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Tumour Electrotherapy Modelling Using Algebraic Topological Method

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Abstract

Electromagnetic therapy is extensively used in various medical treatments of tumour cells. Different studies reveal that electrotherapy can effectively destroy or shrink tumour cells. Accurate modelling of the electromagnetic interactions with the tumour cells will enable biomedical engineers to develop effective treatment techniques. In this paper, we present a radically different non-mainstream approach based on algebraic topology to model this type of problems. We discuss the basic modelling framework for simulating electric field interactions with tumour cells and suggest a test problem to study such interactions at different skin-depths.

1 Introduction

Biomedical application of electromagnetic fields (electrotherapy) is a proven technique to effectively destroy or shrink tumour cells. Accurate modelling of such electromagnetic interactions is important for developing precise treatment procedures for curing tumours. Several analytical and numerical tools can be used to model electromagnetic interactions [1–9]. In particular, comparisons between numerical and analytical methods to model different electric field and potential distributions around tumour cells were done in [10–16]. In this paper, we are presenting a radically different approach for modelling electric field interactions with tumour cells using a *non-mainstream* tool called *algebraic topological method* - ATM [17–26]. Compared to the traditional methods like finite-element or finite-difference methods, this method has several advantages. For example, in the case of ATM, we use only physically measurable scalar quantities *without the need for differential equations and field vectors*. In addition, we directly get the discrete formulation, which can be easily translated into computational algorithm.

2 Electric field - tumour interaction: test problem

By accurately controlling the electric field distributions generated by needle electrodes, we are able to concentrate electric fields in places where it is precisely needed for efficient treatment of tumour cells. Hence, the electrode configuration is a crucial factor, which needs detailed investigation. Calculating potential and electric field distributions essentially involve (analytical or numerical) procedure for solving the *Poisson* equation. A real-time example of needle electrodes placed on human body is shown on the left-hand side of Fig. 1. We will analyse two needle electrode configurations, namely, circular and elliptical, with hypothetical tumours located in four points as illustrated on the righthand side of Fig. 1. In this test problem, all tumour cells are assumed to be located in the same (top) layer of the human body. Of course, this is rather a simplification only for modelling the field distribution on the top layer. A more generalized case of tumours located at different depth will be considered in the ATM modelling.

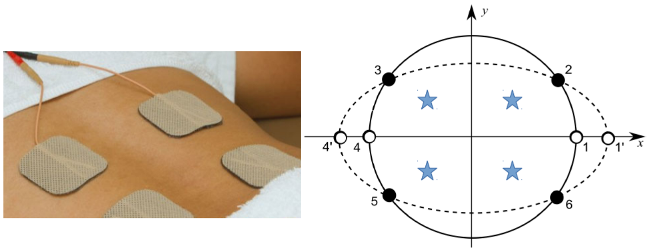

Figure 1. Left: An example of electrotherapy treatment. Right: Circular and elliptical needle electrode configurations with four different tumour locations (marked as stars).

3 Algebraic topological method (ATM) modelling

For modelling the electric field-tumour interactions using ATM, we start with the Poisson equation given by

$$\nabla^2 \varphi = \rho \varepsilon^{-1} \, . \tag{1}$$

In Eqn. 1 φ, ρ, and ε denote electric potential, volume charge density, and permittivity, respectively. The model we use for permittivity is very critical for accurately simulate potential and electric field distributions. There are many ongoing research in this area and we use the outcomes from these research in our modelling [27–30]. The ATM equivalent of the Poisson equation is written as

$$\sum_l \tilde{a}^3_{ml} \, \Psi(\tilde{s}^2_l, t_n) \quad = \quad Q_c(\tilde{s}^3_m, t_n) \tag{2}$$

where $\tilde{a}^3_{ml}$ is the *coboundary* operator, Ψ is the *electric flux* and Q_c is the *charge content* as defined in [18]. The beauty of ATM lies in the simplicity and power of two mathematical tools, namely *boundary* and *coboundary* operators.

In ATM, we discretize the domain into cells (volumes), which could be of any shape. Ideally, we go for tetrahedral cells to best model a *multiscale* problem. In the case of electric field-tumour interactions modelling, we consider a 3-layered human skin and the tumours are assumed to be located at different depths (layers) as depicted in the Fig. 2. The reason for choosing different tumour depths is to study the penetration of electric field at different skin-depths. This way, we can evaluate the efficacy of certain electrode configurations. We employ highly unstructured tetrahedral grid and the tumour regions are meshed very finely compared to the surrounding regions.

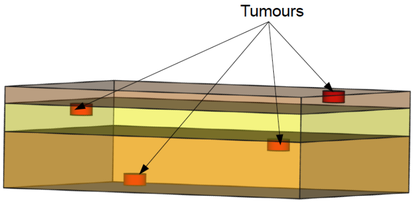

Figure 2. Cut-through-view of a 3-layered human skin with 4 hypothetical tumour regions at different skin depths.

Like in many numerical modelling methods, we have to truncate the region using accurate domain truncation techniques. The most common approach is to employ absorbing boundary conditions (ABCs) [31, 32] or perfectly matched layers (PML) [33–35]. Currently we go for simple one-way ABC, however, highly accurate PMLs are topics of further research in the ATM. Already existing conformal time-domain methods [36–45] are excellent candidates for ATM domain truncation.

4 Tuning electric potential distribution

By varying the potential applied to these needle electrodes one can fine-tune the distribution of electric potential. We have shown typical potential distributions due to circular and elliptical electrode configurations in Fig 3. The difference between two configuration potentials is also given. This difference plot shows how one can fine-tune the electric field in the regions of interest by varying the electrode configuration and the applied electrode potential. These results are only valid for superficial tumour locations. The field distributions will vary as we go to different skin-depths. As a truly multiscale method, the ATM can easily handle the complexities involved in modelling human tissues with tumours. Being a rather non-mainstream method, we need to further test and compare capabilities of ATM, especially with the advanced higher-order discontinuous Galerkin method [46–48]. One can relate various differential-calculus-based methods and ATM formulation as discussed in [49].

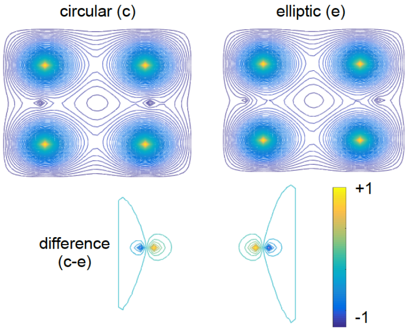

Figure 3. Potential distribution for circular and elliptical electrode configurations and the difference plot between the two configurations.

5 Summary

We have presented the direct discrete formulation of ATM without the need for field vectors and differential calculus. As discussed, the ATM has more advantages than traditional numerical and analytical tools. Compared to finite-element method, we can model multiscale features accurately with less computational resource requirements. Compared to finite-difference method, we can model complex geometries accurately. In various experiments employing structured grids, the numerical accuracy of ATM is comparable to that of standard finite-difference method. However, if one has to use ATM to its best potential, we need to use them on highly unstructured inhomogeneous grids. We have shown how these features make them ideal candidate for modelling electric field-interactions with tumour cells.

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