URSI AP-RASC 2019, New Delhi, India; 09 - 15 March 2019

# Effect Of Anisotropic Pressure On Electron Acoustic Shock Waves In Magneto-rotating Plasma

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## Abstract

Nonlinear electron acoustic shock waves (EASWs) in dissipative magneto-rotating electron-positron-ion (e-p-i) plasmas containing cold dynamical electrons, superthermal electrons and positrons have been analyzed in the stationary background of massive positive ions. The Korteweg de Vries Burger (KdVB) equation which describes the dynamics of the nonlinear shock structures is derived by using small amplitude reductive perturbation technique. The quantitative analysis of different physical parameters on the shock structures is presented here. The present work may be employed to explore and to understand the formation of electron acoustic shock structures in the space and laboratory plasmas with superthermal electrons and positrons under magneto-rotating effects, especially in auroral zone, Van Allen radiation belts, planetary magnetospheres, pulsars, etc.

## 1 Introduction

The study of electron acoustic waves (EAWs) has grown to climax in the past many years because of their presence in various plasma systems ranging from laboratory generated plasmas to numerous astrophysical/space plasmas [1]. These waves are the result of two distinct electron components at different temperatures. Depending upon the temperature difference, the cold electrons become inertial and hot electrons provide the necessary pressure to develop the restoring force for the EAWs to exist likewise the ion acoustic waves (IAWs) in electron-ion plasma where the inertia is provided by the massive ions and the inertialess electrons provide the restoring force. Moreover, in the dynamics of EAWs the ions are assumed to form only a stationary background because of their larger dynamical time scale as compared to that of electrons. In addition to electrons and ions, positrons also constitute an essential component because of their presence in different plasma systems including both terrestrial and astrophysical space plasmas to form electron-positron-ion (e-p-i) plasma where the presence of positrons can lead to a change in the restoring force that plays the role of backbone to control the oscillations underpinning the electrostatic waves. Some useful work has already been done to explore the properties of linear and nonlinear electrostatic EAWs. Han et al. [2] has studied the EASWs and shock waves in a dissipative, nonplanar space plasma with superthermal hot electrons and has found that only negative potential solitary waves exist.

The observations made by the Fast Auroral SnapshoT (FAST) in geotail, intermediate auroral region (altitude$<$ 4000 km), and the polar observations at higher altitude ($2R_E < altitude < 8R_E$, $R_E$ being the radius of earth) conforms the existence of EAWS in different regions of magnetosphere where a significant amount of highly energetic superthermal electrons and positrons in addition to cold electrons and heavy ions has been observed. The source of these energetic particles is the solar wind and cosmic rays. In various parts of earth's magnetosphere, such EAWs under strong excitations lead to the existence of various nonlinear coherent structures including double layers, shocks, solitons, turbulence, electron holes, etc. Plasmas in space or in laboratory may contain substantially high energy particles, which can be effectively modeled by the Lorentzian distribution or simply the kappa distribution. The occurrence of such non-equilibrium distributions of different plasma components are common in collisionless space plasma environments. The spacecraft measurements of plasma velocity distribution reveal that the solar wind, the planetary magnetosphere and the magnetosheaths contain the high energy particles in the tail of the velocity distribution. The high energy tail appearing in the corresponding distribution function is conveniently modelled by a nonthermal distribution function. Vasylinuas [3] was the first to discuss the general form of the kappa distribution and its relation to the Maxwellian distribution. In view of the significant role of superthermal distribution, the study of nonlinear waves has been reported in various kinds of plasmas containing superthermal particles [4, 5]. Coriolis force plays a dominant role in cosmic phenomena, and in many other plasma environments including rotating plasma in laboratory devices as well as in space plasmas. Several workers have attempted to examine the nature of wave propagation in rotating plasmas including Coriolis force. Most of the astrophysical plasma environments (for instance, neutron stars, pulsars, quasars, black-hole magnetospheres, etc.) and in tokamak, the plasma rotates quite rapidly and should be strongly magnetized. The physics of such plasma systems can be coined in terms of non-inertial (rotating) frames. To the best of our knowledge, no investigation has been reported for magneto-rotating e-p-i-plasma consisting of nonthermal electrons and positrons following a non-Maxwellian distribution along with cold electrons being dynamical species and massive ions to provide a stationary background for the

EASWs. The paper is organized as follows: Sec. 2 presents the fluid model for given plasma system. The derivations of KdV-B equation is illustratd in Sec. 3. Numerical analysis and discussion are presented in Sec. 4. The last Sec. 5 is devoted to the conclusions.

## 2 Fluid Model

We consider a plasma with four components, namely cold electrons-fluid, inertialess hot electrons and hot positrons components with suprerthermal (non-Maxwellian) distribution, and uniform stationary ions background. The cold electrons fluid governing the linear and non-linear dynamics of electron acoustic shocks feels the effect of hot superthermal electron. The model is electrostatic in which the ambient magnetic field is along z-axis. The plasma is rotating with frequency $\Omega$ about the axis of rotation which makes an angle $\theta$ with the magnetic field direction. The dynamics of $EASWs$ are characterized by the following set of normalized fluid equations (continuity, momentum and Poisson):

$$\partial_t n_c + \partial_x (n_c u_{cx}) + \partial_y (n_c u_{cy}) + \partial_z (n_c u_{cz}) = 0 \tag{1}$$

$$\partial_t u_{cx} + u_{cx}\partial_x u_{cx} + u_{cz}\partial_z u_{cx} = \partial_x \phi - \Gamma u_{cy} - \frac{\sigma_\perp}{n_c}\partial_x n_c + \eta_{c\perp}\nabla^2 u_{cx} \tag{2}$$

$$\partial_t u_{cy} + u_{cx}\partial_x u_{cy} + u_{cz}\partial_z u_{cy} = \Gamma u_{cx} - \frac{\sigma_\perp}{n_c}\partial_y n_c + 2\Omega_x u_{cz} + \eta_{c\perp}\nabla^2 u_{cy} \tag{3}$$

$$\partial_t u_{cz} + u_{cx}\partial x u_{cz} + u_{cz}\partial z u_{cz} = \partial_z \phi - 2\Omega_x u_{cy} + \eta_{c\parallel}\nabla^2 u_{cz} - 3\sigma_\parallel n_c \partial_z n_c \tag{4}$$

$$\nabla^2\phi = 1 - \alpha + n_c\sigma - \delta + \phi\left[c_1 + \alpha\gamma d_1\right] + \frac{\phi^2}{2}\left[c_2 - \alpha\gamma^2 d_2\right] + \ldots \tag{5}$$

Here the parameters $c_1 = \left(\frac{\kappa_e - \frac{1}{2}}{\kappa_e - \frac{3}{2}}\right)$ and $c_2 = \left(\frac{\kappa_e^2 - \frac{1}{4}}{\left(\kappa_e - \frac{3}{2}\right)^2}\right)$ are functions of $\kappa_e$, whereas $d_1 = \left(\frac{\kappa_p - \frac{1}{2}}{\kappa_p - \frac{3}{2}}\right)$ and $d_2 = \left(\frac{\kappa_p^2 - \frac{1}{4}}{\left(\kappa_p - \frac{3}{2}\right)^2}\right)$ are functions of $\kappa_p$. where $\alpha = \frac{n_{op}}{n_{oh}}$, $\sigma = \frac{n_{oc}}{n_{oh}}$, $\delta = \frac{n_o}{n_{oh}}$, and $\gamma = \frac{T_h}{T_p}$. New dimensionless parameters used for normalization are; $\Omega_c = \frac{\Omega_c}{\omega_{ph}}$, $\Omega = \frac{\Omega}{\omega_{ph}}$, $u_c = \frac{u_c}{u_{th}}$, $x = \frac{x}{\lambda_D}$, and $\eta_{c\parallel\perp} = \mu_{c\parallel,\perp}\frac{\omega_{ph}}{u_{th}^2}$. Where electron plasma wave frequency is given as $\omega_{ph} = \left(\frac{4\pi n_{oh} e^2}{m_e}\right)^{\frac{1}{2}}$, thermal velocity is $u_{th} = \left(\frac{T_h}{m_e}\right)^{\frac{1}{2}}$ and Debye length is $\lambda_D = \left(\frac{T_h}{4\pi n_{oh} e^2}\right)^{\frac{1}{2}}$. The external magnetic field introduce an anisotropy in the electron pressure which splits the pressure tensor into two components in the directions perpendicular and parallel to the direction of magnetic field. The anisotropic pressure is modeled by using double adiabatic or Chew-Goldberger-Low (CGL) theory [7] which is given by the following relation,

$$\tilde{P} = p_\perp \hat{I} + \left(p_\parallel - p_\perp\right)\hat{z} \tag{6}$$

where $\hat{I}$ is the unit tensor and $\hat{z}$ is the unit vector along the external magnetic field. The parallel and perpendicular componentes of the pressure are given as:

$$p_\perp = p_{\perp o}\left(\frac{n_c}{n_{oc}}\right) \qquad and \qquad p_\parallel = p_{\parallel o}\left(\frac{n_c}{n_{oc}}\right)^3 \tag{7}$$

Here, $p_{\parallel o} = n_{oc}T_{\parallel c}$ and $p_{\perp o} = n_{oc}T_{\perp c}$ represent the equilibrium cool electron pressures in perpendicular and parallel directions. In isotropic case $p_\parallel = p_\perp$, and it leads to the relation: $\nabla.\tilde{P} = \nabla p$. The difference in the parallel and perpendicular temperatures also introduces anisotropy in the kinematic viscosity of the cold electrons fluid. Here $\sigma_\perp = \frac{T_{\perp c}}{T_h}$ and $\sigma_\parallel$ and $\sigma_\parallel = \frac{T_{\parallel c}}{T_h}$.

## 3 Derivation of KdV-Burgers equation:

In order to derive the non linear KdV-Burger equation, we have to employ the reductive perturbation method (RPM) by defining the new stretched independent variables as.

$$\xi = \varepsilon^{\frac{1}{2}}\left(l_x x + l_z z - \lambda t\right) \qquad and \qquad \tau = \varepsilon^{\frac{3}{2}} t \tag{8}$$

Here $l_x$, and $l_z$ are direction cosines of the wave vector in the directions perpendicular and parallel to the magnetic field, $\lambda$ is the phase velocity of the electron acoustic wave (EAW) and $\varepsilon$ is a small parameter ($0 < \varepsilon < 1$) which determines the weakness of the nonlinearity. The physical quantities $n_c$, $u_{cx}$, $u_{cy}$, $u_{cz}$, $\phi$,and $\eta_{c\parallel,\perp}$ are expanded in terms of $\varepsilon$. The expansions used are given as:

$$n_c = 1 + \varepsilon n_c^{(1)} + \varepsilon^2 n_c^{(2)} + \ldots. \tag{9}$$

$$u_{cx} = \varepsilon^2 u_{cx}^{(1)} + \varepsilon^3 u_{cx}^{(2)} + \ldots \tag{10}$$

$$u_{cy} = \varepsilon^{\frac{3}{2}} u_{cy}^{(1)} + \varepsilon^{\frac{5}{2}} u_{cy}^{(2)} + \ldots \tag{11}$$

$$u_{cz} = \varepsilon^1 u_{cz}^{(1)} + \varepsilon^2 u_{cz}^{(2)} + \ldots \tag{12}$$

$$\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \ldots \tag{13}$$

$$\eta_{c\parallel} = \varepsilon^{\frac{1}{2}}\eta_{c\parallel o} \quad and \quad \eta_{c\perp} = \varepsilon^{\frac{1}{2}}\eta_{c\perp o} \tag{14}$$

Due to anisotropy induced by external magnetic field, the parallel component of fluid electron $u_{cz}$ is stronger than the perpendicular components $u_{cx}$ and $u_{cy}$. This effect is noticable from the powers of the $\varepsilon$ that is, $u_{cz}$ contain lower power of $\varepsilon$ as compared to $u_{cx}$ and $u_{cy}$. Substituting Eqs. (8) and (9)-(14) in Eqs. (1)-(5) and collecting different powers of $\varepsilon$. For zeroth order in $\varepsilon$, we get the charge neutrality condition as $1 + \sigma = \delta + \alpha$. The phase velocity of electron acoustic wave (EAW)is: $\lambda^2 = \lambda_c^2 + \left(3\sigma_\parallel l_z^2 - \frac{2\sigma_\perp l_x l_z \Omega_x}{\Omega - 2\Omega_z}\right)$. Where parameters $Q = \left(\frac{c_1 + \alpha\gamma d_1}{\sigma}\right)$ and $\lambda_c = \frac{1}{Q}\left[l_z^2 - \frac{2 l_x l_z \Omega_x}{\Omega - 2\Omega_z}\right]^{\frac{1}{2}}$. Next higher order after some calculation yield KdVB equation as

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} + C\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = 0 \tag{15}$$

Here, $A = \frac{-1}{2\lambda\sigma Q}\left[\lambda_c^2\alpha + \sigma Q^2\left(3\lambda^2 + 3\sigma_\parallel l_z^2 + \frac{2\sigma_\perp l_x l_x \Omega_x}{\Omega - 2\Omega_z}\right)\right]$, $B = \frac{1}{2\lambda\sigma Q}\left[\lambda_c^2 + \frac{\lambda^2(1+\sigma_\perp Q)}{(\Omega - 2\Omega_z)^2}\left(l_x^2 + \frac{2\sigma_\perp l_x l_x \Omega_x}{\Omega - 2\Omega_z}\right)\right]$, and $C =$

$\frac{-\eta_{c\|o}}{2}$ represent the nonlinear, dispersive and dissipative coefficients respectively. We have used the following analytical solution of the KdVB equation:

$$\phi = \phi_o \left[ \frac{1}{\cosh^2\left(\frac{\xi - \frac{6C^2\tau}{25B}}{\Delta}\right)} + 2\left(1 - \tanh\left(\frac{\xi - \frac{6C^2\tau}{25B}}{\Delta}\right)\right) \right] \tag{16}$$

Where $\phi_o = \frac{3C^2}{25AB}$ and $\Delta = \frac{10B}{C}$ represent the maximum amplitude and width of the shock structure.

## 4 Results and Discussions:

In this section we investigate the dependence of the electron acoustic shocks in a magnetized dissipative e-p-i plasma on relevant plasma parameters such as coriolis force, magnetic field, cold electron to hot electron temperature ratio $\sigma_\perp$ and $\sigma_\|$, kinematic viscosity $\eta_{c\|}$ and $\eta_{c\perp}$, and superthermal distribution parameters $\kappa_e$ and $\kappa_p$, along with other normalized physical parameters of interest including $\alpha$ and $\sigma$. We have selected the appropriate range $n_{oc} \sim (0.1-0.4)cm^{-3}$, $n_{op} \sim (1.5-3)cm^{-3}$, $n_{oh} \sim (1.5-3)cm^{-3}$, $T_h \sim (200-1000)eV$, $T_p \sim (200-1000)eV$ to satisfy various plasma systems from laboratory level to astrophysical space plasmas [8].

Fig. 1 depicts the variation of phase velocity $\lambda$ with superthermal index for hot electrons (via $\kappa_e$) for different values of parallel temperature ratio ($\sigma_\| = \frac{T_{c\|}}{T_h}$) and perpendicular temperature ratio ($\sigma_\perp = \frac{T_{c\perp}}{T_h}$). Overall, it can be seen from the figure that phase velocity increases with $\kappa_e$ (i.e., decrease in the supthermality of electrons). It is also seen that the phase velocity escalates with increase in $\sigma_\|$ and $\sigma_\perp$. It is also found numerically that the phase velocity decreases with decrease in both $\gamma$ and $\Omega_c$. It is apparent from these variations that dispersion properties are effectively modified in the presence of the magneto-rotating effects and therefore, the shock structures would also be modified by these parameters because shocks structures are dependent on nonlinearity, dispersive and dissipative factors. Now, we will proceed to see the variation in the shock structure by the modification of shock strength $\phi$ with various parameters.

Fig. [2] depicts the variation of shock structure strength $\phi$ as a function of $\xi$ for different values of the kinematic viscosity $\eta_{c\|o}$. Shock structure depends on the dissipative coefficient and thus is a function of dissipative parameter $\eta_{c\|o}$. It can be noted from the Fig. [2], that in the absence of the dissipative coefficient there is no shock structure. With the increase in the kinematic viscosity, the shock structure becomes more and more prominent. The width of the shock structure decreases and the magnitude of the shock strength increases dramatically. This impact can be understood by viewing that shock structure parameter $\phi$ depends uppon

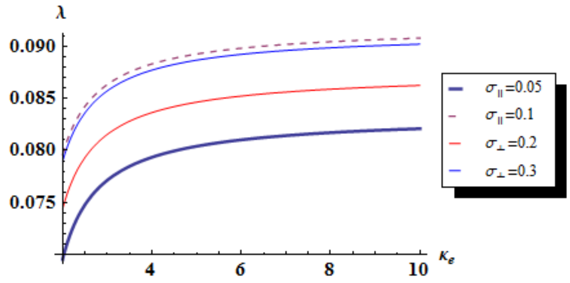


**Figure 1.** Variation of $\lambda$ (Phase velocity) with $\kappa_e$ for different values of $\sigma_\|$ and $\sigma_\perp$ taking $\gamma = 10$, $\alpha = 0.1$, $\kappa_p = 3.5$, $\sigma = 0.7$, $l_z = 0.1$, $\theta = 10$, and $\Omega = 0.3$.

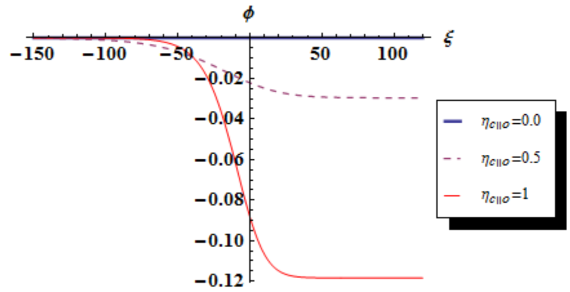


**Figure 2.** Variation of $\phi$ (shock strength) with $\xi$ for different values of $\eta_{c\|o}$, taking $\gamma = 10$, $\sigma_\| = 0.05$, $\sigma_\perp = 0.1$, $\kappa_e = 3.5$, $\kappa_p = 3.5$, $l_z = 0.1$, $\theta = 10$, $\Omega_c = 0.3$

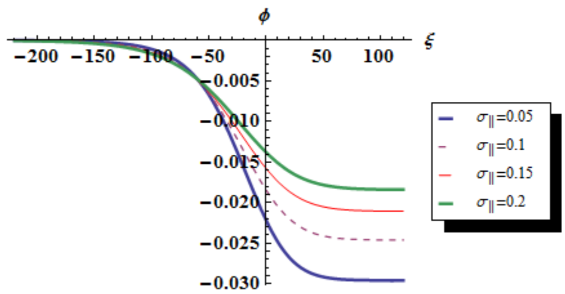


**Figure 3.** Variation of $\phi$ (shock strength) with $\xi$ for different values of $\sigma_\|$, taking $\gamma = 10$, $\sigma_\perp = 0.1$, $\kappa_e = 3.5$, $\kappa_p = 3.5$, $l_z = 0.1$, $\theta = 10$, $\Omega_c = 0.3$, $\eta_{c\|o} = 0.5$.

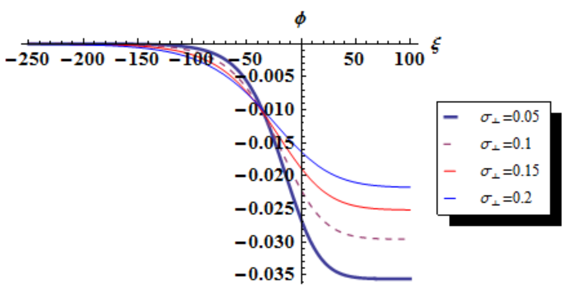


**Figure 4.** Variation of $\phi$ (shock strength) with $\xi$ for different values of $\sigma_\perp$, taking $\gamma = 10$, $\sigma_\| = 0.05$, $\kappa_e = 3.5$, $\kappa_p = 3.5$, $l_z = 0.1$, $\theta = 10$, $\Omega_c = 0.3$, $\eta_{c\|o} = 0.5$.

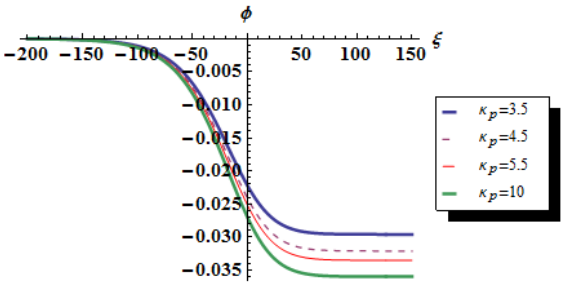


**Figure 5.** Variation of $\phi$ (shock strength) with $\xi$ for different values of $\kappa_p$, taking $\gamma = 10$, $\sigma_{\|} = 0.05$, $\sigma_{\perp} = 0.1$, $\kappa_e = 3.5$, $l_z = 0.1, \theta = 10$, $\Omega_c = 0.3$, $\eta_{c\|o} = 0.5$.

the dissipative coefficient $C$, which further depends on the $\eta_{c\|o}$. Fig. [3] and Fig. [4] depicts the variation of shock structure strength $\phi$ as a function of $\xi$ for different values of parallel ($\sigma_{\|}$) and perpendicular ($\sigma_{\perp}$) temperature ratios respectively. It can be seen from Fig. [3], with the increase in the parallel temperature component $\sigma_{\|}$ the magnitude of the shock strength decrease. Though, width remains same for increasing $\sigma_{\|}$. However, for increasing perpendicular temperature component $\sigma_{\perp}$ the width of the shock structure decreases and the magnitude of the shock strength increases and become more negative. We have analyzed the variation of the shock strength $\phi$ with $\kappa_p$ in Fig. [5]. It can be seen that as the distribution changes from non-Maxwellian to Maxwellian distribution the shock strength and width of shock structure both increases.

## 5 Conclusion:

We have studied the propagation of electron acoustic shock waves in magnetized, rotating, dissipative, superthermal e-p-i plasma. The plasma is composed of cool inertial electrons, hot electrons and hot positrons obeying superthermal distribution, and inertialess uniform background ions. Small reductive perturbation method is employed to derive the Korteweg de Vres Burger (KdVB) equation which describes the dynamics of the electron acoustic shock structures in the multicomponent plasma. The results shows the strong dependence on the plasma parameters which are summarized here.

• The phase speed of the electron acoustic shock shows strong dependence on the parallel and perpendicular components of the inertial cool electron. This in turn leads to the variation in the shocks potential at different cool electron temperatures.

• With increasing the strength of coriolis force the phase velocity increases. This further modifies the shock structure for different values of the coriolis force.

• With the increase in the strength of external magnetic field, phase velocity of electron acoustic shock wave decreases.

• The strength of the electron acoustic shock structure increases with the increasing superthermal parameter of hot positrons $\kappa_p$ as well as with increasing superthermal parameter of hot electrons $\kappa_e$.

• Finally, the shock strength increases with increasing the dissipative coefficent $C$ which is solely dependent on the parallel component of kinematic viscosity of cool electron $\eta_{c\|o}$.

The present investigation can be a useful knot to have a keen understanding of the formation of EA shock waves in auroral zone, Van Allen radiation belts, planetary magnetospheres, Earth's ionosphere, pulsars, quasars, black-hole magnetospheres, etc.

## 6 Acknowledgements


Kuldeep Singh is thankful to the UGC for BSR fellowship award. The authors also acknowledge the support by DRS-II (SAP) No. F 530/17/DRS-II/2015(SAP-I) UGC, New Delhi.


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