \frac{1}{\gamma_2}$ and $C = \gamma_3 B$ with

$$\gamma_1 = \frac{3\alpha_1^2 \mu_b (v_{ph} - u_{b0})^2 + 3\sigma_b \alpha_1^3 \mu_b}{((v_{ph} - u_{b0})^2 - 3\sigma_b \alpha_1)^3} + \frac{3(v_{ph}^2 + \sigma_i)}{(v_{ph}^2 - 3\sigma_i)^3} + 2\mu_e h_2, \tag{20}$$

$$\gamma_2 = \frac{2\alpha_1 \mu_b (v_{ph} - u_{b0})}{(v_{ph} - u_{b0})^2 - 3\sigma_b \alpha_1} + \frac{2v_{ph}}{(v_{ph}^2 - 3\sigma_i)^2}, \tag{21}$$

and

$$\gamma_3 = \frac{\eta_{i0}}{(v_p^2 - 3\sigma_i)^2}. \tag{22}$$

By employing the tanh method and using the transformation $\xi = k(x - u\tau)$, where k is the wave number and u is the velocity of the frame of reference, the shock wave solution in terms of independent variable ξ , can be expressed as

$$\phi = \frac{12B}{A}\left(1 - tanh^2(\xi)\right) - \frac{36C}{15A} tanh(\xi) \tag{23}$$

4 Numerical analysis

The dynamics of ion acoustic shock waves and their characteristics variation with the various plasma parameters are investigated numerically in this section. We have observed that the beam parameters (such as number density, temperature and beam velocity), kinematic viscosity of ions as well as the concentration of non-Maxwellian electrons exhibit significant variations in the shock wave amplitude and steepness in the considered plasma system. The numerical analysis has been carried out with the data obtained from DE-1 satellite observations in the polar cap region where the presence of warm He^+ beam was detected [9]. The nonlinear, dissipation and dispersion coefficients are strongly dependent on the various plasma parameters, due to which change in any of the parameter causes a significant change in the given coefficients, which further modify the characteristics of IA shock waves.

Fig 1 depicts the variation in the amplitude of the shock wave profile for different values of concentration of positive ion beam (via μ_b) with the fixed values of nonthermal parameter α and spectral index κ_e. It is obvious that the increase in the ion beam concentration causes a vital reinforcement of shock wave by strengthening it in the system and hence both the amplitude and steepness of the shock waves increases with increase in the value of μ_b. Fig. 2 explores the effect of ion kinematic viscosity (via η_{i0}) on the 3D profile of the IA shock waves. If the dissipative coefficient C is negligible, in comparison with the nonlinear A and dispersive B coefficients, the solitary structure will be established by balancing the effects of dispersion and nonlinearity. On the other hand, if the coupling becomes very strong, then the shock waves will appear and same effect is depicted in Fig. 2. The amplitude of the IA shock waves rises with the increasing value of the kinematic viscosity

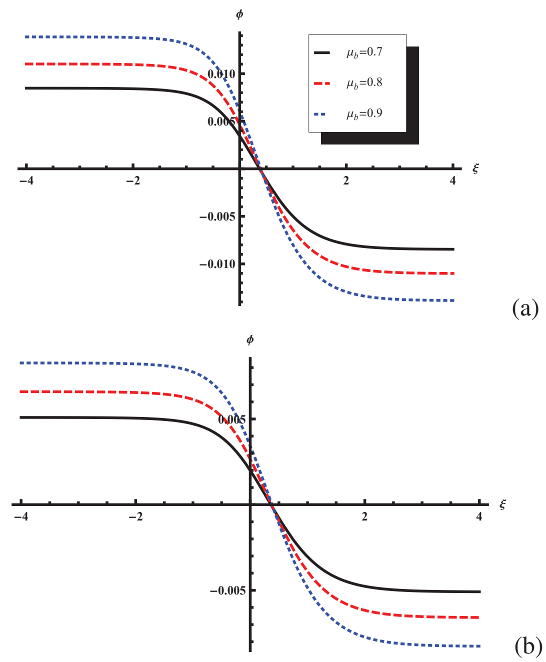

Figure 1. Variation of IA shock wave profile ϕ vs. ξ for different values of μ_b (a) cairns distribution, (b) kappa distribution with $\mu_e = 0.1$, $\sigma_b = 0.3$, $\sigma_i = 0.1$, $\alpha_1 = 0.25$, $u_b = 0.85$, $\eta_{i0} = 0.4$.

for ions. As the value of the kinematic viscosity approaches to zero, we get the profile of IA solitary waves (figure not shown).

The variation of shock wave profile with the nonthermal parameter α is depicted in Fig. 3. The amplitude of the shock wave decreases with the rise in the value of nonthermal parameter (via α). It can be concluded that the addition of the nonthermal electrons opposes the propagation of the high amplitude shock waves in the given plasma system. The variation of the shock wave profile with the varying value of ion beam velocity (via u_{b0}) is depicted in Fig. 4. It is noted that the amplitude of the shock structures are sensitive to the ion beam velocity only in a very small range and

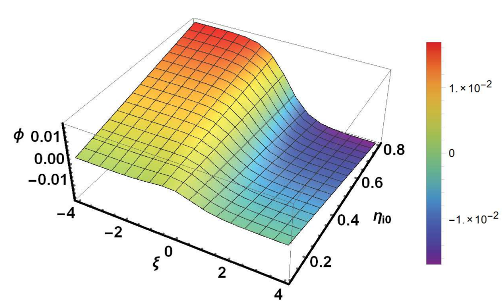

Figure 2. Variation of 3D shock wave profile for kinematic viscosity η_{i0} with spectral index $\kappa_e = 3.5$ and other parameters same as shown in the caption of Fig. 1.

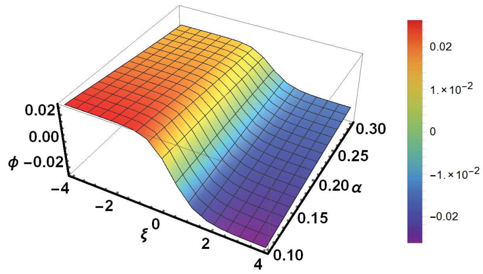

Figure 3. Variation of 3D shock wave profile for nonthermal parameter α with other parameters same as shown in the caption of Fig. 1.

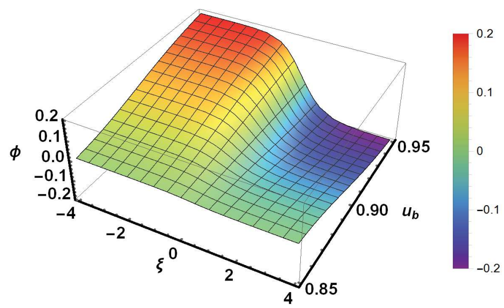

Figure 4. Variation of 3D shock wave profile for ion beam velocity u_{b0} with other parameters same as shown in the caption of Fig. 1.

increases with the increasing value of u_{b0}.

5 Conclusions

The propagation characteristics of ion acoustic shock waves are investigated in an unmagnetized plasma consisting of positive ion fluid and non-Maxwellian distributed electrons penetrated by positive ion beam. The KdVB equation has been derived by using the reductive perturbation technique. The impact of various plasma parameters (such as ion beam concentration (via μ_b), superthermality of electrons (via κ_e), nonthermal parameter (via α), ion kinematic viscosity (via η_{i0}), ion beam temperature (via σ_b) etc.) on the characteristics of IA shock waves has been analyzed numerically. Both polarity shock structures are formed. It is found that the with increase in the concentration of number density of ion beam, there is rise in the amplitude of shock waves for the fixed value of spectral index κ_e and nonthermal parameter α. Also, the amplitude of the shock wave profile increases with the rising value of ion kinematic viscosity η_{i0}. The findings of present investigation may be helpful in understanding the nonlinear structures in the polar cap region where plasma contains nonthermal electrons, and beam of positive ions (e.g., He^+, O^+) and other space/astrophysical environments.

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