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# Low-Frequency Stable SPIE-PMCHWT Formulation for Multi-region Dielectric Problems

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## Abstract

In this work the surface-based Separated Potential Integral Equation (SPIE) approach is extended to mitigate low-frequency breakdown issues associated with multi-region dielectric problems and penetrable scatterers. The formulation appends electric and magnetic potentials as unknowns in addition to existing electric and magnetic current-density unknowns in a PMCHWT-based formulation. The proposed SPIE-PMCHWT methodology is demonstrated to be stable at low-frequencies.

## 1. Introduction

Method of Moments (MoM) [1], in conjunction with fast solver algorithms, is a popular and widely used computational technique for solving a large-range of 3D full-wave electromagnetic problems. The applications include radiation analysis of platform mounted antennas, scattering from electromagnetic structures designed for stealth, electrical parameter analysis for Radio-Frequency (RF) and microwave passive circuits, signal and power integrity analysis for chip-package-board systems, electromagnetic interference (EMI) and compatibility (EMC), inverse scattering and non-invasive detection for bio-medical RF imaging etc. In most of these practical day-to-day applications, analysis of dielectrics and penetrable scatterers form an important part of MoM modeling. Volume Integral Equation (VIE) with Schaubert–Wilton–Glisson (SWG) basis functions [2] involve tetrahedral mesh elements for volumetric MoM-based dielectric modeling. Although, well suited for inhomogeneous dielectric problems, the VIE method requires more mesh elements to model the volume of the geometry as compared to its surface-based counterparts, and hence is not appropriate for modeling piecewise-homogeneous dielectrics in terms of computational efficiency. On the other hand, the Poggio–Miller–Chang–Harrington–Wu–Tsai (PMCHWT) method [3] based on the surface equivalence principle, is a preferred surface based formulation for handling piecewise-homogeneous dielectric problems, often used with Rao-Wilton-Glisson (RWG) [4].

The PMCHWT operator however suffers from two major breakdown phenomena related to system matrix ill-conditioning: (a) low-frequency breakdown [6] and (b) dense mesh/discretization breakdown. The condition number of PMCHWT worsens as the frequency approaches zero, a phenomenon termed as low-frequency breakdown of PMCHWT. To stabilize PMCHWT at low-frequencies, loop-tree/star decomposition involving inexact Helmholtz splitting is often performed [5]. However, the loop-tree decomposition is plagued by several bottlenecks, the major being the computational inefficiency to extract global loops for geometries with multiple holes and handles.

To alleviate the existing low-frequency dielectric solution bottlenecks, a Separated Potential Integral Equation (SPIE) approach [6] has been extended to stabilize the PMCHWT method for multi-region composite dielectric objects and scatterers. Region based charge neutrality enforcement and potential referencing scheme is proposed for forming a well-conditioned, stable and accurate system.

## 2. Generic SPIE-PMCHWT Formulation for Multi-Region Problems

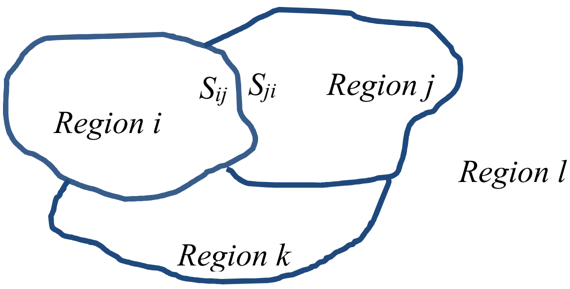



Fig. 1 Multi-region dielectric problem

As shown in Fig. 1, let *i, j, k* and *l* (background region) be different regions representing a generic multi-region dielectric problem. For an interface between region *i* ($R_i$) and region *j* ($R_j$), the common surface is defined as $S_{ij}$ which is same as $S_{ji}$. Let a surface charge (electric or magnetic) which produces correct field inside region *i* and null field outside it be defined as $q^{i,j}_{T_n}$. Here the charge is associated with triangular element $T_n$, placed on surface $S_{ij}$, belonging to both $R_i$ and $R_j$, producing correct field inside $R_i$ and null field inside $R_j$. In addition, by enforcing equivalence theorem, boundary condition and charge-current continuity equation it follows that:

$$q^{j,i}_{T_n} = -q^{i,j}_{T_n}, \forall\, T_n \in S_{ij} \tag{1a}$$

Also, from the summation of charges on $S_{ij}$ it follows that:

$$\sum_n q^{j,i}_{T_n} = -\sum_n q^{i,j}_{T_n}, \forall\, T_n \in S_{ij} \tag{1b}$$

$$
\begin{bmatrix}
j\omega\sum_{m=1}^{N_R} S^T_{m_{L\to G}}(\mu_m L_{V_m})_L S_{m_{G\to L}} & \sum_{m=1}^{N_R} S^T_{m_{L\to G}}(\alpha_m)_L S_{m_{G\to L}} & -A_{gen} & 0 \\
-\sum_{m=1}^{N_R} S^T_{m_{L\to G}}(\beta_m)_L S_{m_{G\to L}} & j\omega\sum_{m=1}^{N_R} S^T_{m_{L\to G}}(\varepsilon_m C_{V_m})_L S_{m_{G\to L}} & 0 & -A_{gen} \\
\begin{Bmatrix} P_1^{\varepsilon} A_1^T \\ P_2^{\varepsilon} A_2^T \\ \cdots\cdots \\ P_{N_R}^{\varepsilon} A_{N_R}^T \end{Bmatrix} & 0 & -j\omega\begin{Bmatrix} D_1^{\varepsilon} \\ D_2^{\varepsilon} \\ \cdots\cdots \\ D_{N_R}^{\varepsilon} \end{Bmatrix} & 0 \\
0 & \begin{Bmatrix} P_1^{\mu} A_1^T \\ P_2^{\mu} A_2^T \\ \cdots\cdots \\ P_{N_R}^{\mu} A_{N_R}^T \end{Bmatrix} & 0 & -j\omega\begin{Bmatrix} D_1^{\mu} \\ D_2^{\mu} \\ \cdots\cdots \\ D_{N_R}^{\mu} \end{Bmatrix}
\end{bmatrix}
\begin{bmatrix} I \\ K \\ \begin{Bmatrix} \Phi_1 \\ \Phi_2 \\ \cdots\cdots \\ \Phi_{N_R} \end{Bmatrix} \\ \begin{Bmatrix} \Psi_1 \\ \Psi_2 \\ \cdots\cdots \\ \Psi_{N_R} \end{Bmatrix} \end{bmatrix}
=
\begin{bmatrix} V_e \\ V_h \\ 0 \\ 0 \end{bmatrix}
\tag{2}
$$

For a generic scenario involving multiple penetrable dielectrics with partial interfaces and junctions as shown in Fig. 1, the electric and magnetic potentials $\phi$ and $\psi$ respectively are expanded across all regions as shown in (2), which represents the proposed SPIE-PMCHWT equation. $A_{gen}$ is a generic incidence matrix that maps between the triangles and edges in the system. I and K are the electric and magnetic currents respectively. The total number of regions is given by $N_R$. In addition, vectors $\Phi_m$ and $\psi_m$ for a region $m$ contains the potentials on all the triangles in that region. The subscript $G$ specifies global index in the system while $L$ stands for local index specific to a particular region. A selector matrix $S_m$ and its transpose for $m^{th}$ region help in the conversion from global frame to local frame of reference (G→L) and vice versa (L→G).

In addition, for RWG basis function $f$:

$$L_{v_m}(i,j) = \frac{1}{4\pi}\int f_i.\int G_m(r_i;r_j) f_j ds_i ds_j \,, \forall f_i, f_j \in I \tag{3}$$

$$C_{v_m}(i,j) = \frac{1}{4\pi}\int f_i.\int G_m(r_i;r_j) f_j ds_i ds_j \,, \forall f_i, f_j \in K \tag{4}$$

$$\alpha_m(i,j) = \frac{1}{4\pi}\int f_i.\int \nabla\times G_m(r_i;r_j) f_j \, ds_i ds_j, \forall f_i \in I, f_j \in K \tag{5}$$

$$\beta_m(i,j) = \frac{1}{4\pi}\int f_i.\int \nabla\times G_m(r_i;r_j) f_j \, ds_i ds_j, \forall f_i \in K, f_j \in I \tag{6}$$

$$V_e(i) = \int \vec{E}_{inc}.f_i \, ds_i \,, \forall f_i \in I \tag{7}$$

$$V_h(i) = \int \vec{H}_{inc}.f_i \, ds_i \,, \forall f_i \in K \tag{8}$$

In (3)-(6), Green's function $G$ is given as:

$$G_m(r_i;r_j) = \frac{e^{-jk_m|r_i-r_j|}}{|r_i-r_j|} \tag{9}$$

where $k_m$ is the wavenumber of $m^{th}$ medium, $\mu_m$ s and $\varepsilon_m$ s are the respective permeability and permittivity of different mediums, $s_i$ and $s_j$ represent the domain of the test and source basis, $r_i$ and $r_j$ represent the position vector of any point on $s_i$ and $s_j$ respectively.

## 3. Charge Neutrality and Potential Referencing

For eliminating redundancies in the system at low-frequencies and hence forming a stable system, each rank-deficient block in the SPIE-PMCHWT generic system (2) is subjected to region-based charge neutrality enforcement. The number of charge neutralities required to be enforced is governed by the rank deficiencies for every region. The number of spanning trees extracted in each region is the number of charge neutrality conditions that needs to be enforced. Let a generic case of multi-region composite dielectric objects involving partial-interfaces, junctions, and completely submerged disjoint objects be considered for illustration (Fig. 2). A two-dimensional view is shown for simplicity, although the objects are entirely 3D in nature.

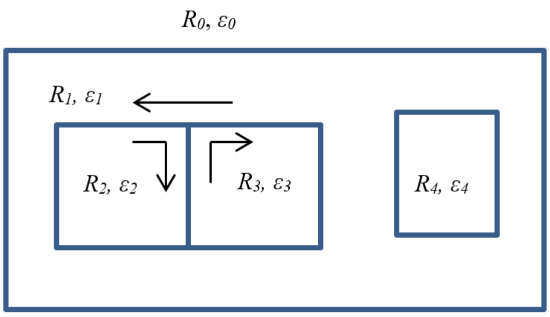



Fig. 2. Generic case- disjoint bodies, partial interfaces, junctions

Here region $R_0$ is the background region, typically considered as free space. $R_1$ and $R_4$ are disjoint dielectric bodies. $R_2$ and $R_3$ together form composite dielectrics with a partial interface $S_{23}$ resulting in junctions. Figure 3 below illustrates number of spanning trees and their extraction in each region (marked by red).

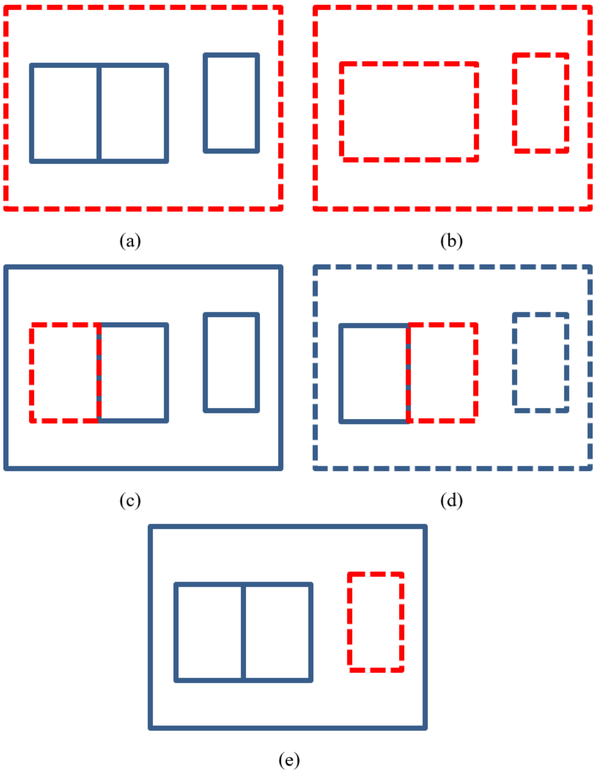



Fig. 3 Number of spanning trees and their extraction in each region (a) Region $R_0 - 1$ (b) Region $R_1 - 3$ (c) Region $R_2 - 1$ (d) Region $R_3 - 1$ (e) Region $R_4 - 1$

The rank-deficient blocks further require potential referencing [6] to be enforced in addition to charge neutrality, in order to make them full-ranked systems. In other words, potential on one of the triangles across each spanning tree in a region is treated as a reference potential, and the potentials of all the other triangles in that spanning tree is expressed as potential difference with the reference potential. This eliminates the redundant potentials in the system without losing any information, and hence mitigates the rank deficiencies.

## 4. Numerical Examples

An illustrative example containing multi-region dielectrics is considered in Fig. 4.

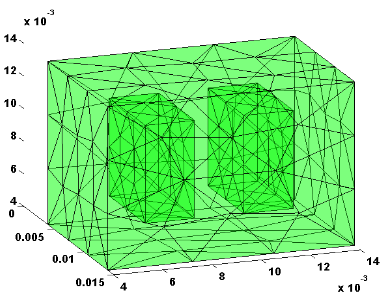



Fig. 4. Dielectric objects completely submerged in another dielectric

The geometry is excited with a plane wave from elevation angle $\theta = \pi/2$ and azimuth angle $\phi = 0$. The condition number plot for the proposed well-conditioned full-ranked SPIE-PMCHWT matrix for the structure as plotted against frequencies is depicted in Fig. 5.

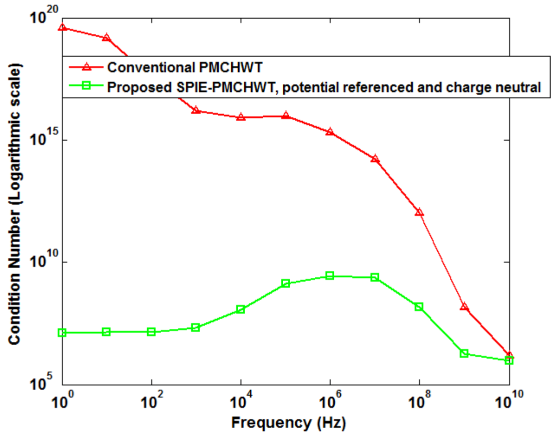



Fig. 5. Proposed SPIE-PMCHWT method applied to structure in Fig. 4

In the next example, a fighter aircraft F5 like model as shown in Fig. 7 is taken for demonstration. The aircraft is assumed to be composed of a material of relative permittivity 9. Figure 6 plots the condition number across frequencies.

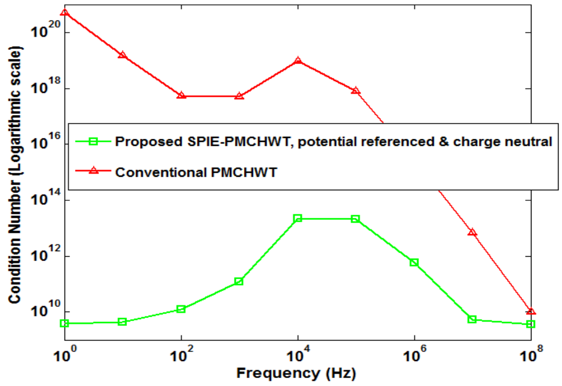



Fig. 6. SPIE-PMCHWT method applied on dielectric F5 aircraft like model- Condition number

The electric and magnetic surface current distribution on the aircraft at 1 Hz when illuminated with plane-wave from top and solved using proposed SPIE-PMCHWT is shown in Fig. 7.

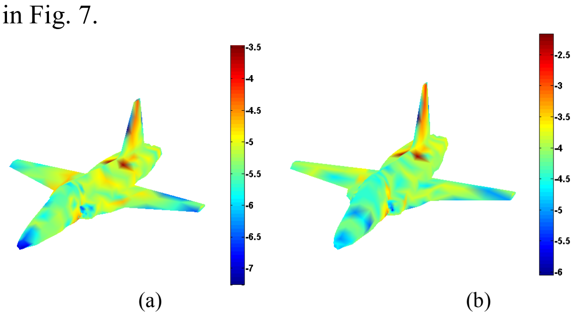
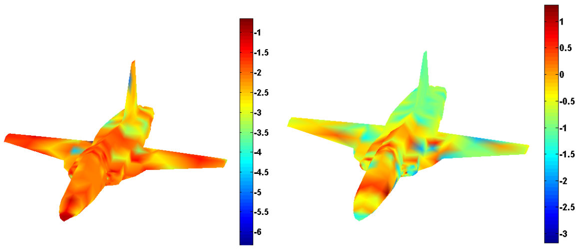



Fig. 7. Surface current distribution in logarithmic scale (a) Electric (Amperes) (b) Magnetic (Volts)

Figure 8 plots the zones of errors in currents on the aircraft and their relative magnitudes in logarithmic scale when the geometry is solved using conventional PMCHWT method.




(a) (b)

Fig. 8. Relative error zones for currents in logarithmic scale when solved using conventional PMCHWT for (a) Electric Current (b) Magnetic Current

## 5. Conclusion

In this work, a low-frequency stable SPIE-PMCHWT formulation to tackle multi-region dielectric bodies is proposed. The formulation is demonstrated to be stable in the low-frequency regime.

## 6. References

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