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Regression Methods of Obtaining Angular Superresolution

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Abstract

New methods of signal processing based on nonlinear regression methods are presented. They allow us to restore images of individual objects of group targets with superresolution at signal-to-noise ratios that are significantly lower than those provided by the known methods.

1 Introduction

Angular superresolution allows separate observation of individual targets as part of group targets, as well as obtaining detailed images of the objects under study.
The problem of obtaining superresolution belongs to the class of inverse problems. Attempts to exceed the Rayleigh criterion due to digital signal processing often lead to instability in solutions and significant errors.
There are many known methods for achieving superresolution, some of them are reported in [1-2]. These techniques that employ solution to inverse problems are not universal and to a big extent far from being efficient. At a relatively high noise level, their applications may lead to significant errors. Basically, they start to work successfully at a signal-to-noise ratio (SNR) of at least 20-25 dB. The performance of many of the above methods is not sufficient to restore the characteristics of objects in real time. Another drawback is that the methods for obtaining superresolution, in particular the popular MUSIC and ESPRIT, use narrowband signals and are inefficient for ultra-wideband radars.

2 Formulation of the Problem

A number of methods, called algebraic, make it possible to effectively use the available *a priori* information and obtain solutions with superresolution in real time with a higher level of random components [3-5].
The angular resolution of radar based on the Rayleigh criterion can be represented as:

$$\delta\theta \cong \lambda/D \tag{1}$$

which coincides with the width of the directional pattern (DP) at half power $\theta_{0,5,}$, where D is the antenna size and λ is the wavelength.
The signal $U(\alpha)$ received at sector scanning is a linear integral transformation:

$$U(\alpha) = \int_\Omega f(\alpha - \varphi) I(\varphi)\, d\varphi \tag{2}$$

where Ω is the angular region of the location of the radiation sources, $f(\alpha)$ is a DP, and $I(\varphi)$ is the angular distribution of the amplitude of the reflected (or radiated) signal. The problem is to restore the image of the source of signals $I(\varphi)$ with superresolution. The search for angular distribution $I(\varphi)$ reduces to an approximate solution of linear integral Fredholm equation of the first kind (2) of the convolution type.

3 Method of Solution

Assume that it is known in advance that a radar target is a group target consisting of two objects. Given the received scanning signal $U(\alpha)$ it is required to find the angular positions of the point objects α_1 and α_2 and the amplitudes of the signals radiated by them, I_1 and I_2.
Based on the *a priori* information, the group target under study can be described as a superposition of delta functions:

$$I(\alpha) = I_1\delta(\alpha - \alpha_1) + I_{21}\delta(\alpha + \alpha_2) \tag{3}$$

Thus, the problem of image reconstruction is parameterized and is reduced to the search for four unknowns.
Determination of unknown parameters can be carried out as a search for a minimum of a function of four variables in the form of minimizing the mean-square error

$$U(\alpha) = \int_\Omega U(\alpha) U_R(\alpha, \alpha_{1,} \alpha_{2,} I_1, I_{12})\, d\varphi \tag{4}$$

Direct numerical search for the minimum for heterogeneous variables $I_{1,2}$ and $\alpha_{1,2}$ reduces to solving the multi-extremal problem and does not guarantee obtaining global minimum.
An algorithm based on regression methods is significantly more efficient, specifically in the cases of nonlinear regression applied in the problem under study. We introduce the regression function as a mean-square error; the numerical solution is performed using the optimized Levenberg-Marquardt method.

4 Numerical Results

Characteristics of the increase in angular resolution and its limits have been studied using a mathematical model. At first, the classical problem of solving two identical

point objects was solved. At a low noise level (SNR $q <$ 80 dB), the regression analysis allowed to exceed the Rayleigh criterion by more than two orders of magnitude.
We note that the presence of random components in the signal under investigation substantially affects the quality of the approximate solution of the inverse problem under consideration.
Figure 1 shows in logarithmic scale the results as a dependence of the coefficient K_R (indicating the degree of excess of the Rayleigh criterion) on SNR q. For relatively low values of q, the achieved degree of superresolution of KR depends on the specific implementation of noise.

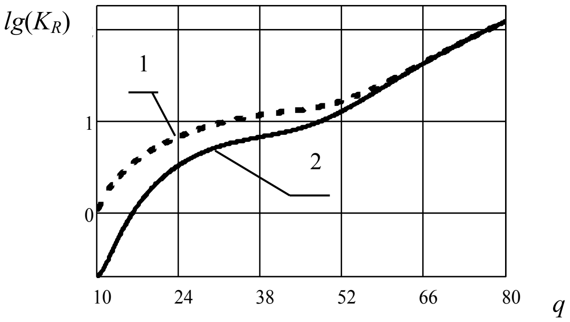

Figure 1. The degree of excess of the Rayleigh criterion against SNR. Curve 1 is the value of K_R achieved at not less than 10%-range of the noise realization. Curve 2 is the boundary of the region above which the K_R values are located in at least 90% -range.

For known methods, increasing the resolution by one half requires an increase in SNR q by an order of magnitude.
The present method is characterized by the separation of two regions. At small q, the achieved degree of superresolution increases with increasing q faster than in the known methods. At large values of q, the growth slows down and becomes comparable with them. Thus, the proposed method provides more pronounced noise-proof.
Suppose now that the group is a target object point with two different amplitudes of reflected signals. The numerical solution of the problem turns out to be more complicated. The stability and adequacy of solutions now depend not only on the angular distance between objects and SNR, but also on the ratio of signal amplitudes from each of the point targets.
Figure 2 shows the results of reconstructing the image of a group target with an angular distance between objects of $0.25\theta_{0,5}$ at SNR of 20 dB and a significant ratio of amplitudes of the reflected signals 1:5.
The obtained results confirm that despite high noise level, the quality of restoring the angular position of the objects remains good. The found positions of the objects and the amplitudes of the signals reflected by each object turned out to be close to the true ones which justifies applicability of the method.

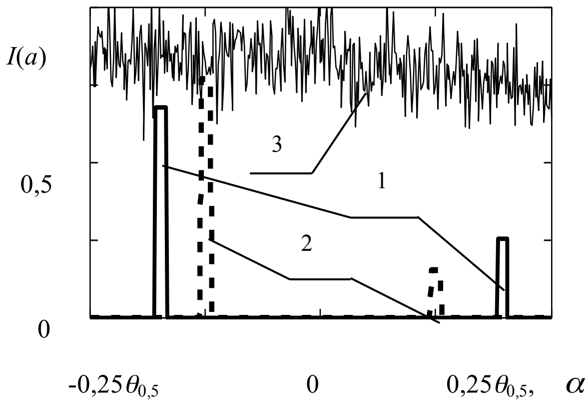

Figure 2. The angular position of the objects of the group target with different amplitudes of the reflected signal: 1 true targets; 2 restored targets; and 3 received signal.

5 Conclusion

A new method for processing signals obtained by locating group targets based on nonlinear regression methods is proposed. The algorithm created on its basis allows one to restore images of individual targets with superresolution.
It has been confirmed that the degree of excess of the Rayleigh criterion is mainly determined by the value of SNR. The results of numerical studies have shown that the minimum required SNR for obtaining a stable solution with superresolution is 10-12 dB, which is substantially less than that provided by the known methods.

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