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Compact design of fractional order *LC* oscillator

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Abstract

This paper deals with the design and realization of fractional order *LC* oscillator by utilizing the concept of fractional-order calculus. To design this oscillatory system, capacitor and inductor of the conventional *LC* oscillator are replaced with the fractance devices (FDs) to achieve additional control over the phase and frequency of the generated waveform. In this work, a recently developed FD approximation technique has been used to improve the compactness of the system. Finally, PSpice simulation has been done to see the accuracy of the designed oscillator.

1. Introduction

Nowadays, fractional order oscillators are getting appreciable attentions among the researchers [1-6]. These non-integer order oscillators possess several attractive advantages over the conventional integer order oscillators, such as more degree of freedom in system designing, precise control over the phase and frequency of the oscillatory output. In fractional order oscillators, fractional energy storing elements are used in places of traditional capacitors and inductors. The fractional energy storing elements are also known as fractance devices (FDs), and can be categorized as fractional order capacitor and fractional order inductor. The current-voltage relations of fractional order capacitor (order α) and fractional order inductor (order β) is given by $(1/C)i = d^{\alpha}v/dt^{\alpha}$ and $(1/\ L)v = d^{\beta}i/dt^{\beta}$ respectively. The SI unit and dimensional formula for fractional order capacitors are $Fs^{\alpha-1}$ and $[\ M^{-1}L^{-2}T^{(3+\alpha)}I^{2}\ ]$ respectively; whereas the same are $Hs^{\beta-1}$ and $[M\,L^{2}T^{(-3+\beta)}I^{-2}]$ respectively for the fractional order inductors [7].

There are several methods to approximate the fractance devices [8-13]; however, the *R-C* ladder network based designs are the most famous and commonly used in various analog realizations. The major limitation of ladder network based design is the requirement of large passive components even for a single FD. Recently, a new approach has been developed to approximate FDs suitable for fractional order sinusoidal oscillator [7]. This approach makes possible to approximate an FD using a pair of passive components only as illustrated in Figure 1. This approach suggests that a fractional order capacitor $(C,\ order{:}\,\alpha)$ or fractional order inductor $(L,\ order{:}\,\beta)$ can be approximate with *R-C* or *R-L* series pair.

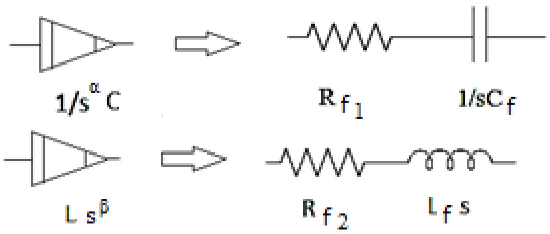

Figure 1. Schemes for approximating fractional order capacitor and fractional order inductor [7].

The numerical values of the circuit elements are obtained from the following relations [7].

Fractional order capacitor:

$$R_{f_1} = \frac{\cos(\alpha\pi/2)}{\omega^{\alpha}C}; \qquad C_f = \frac{\omega^{(\alpha-1)}C}{\sin(\alpha\pi/2)} \tag{1}$$

Fractional order inductor:

$$R_{f_2} = L\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right); \qquad L_f = L\omega^{(\beta-1)}\sin\left(\frac{\beta\pi}{2}\right) \tag{2}$$

where ω is the operating angular frequency (rad/sec) of the fractional order oscillator. Since, this design requires only two passive components, any fractional order oscillator designed using this FDs will have significantly compact physical structure than the other traditional *R-C* ladder based designs. In this work, an attempt has been made to realize fractional order *LC* oscillator using the *R-C* and *R-L* series based FDs.

2. Fractional order LC oscillator

The fractional order *LC* oscillator consists of one fractional capacitor (order: α), one fractional inductor (order: β), and three resisters [14], as shown in Figure 2. The characteristic equation of this oscillator is given as:

$$s^{\alpha+\beta} - \frac{K}{RC}s^{\beta} + \frac{1}{LC} = 0 \tag{3}$$

where $K = R_2/R_1$. The frequency of oscillation is given by (4), while the condition of oscillation is given by (5).

$$f_o = \frac{1}{2\pi}\left[\frac{\sin(\beta\pi/2)}{LC\sin(\alpha\pi/2)}\right]^{\frac{1}{\alpha+\beta}} \tag{4}$$

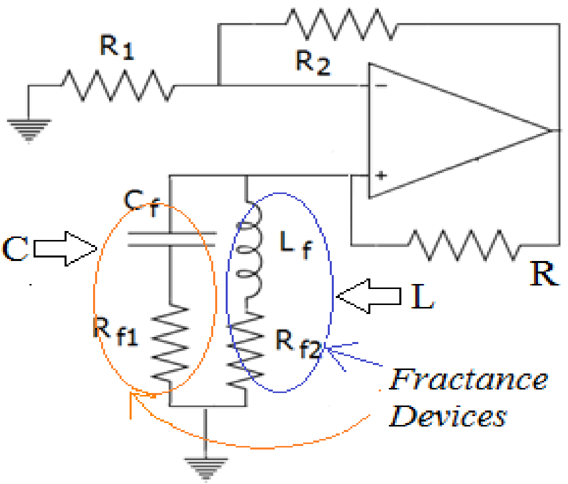

Figure 2. Schematic of fractional order LC oscillator using *R-C* and *R-L* series pair based FDs.

$$K = \frac{RC\sin\left(\frac{(\alpha+\beta)\pi}{2}\right)}{\sin\left(\frac{\beta\pi}{2}\right)}\left[\frac{\sin(\beta\pi/2)}{LC\sin(\alpha\pi/2)}\right]^{\frac{\alpha}{\alpha+\beta}} \tag{5}$$

The phase difference between the voltage across the fractional order capacitor and the current flowing through the inductor is $\beta\pi/2$.

For the special case where $\alpha = \beta$, the above relations can be simplified as :

$$f_o = \frac{1}{2\pi}\left[\frac{1}{LC}\right]^{\frac{1}{2\alpha}} \tag{6}$$

$$K = 2RC\cos\left(\frac{\alpha\pi}{2}\right)\left[\frac{1}{LC}\right]^{\frac{1}{2}} \tag{7}$$

Here, we can see that the frequency of oscillation also depends not only on the numerical value inductor and the capacitor but also on the orders α and β. This additional frequency controlling parameter (α and β) provides an extra degree of freedom in system designing. The variation of the frequency of oscillation with α, β, and *LC* is shown in Figure 3.

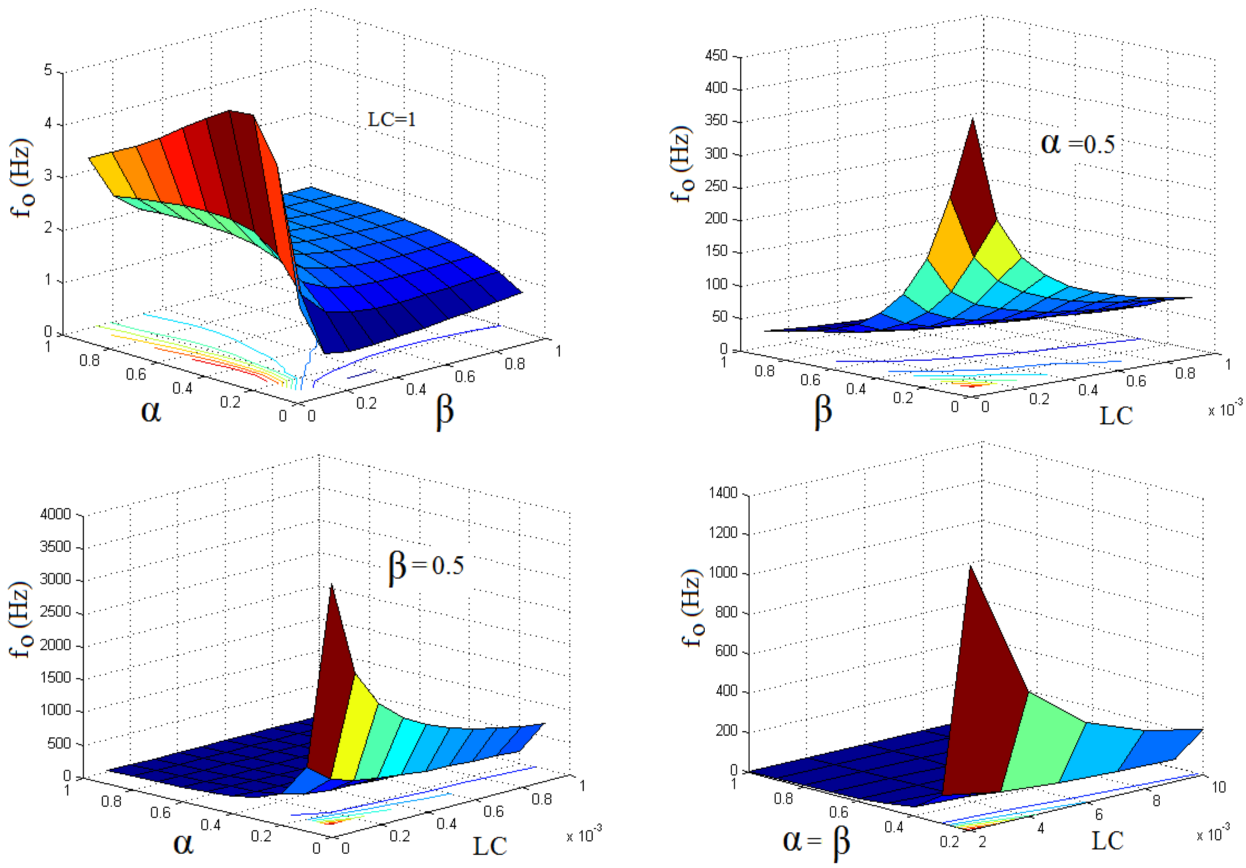

Figure 3. Variation of the oscillation frequency with several different circuit parameters.

3. Simulation and Results

To see the accuracy of the design, the fractional order *LC* oscillator is simulated in PSpice environment. For this, we considered a particular case where $\alpha = 0.6$, $\beta = 0.7$, $C = 10^{-5} Fs^{\alpha-1}$, $L = 100mH^{\beta-1}$ and $R = 100\Omega$. From (4) and (5), the oscillation frequency and the oscillation condition are obtained as $f_o = 7.07kHz$ and $K = 0.615$ respectively. The phase shift between the voltage across the fractional order capacitor and the current through the inductor should be $63°$. In this simulation, the *R-C* and *R-L* pair based FDs have been used. The numerical value of circuit components for approximating fractional order capacitor and the fractional order inductor are $R_{f_1} = 95.63\Omega$, $C_f = 171.0nF$ and $R_{f_2} = 81.35\Omega, L_f = 3.6mH$ respectively. The voltage across the fractional capacitor and the current passing through the fractional order inductor is shown in Figure 4. Frequency of the obtained oscillatory waveform is nearly *7.1 KHz*, and time delay (td) between the two waveforms is nearly *0.026ms*. Therefore, the phase shift is approximately 66°, which is quite close to the theoretical result.

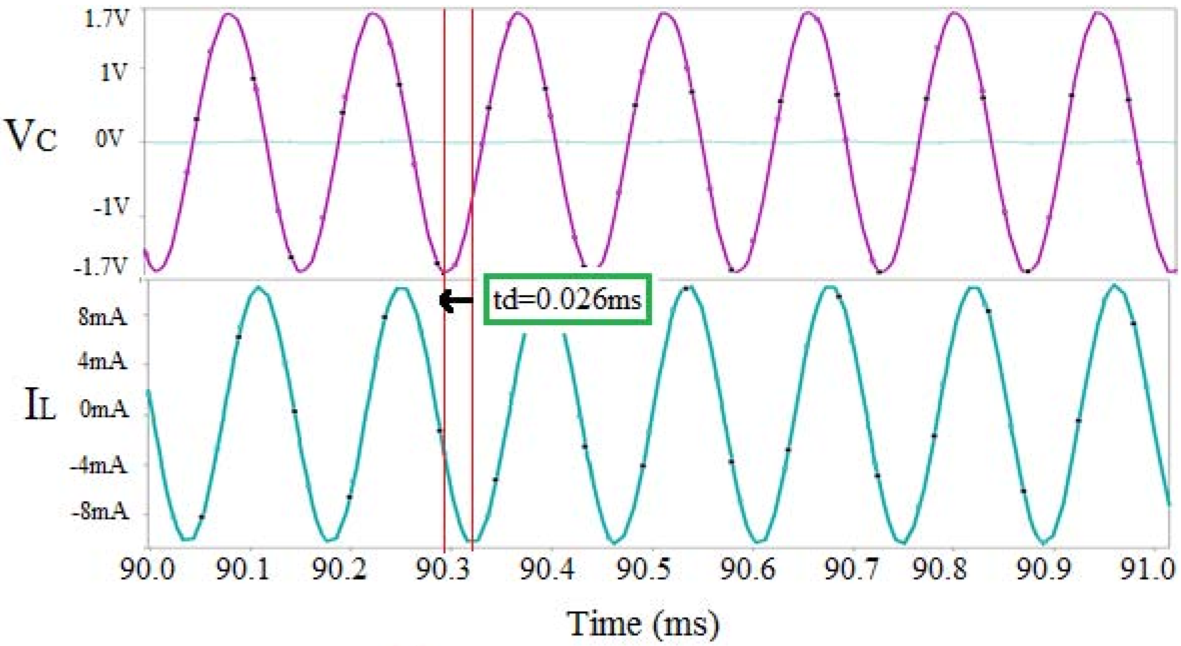

Figure 4. Voltage across the fractional capacitor and the current through fractional inductor obtained by PSpice simulation.

4. Conclusion

A compact design of fractional order *LC* oscillator is presented. Initially, mathematical analysis has done using the numerical simulations in MATLAB environment. Theoretical results obtained by the mathematical analysis have been validated by the PSpice results. In this simulation, the *R-C* and *R-L* series pair based FDs have been used in place of traditional tree or ladder network based FDs. The *R-C* and *R-L* series pair based FDs makes the oscillators significantly compact and power efficient. Hence, such oscillators are extremely useful for various applications where compact and power efficient devices are requisite.

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