

URSI Commission B School  
for Young Scientists

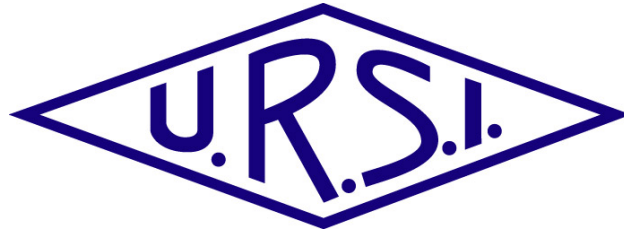
# **Fields and Waves in Metamaterials**

**Lecture Notes**

**August 16-17, 2014**

**Beijing Conference Center  
Beijing, China**





URSI Commission B School  
for Young Scientists

# **Fields and Waves in Metamaterials<sup>1</sup>**

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<sup>1</sup> This School is organized during the “XXXI URSI General Assembly and Scientific Symposium” (URSI GASS 2014), August 16-23, 2014, Beijing, China.



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# Preface

The “URSI Commission B School for Young Scientists” is organized by URSI Commission B and is arranged on the occasion of the “XXXI URSI General Assembly and Scientific Symposium” (URSI GASS 2014) in Beijing, China. This School is a two-day event held during URSI GASS 2014, and is sponsored jointly by URSI Commission B and the URSI GASS 2014 Local Organizing Committee. The School offers a short, intensive course, where a series of lectures will be delivered by a leading scientist in the Commission B community. Young scientists are encouraged to learn the fundamentals and future directions in the area of electromagnetic theory from these lectures.





# Program

## 1. Course Title

Fields and Waves in Metamaterials

## 2. Course Instructor

Professor Nader Engheta  
University of Pennsylvania  
Philadelphia, PA, USA

## 3. Course Program

### Lecture 1

- Date and Time: 14:00-18:00, Saturday August 16, 2014
- Venue: Meeting Room 12, 2nd floor, Conference Building  
Beijing Conference Center, Beijing, China
- Lecture Topics:  
Metamaterials: Basic Principles  
Metasurfaces and Graphene  
Optical Metatronics

### Lecture 2

- Date and Time: 8:00-12:00, Sunday August 17, 2014
- Venue: Meeting Room 12, 2nd floor, Conference Building  
Beijing Conference Center, Beijing, China
- Lecture Topics:  
Extreme-parameter metamaterials  
Metamaterial Guided and Radiating Structures  
Cloaking



# Lecture Abstract

## Fields and Waves in Metamaterials

**Nader Engheta**

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**Web: <http://www.seas.upenn.edu/~engheta/>**

Metamaterials and Plasmonic structures provide mechanisms for controlling and taming electromagnetic fields and waves in unprecedented ways. New directions, novel vistas and new applications are appearing in the horizon in the fields of metamaterials and its 2-D version, metasurfaces. In particular, when the extreme scenarios are considered, e.g., ultrathin structures (graphene), extreme near field (vortex in subwavelength near field), and extreme parameters (epsilon-near zero (ENZ), mu-near-zero (MNZ), epsilon-and-mu-near-zero (EMNZ)), numerous exciting possibilities for the interaction of waves with matter may occur. These may include design of metamaterials for scattering management in numerous applications where low or high scattering is desired, “metafunctional platforms” that can be formed on the metamaterial paradigms, and new functionalities may result from proper combinations of meta-systems and metamaterials. We have been exploring various features and characteristics of these concepts, topics, and directions in metamaterials, and we have been investigating new classes of applications such paradigms may provide. Some of the features of interest include nonlinearity, anisotropy, chirality, non-reciprocity, and non-locality. In this School, we will discuss the following topics:

### **Lecture 1**

*Metamaterials: Basic Principles*

*Metasurfaces and Graphene*

*Optical Metatronics*

### **Lecture 2**

*Extreme-parameter metamaterials*

*Metamaterial Guided and Radiating Structures*

*Cloaking*

# Biographical Sketch of Course Instructor



**Nader Engheta** is the H. Nedwill Ramsey Professor at the University of Pennsylvania in Philadelphia, with affiliations in the Departments of Electrical and Systems Engineering, Bioengineering, Physics and Astronomy, and Materials Science and Engineering. He received his B.S. degree from the University of Tehran, and his M.S and Ph.D. degrees from Caltech. Selected as one of the *Scientific American Magazine 50 Leaders in Science and Technology* in 2006 for developing the concept of optical lumped nanocircuits, he is a Guggenheim Fellow, an IEEE Third Millennium Medalist, a Fellow of IEEE, American Physical Society (APS), Optical Society of America (OSA), American Association for the Advancement of Science (AAAS), and SPIE-The International Society for Optical Engineering, and the recipient of numerous awards for his research including *2014 Balthasar van der Pol Gold Medal from the International Union of Radio Science (URSI)*, *2013 Benjamin Franklin Key Award*, *2013 Inaugural SINA Award in Engineering*, *2012 IEEE Electromagnetics Award*, *2008 George H. Heilmeyer Award for Excellence in Research*, the *Fulbright Naples Chair Award*, *NSF Presidential Young Investigator award*, the *UPS Foundation Distinguished Educator term Chair*, and several teaching awards including the *Christian F. and Mary R. Lindback Foundation Award*, *S. Reid Warren, Jr. Award* and *W. M. Keck Foundation Award*. His current research activities span a broad range of areas including nanophotonics, metamaterials, nano-scale optics, graphene optics, imaging and sensing inspired by eyes of animal species, optical nanoengineering, microwave and optical antennas, and engineering and physics of fields and waves. He has co-edited (with R. W. Ziolkowski) the book entitled “*Metamaterials: Physics and Engineering Explorations*” by Wiley-IEEE Press, 2006. He was the Chair of the Gordon Research Conference on Plasmonics in June 2012.

# **Fields and Waves in Metamaterials**

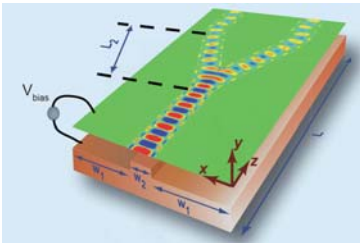
**August 16-17, 2014**

**Professor Nader Engheta  
University of Pennsylvania  
Philadelphia, Pennsylvania  
USA**



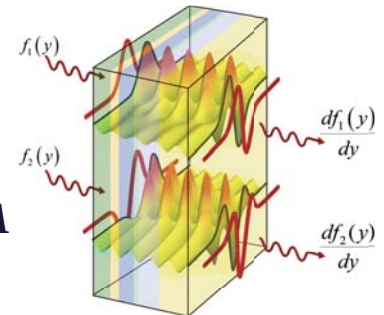
# Fields and Waves in Metamaterials

## Part 1



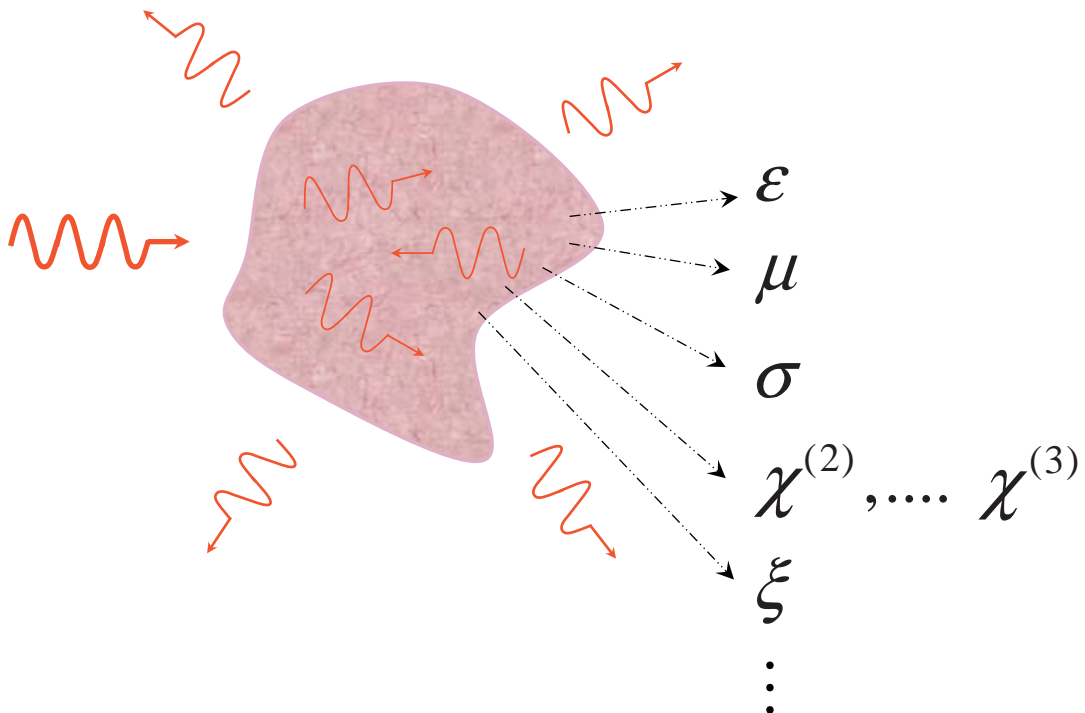
*Nader Engheta*

*University of Pennsylvania  
Philadelphia, PA 19104, USA*

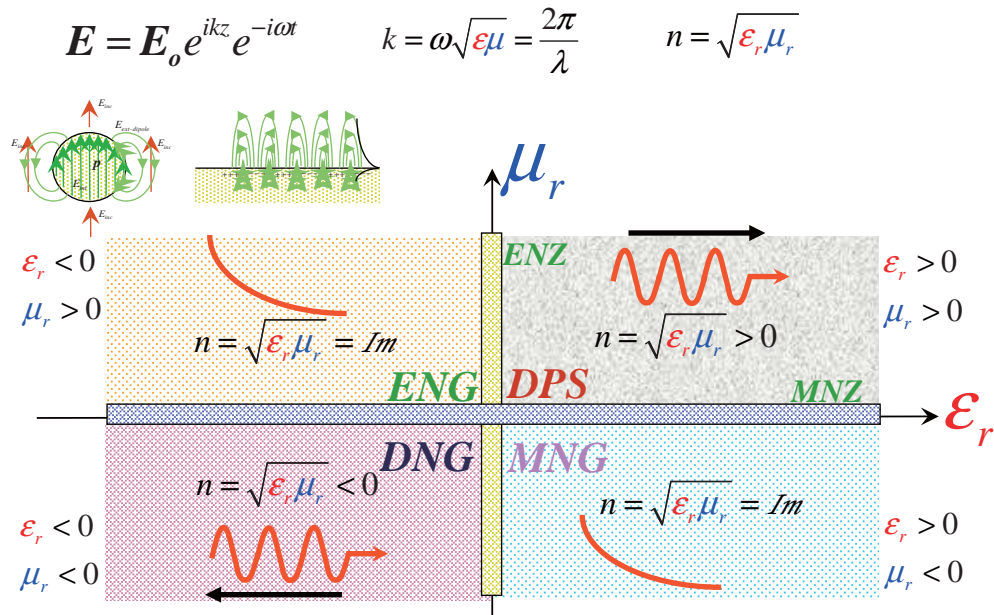


*August 16-17, 2014*

# *Light-Matter Interaction*



# Metamaterials and Plasmonic Phenomena



# “Natural” Materials



PERIODIC TABLE OF THE ELEMENTS







# “Artificially” Engineered Materials

## ● Particulate Composite Materials

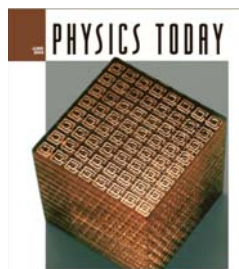
$n_h = \sqrt{\epsilon_{h,r} \mu_{h,r}}$   
 $n_c = \sqrt{\epsilon_{c,r} \mu_{c,r}}$

- Composition
- Alignment
- Arrangement
- Density
- Host Medium
- Geometry/Shape

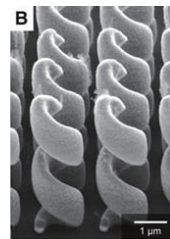
## Metamaterials Samples (2000-2013)



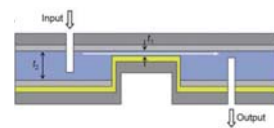
Smith, Schultz group (2000)



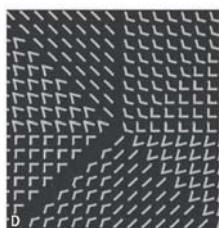
Boeing group



Wegener group (2009)



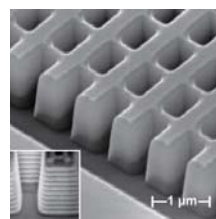
Atwater group (2007)



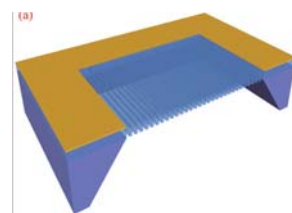
Capasso group (2011)



Shalaev group (2011)



Zhang group (2008)



Engheta group (2012)

# Metamaterial Applications (2000-2013)



*Cloaking*

*Ultrathin Cavities*

*Perfect Lens*

*Transformation Optics*

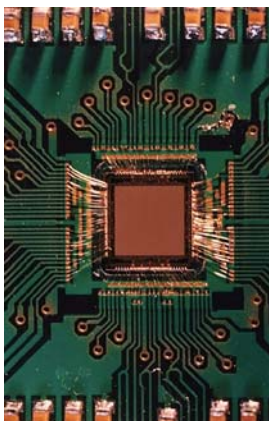
*Hyperlens*

*ENZ & MNZ*

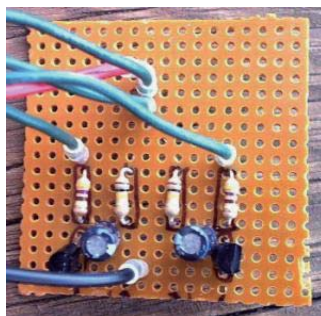
*Metasurfaces*

*Metatronics*

# Electronic Modules



[http://www.imrc.hw.ac.uk/New\\_versions/Home\\_files/Microelectronics.jpg](http://www.imrc.hw.ac.uk/New_versions/Home_files/Microelectronics.jpg)



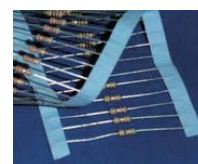
*C*

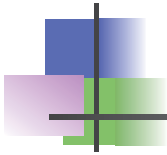


*L*

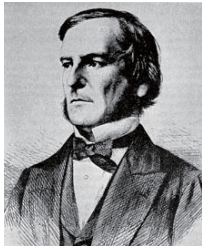


*R*



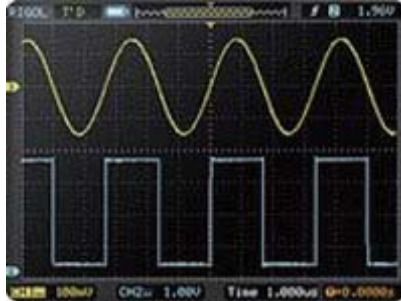


# Analog vs Digital

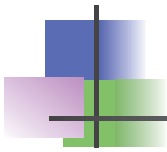
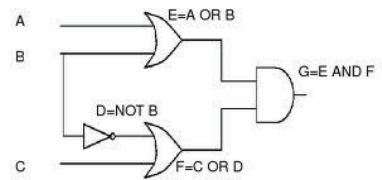


George Boole

$$f(t) = \sin(\omega t)$$

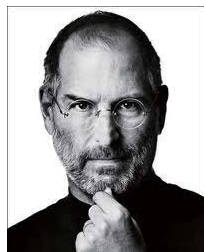


$$u(t) = 1001110\dots$$



# iPhone

# DOS



```

Volume in drive A is BOOTDISK
Volume Serial Number is 3505-18E3
Directory of A:\

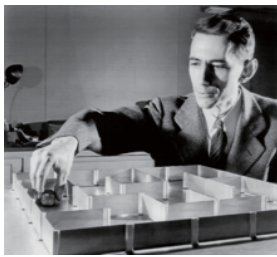
COMMAND  COM          93,812  08-24-96  11:11a
AUTOEXEC BAT           13  11-14-02  12:37p
CONFIG   SYS            0  05-20-07  3:06a
          3 file(s)          93,825 bytes
          0 dir(s)        1,147,392 bytes free

A:\>c:
C:\>nvflash turbo.rom_
  
```

[http://10.gstatic.com/images?q=tbn:ANd9GcQ2jC\\_aCeZHKyjVou0Q\\_xOq0LG3FkyuW963\\_OLq cM07rd4EHAUsA](http://10.gstatic.com/images?q=tbn:ANd9GcQ2jC_aCeZHKyjVou0Q_xOq0LG3FkyuW963_OLq cM07rd4EHAUsA)



# Claude Shannon & Channel Capacity

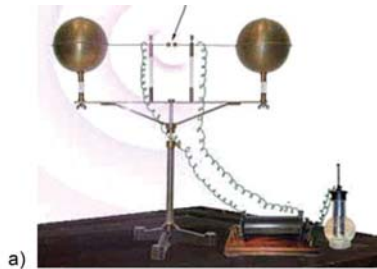


C. Shannon

$$\text{Channel Capacity} = B \log_2 \left( 1 + \frac{S}{N} \right)$$



# Development of Antennas



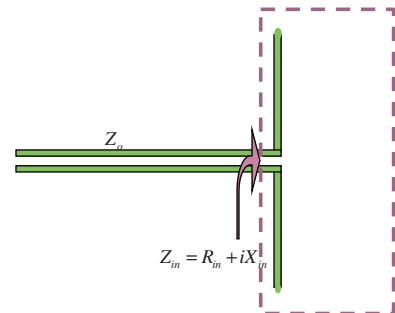
From: <http://www.sparkmuseum.com>



R. W. P. King



S. A. Schelkunoff





# How about Metamaterials?



## Complexity vs Simplicity

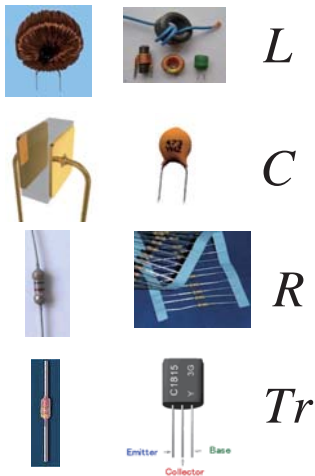
*Modularization*  
*Parameterization*  
*Conceptualization*  
*Cross Breeding*



*Metamaterials*

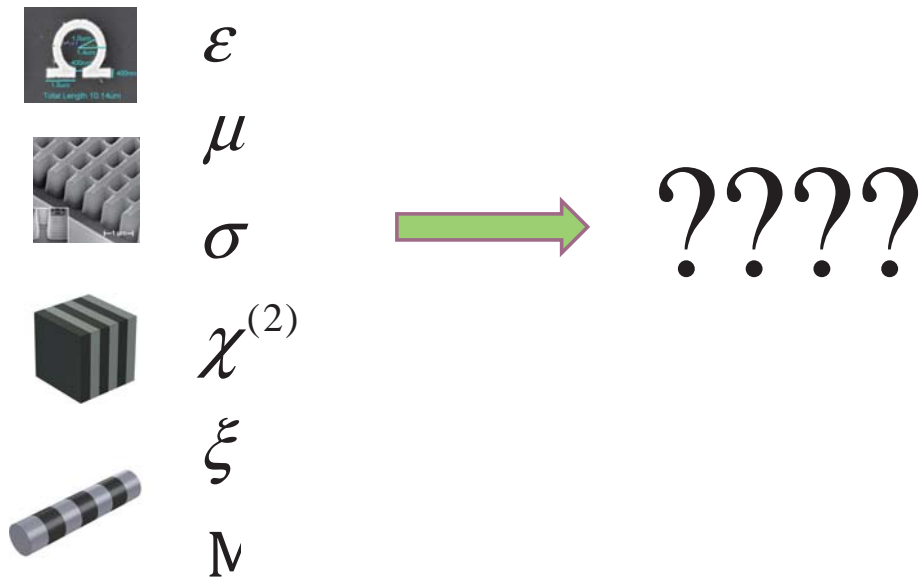


# Metamaterial Gadgets?

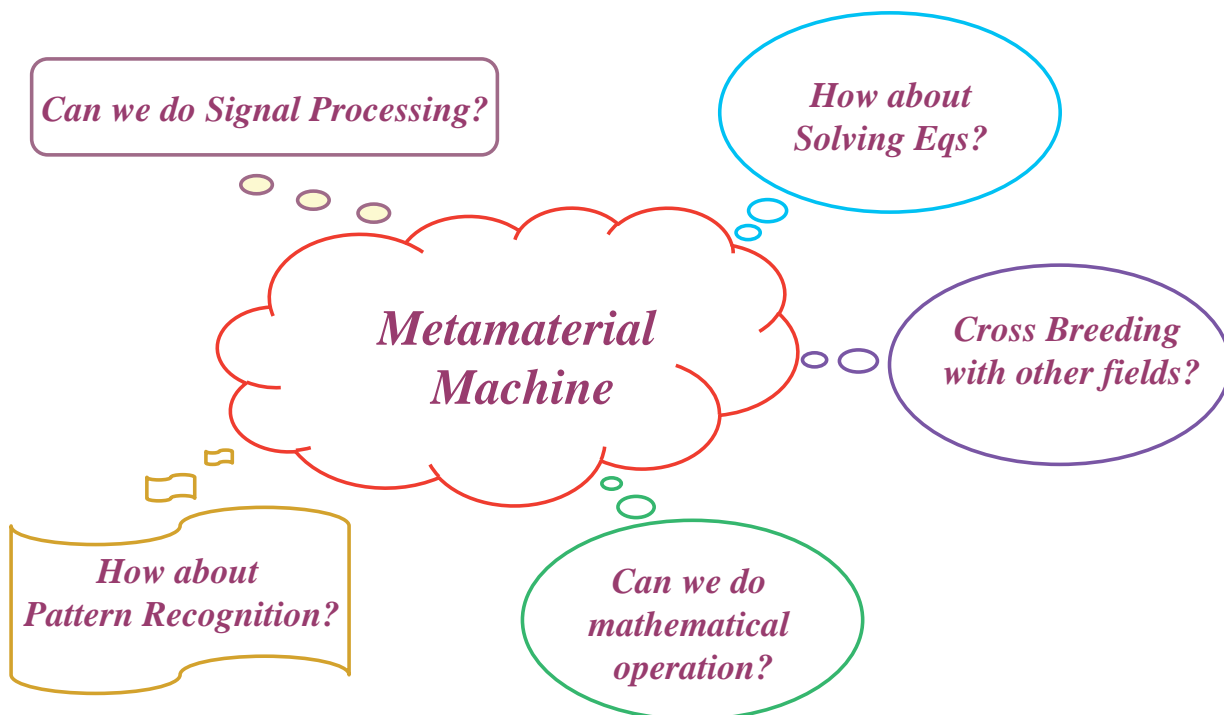


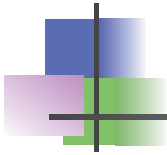


# Metamaterial Gadgets?



# Metamaterial “Machines”?





# Cross Breeding: Photonics vs Electronics



*Photonics/  
Microwaves*



*Concept of  
Metamaterials*



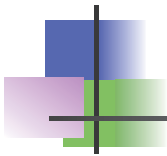
*Electronics*



*“Metatronics”*



*Building Blocks for  
Metamaterials*

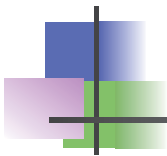


## *“Modular Blocks” in electronics*

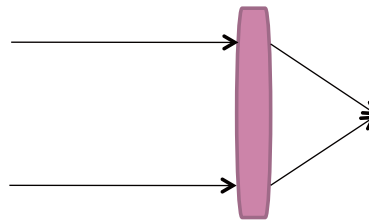




# "Building Blocks" in Optics?

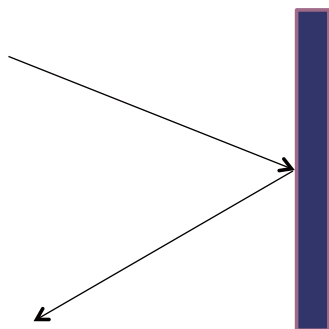


*Waveguide*

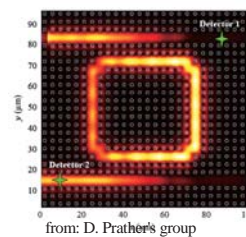


*Lens*

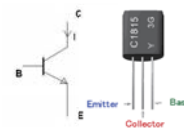
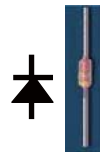
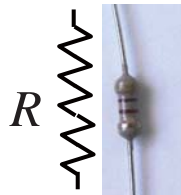
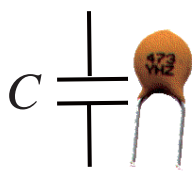
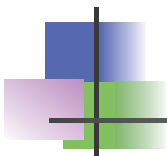
*Optics*



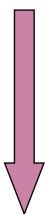
*Mirror*



# "Lumped" Circuit Elements in Nanophotonics?



*Radio Frequency (RF) electronics*



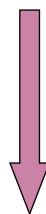
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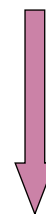
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?



?

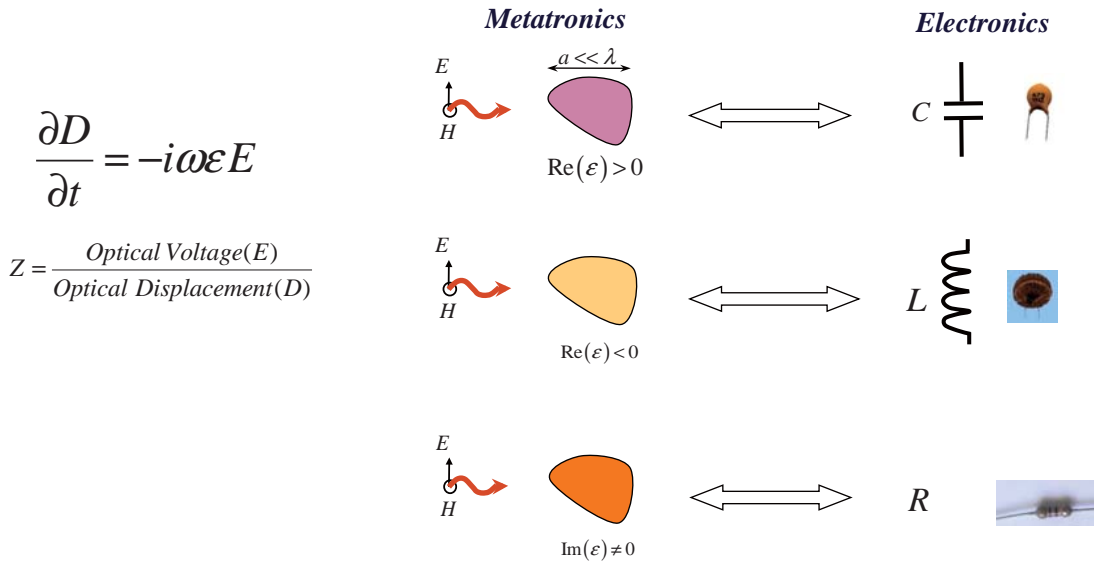


?

*Nano-Optics*

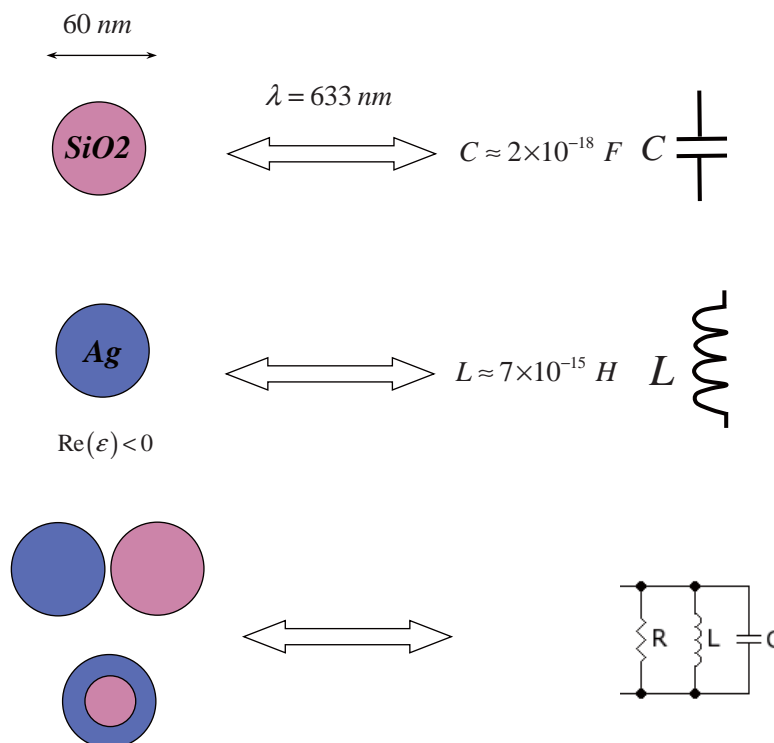


# Optical Lumped Circuit Elements: Modular Blocks



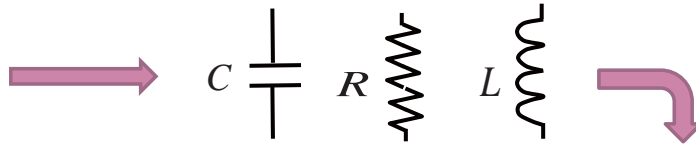
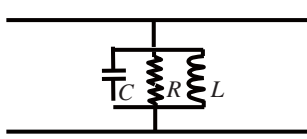
Engheta, *Science*, 317, 1698 (2007) Caglayan, Hong, Edwards, Kagan, Engheta, *Phys. Rev. Lett.* (2013)  
 Engheta, *Physics World*, 23(9), 31 (2010) Sun, Edwards, Alu, Engheta, *Nature Material*, March 2012  
 Engheta, Salandrino, Alu, *Phys. Rev. Lett.* 95 (2005)

## Examples

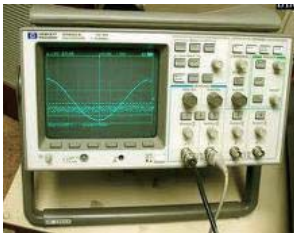
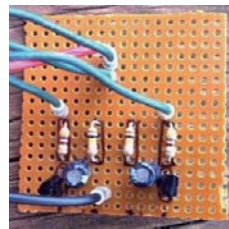




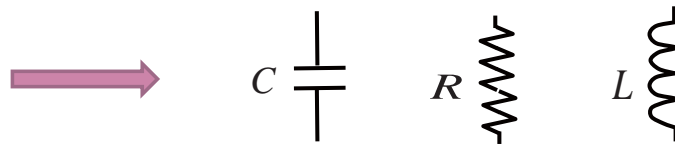
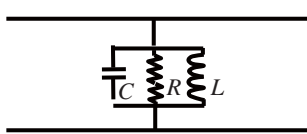
# Electronic Circuit Design?



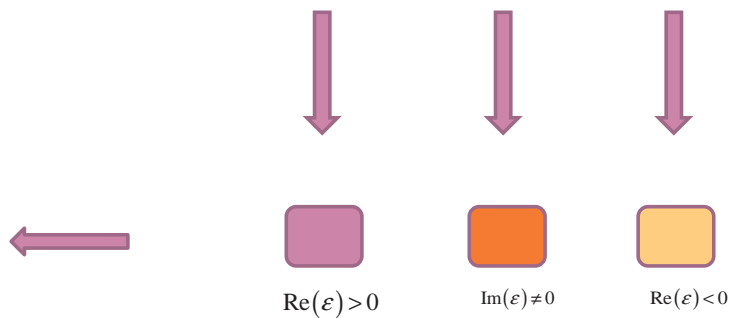
*Circuit Formulas*



# Can we do this in Nano-Optics?

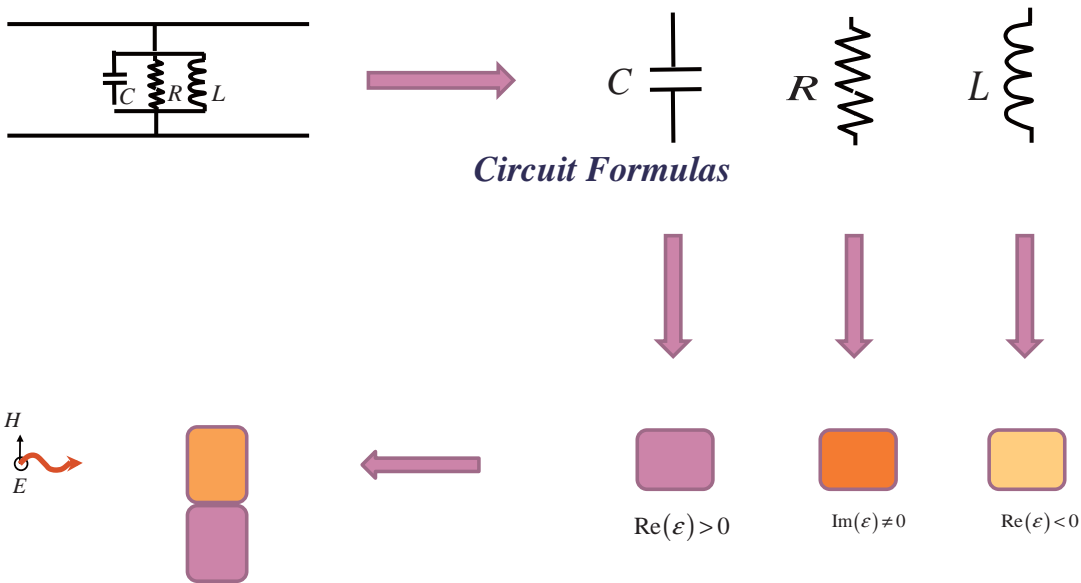


*Circuit Formulas*

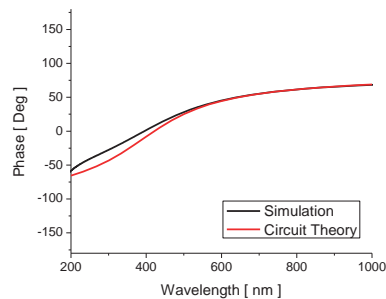
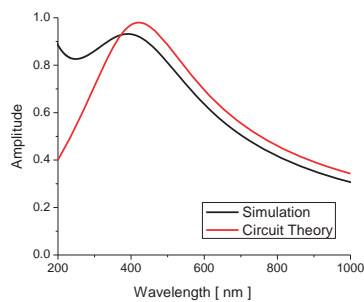
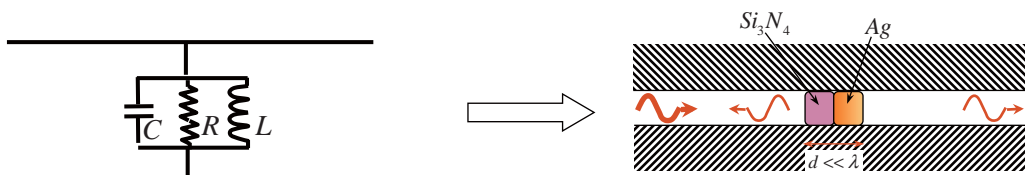




# Can we do this in Nano-Optics?

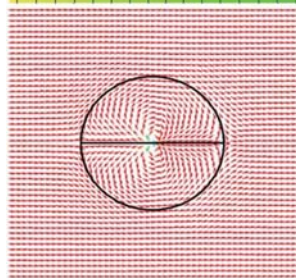
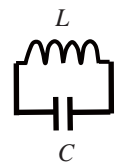
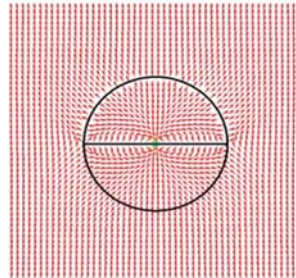
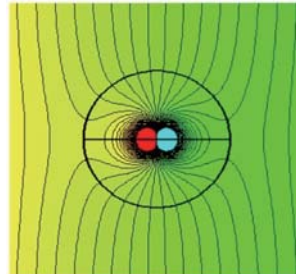
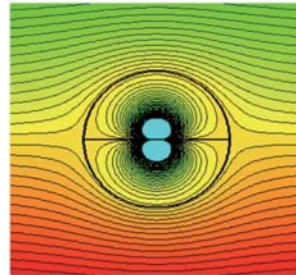
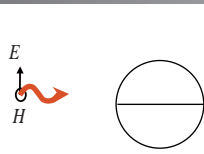
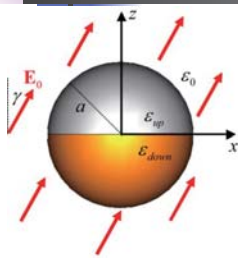


# Optical Filter with Nanorods



# “Stereo-Circuits”

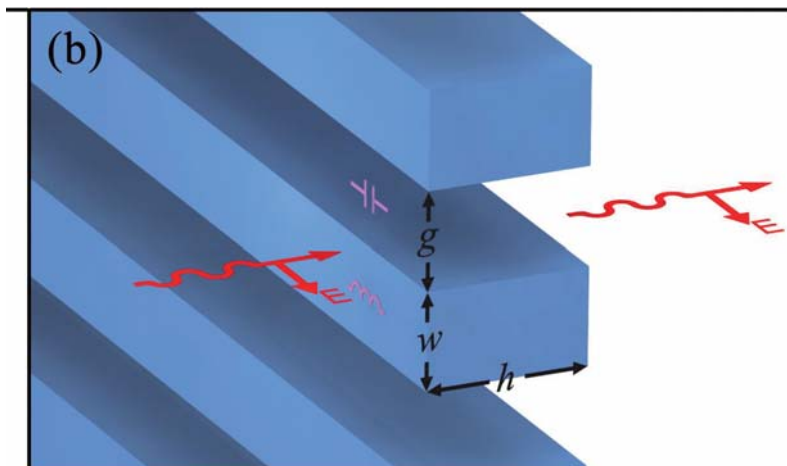
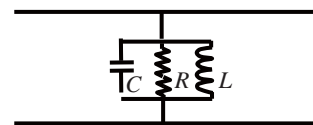
## Different “Circuits” for Different “Views”



Salandrino, Alu, Engheta, *JOSAB*, Part 1, 2007  
 Alu, Salandrino, Engheta, *JOSAB*, Part 2, 2007

Alu and Engheta, *New Journal of Physics*, 2009

## Experimental Verification at IR



$W = 75\text{nm}, 125\text{nm}, 225\text{nm}$

$g = 75\text{nm}$

$h = 175\text{nm}, 250\text{nm}, 325\text{nm}$

Y. Sun, B. Edwards, A. Alu, and N. Engheta, *Nature Materials*, March 2012



# Experimental Verification at IR

## Circuit Theory Model

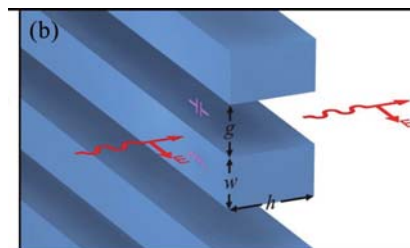
$$Z_{\text{wire}}^{\text{par}} \equiv \frac{i}{\omega h w \epsilon_{\text{Si}_3\text{N}_4}}$$

$$Z_{\text{air-gap}}^{\text{par}} \equiv \frac{i}{\omega h g \epsilon_{\text{air}}}$$

$$Z_{\text{equivalent}}^{\text{par}} \equiv \frac{Z_{\text{wire}}^{\text{par}} \cdot Z_{\text{air-gap}}^{\text{par}}}{Z_{\text{wire}}^{\text{par}} + Z_{\text{air-gap}}^{\text{par}}}$$

$$T^{\text{par}} = \left| \frac{Z_{\text{equivalent}}^{\text{par}}}{Z_{\text{equivalent}}^{\text{par}} + \left[ \eta_o / (2(W + g)) \right]} \right|^2$$

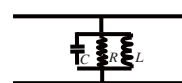
Y. Sun, B. Edwards, A. Alu, and N. Engheta, *Nature Materials*, March 2012



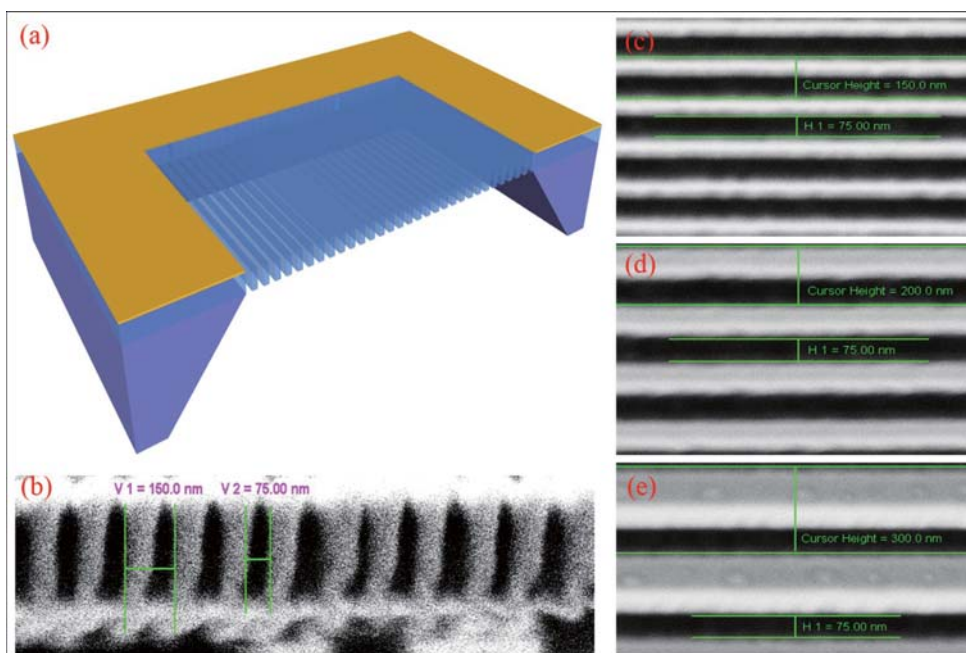
$g = 75\text{nm}$

$h = 175\text{nm}, 250\text{nm}, 325\text{nm}$

$W = 75\text{nm}, 125\text{nm}, 225\text{nm}$



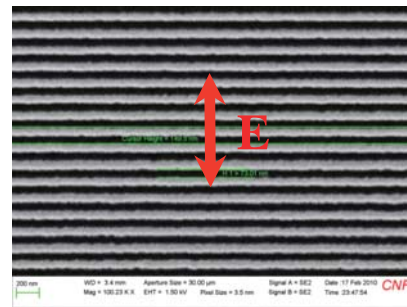
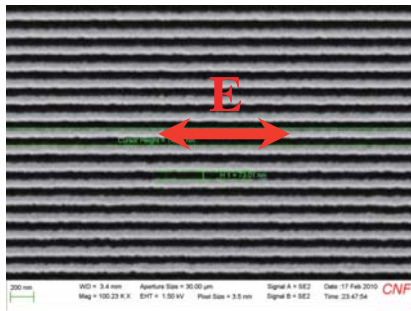
## Our Samples



Y. Sun, B. Edwards, A. Alu, and N. Engheta, *Nature Materials*, March 2012



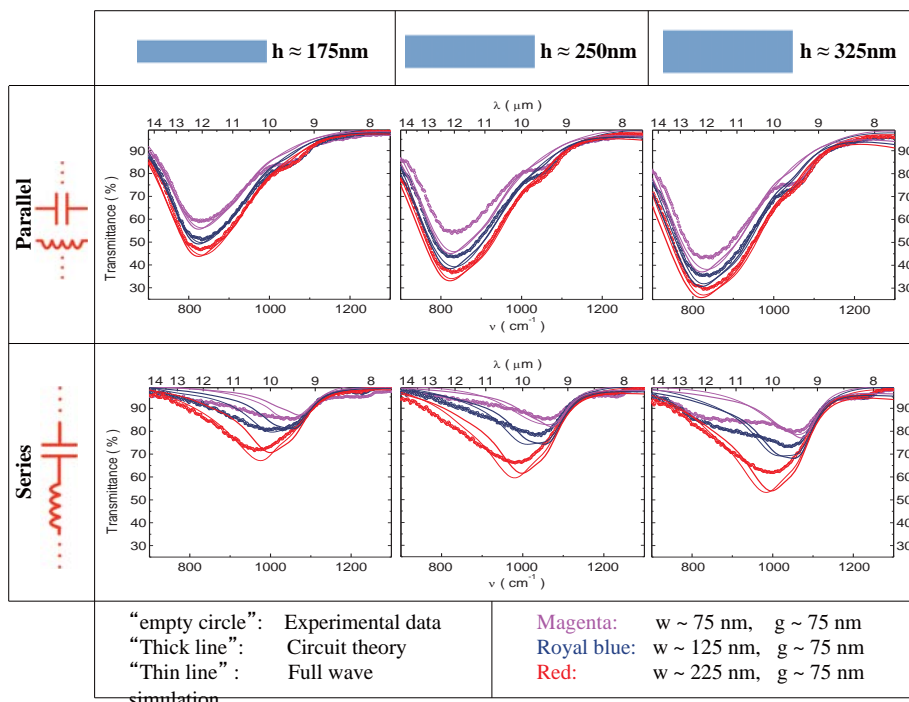
# "Parallel" and "Series" Optical Circuits



Y. Sun, B. Edwards, A. Alu, and N. Engheta, *Nature Materials*, March 2012

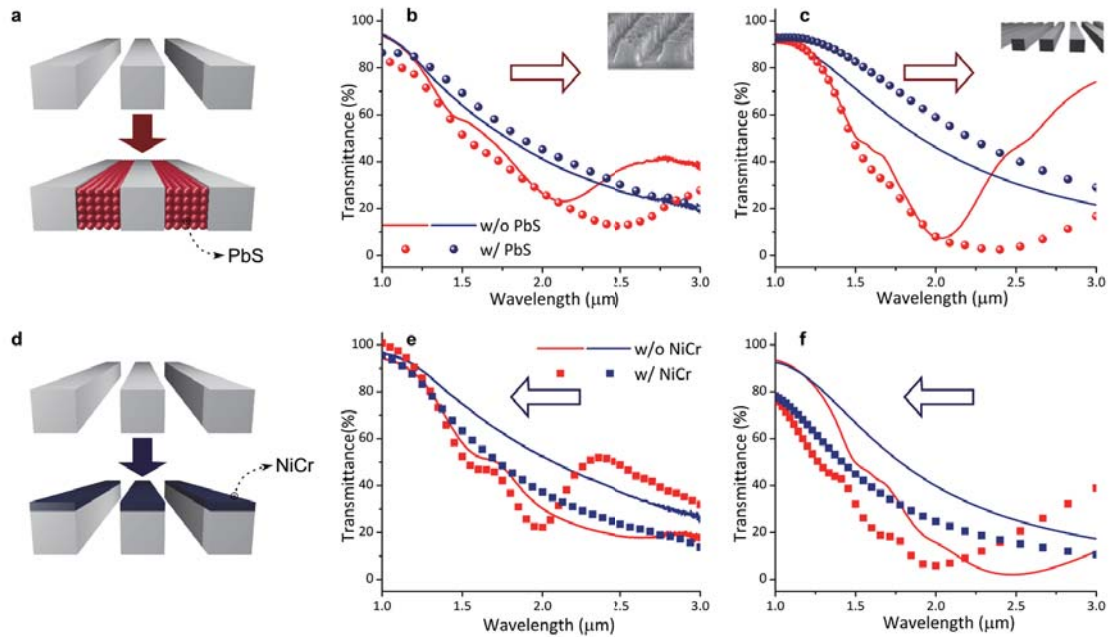


## Collective Results



Y. Sun, B. Edwards, A. Alu, and N. Engheta, *Nature Materials*, March 2012

## *TCO NIR Metatronic Circuits Fabrication and Experimental Results*

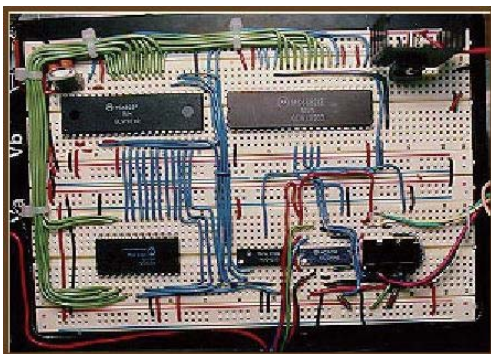


Caglayan, Hong, Edwards, Kagan, Engheta, *Phys. Rev. Lett.* **111**, 073904 (2013)

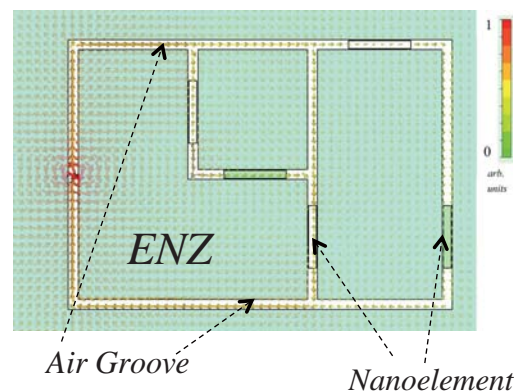
## *Nano-Optics Circuit Boards*



### *Electronic Circuit Board*

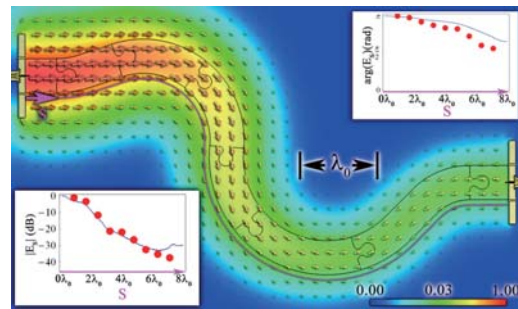
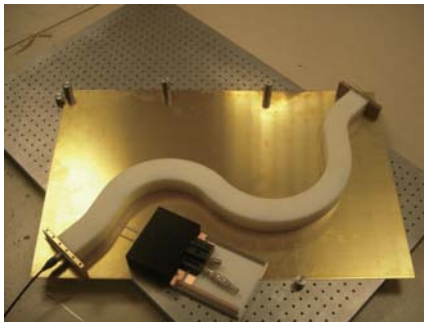
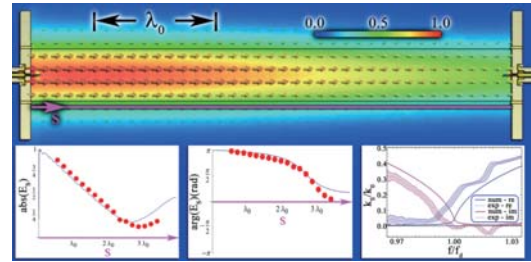
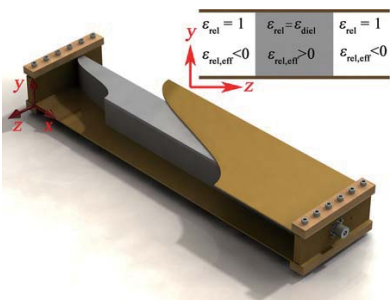


### *Metatronic Circuit Board*



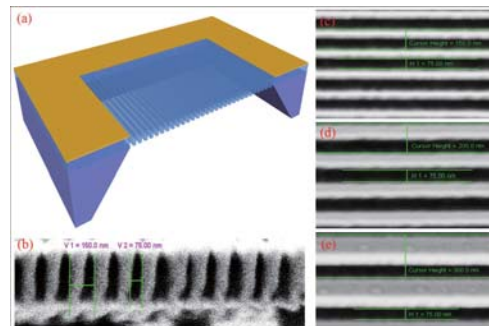
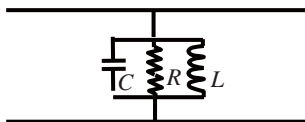
Alu and Engheta, *Phys. Rev. Lett.*, 2009

# Experimental Verification of Displacement-Current Wire



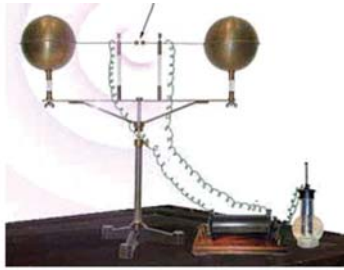
B. Edwards and N. Engheta, *Physical Review Letters*, May 7, 2012

# From a "Filter" to a "Filter"



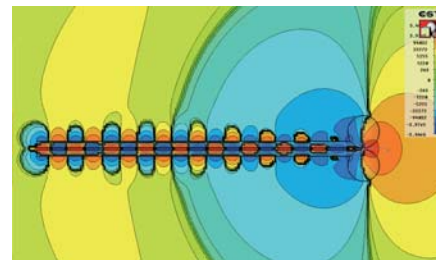
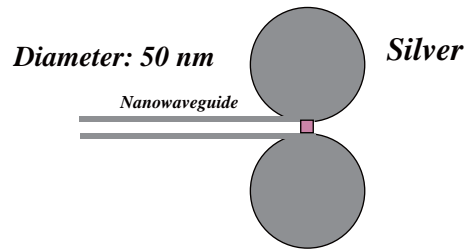


# From an "Antenna" to an "Nanoantenna"



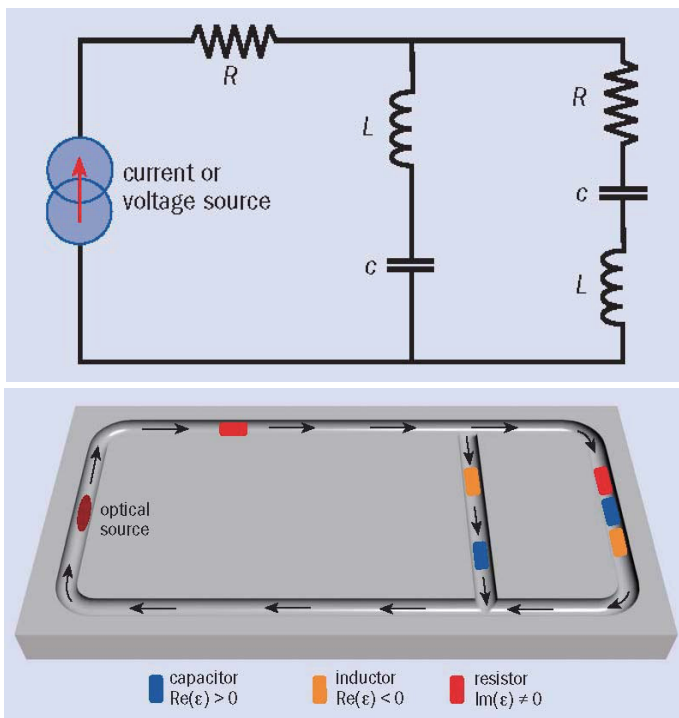
a)

From: <http://www.sparkmuseum.com>

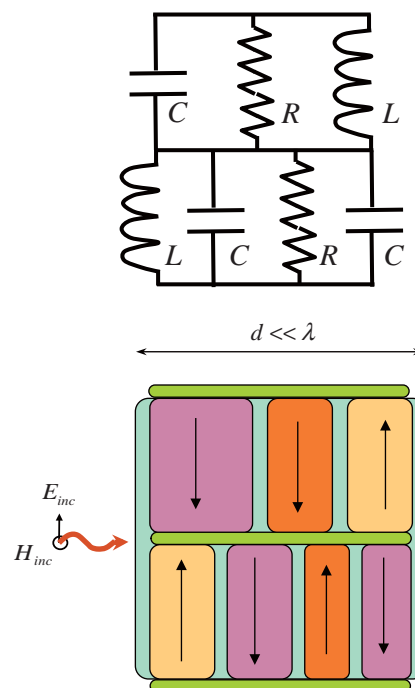


A. Alu and N. Engheta, *Phys. Rev. B*, 2008

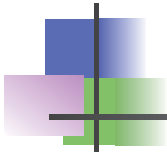
# Optical Metatronics



Engheta, *Physics Worlds*, 23(9), 31 (2010)



Engheta, *Science*, 317, 1698 (2007)



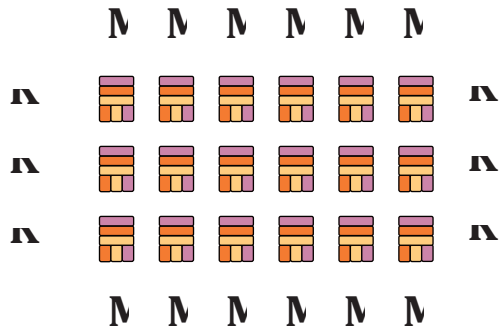
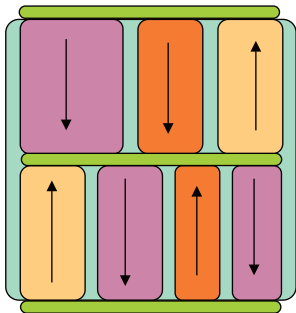
# Metatronics vs Metamaterials



*Metatronics*



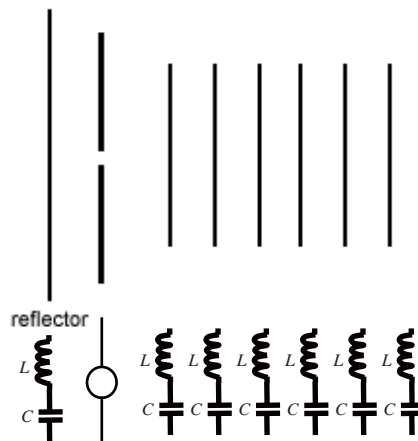
*Building Blocks for Metamaterials*



# Yagi-Uda Antennas

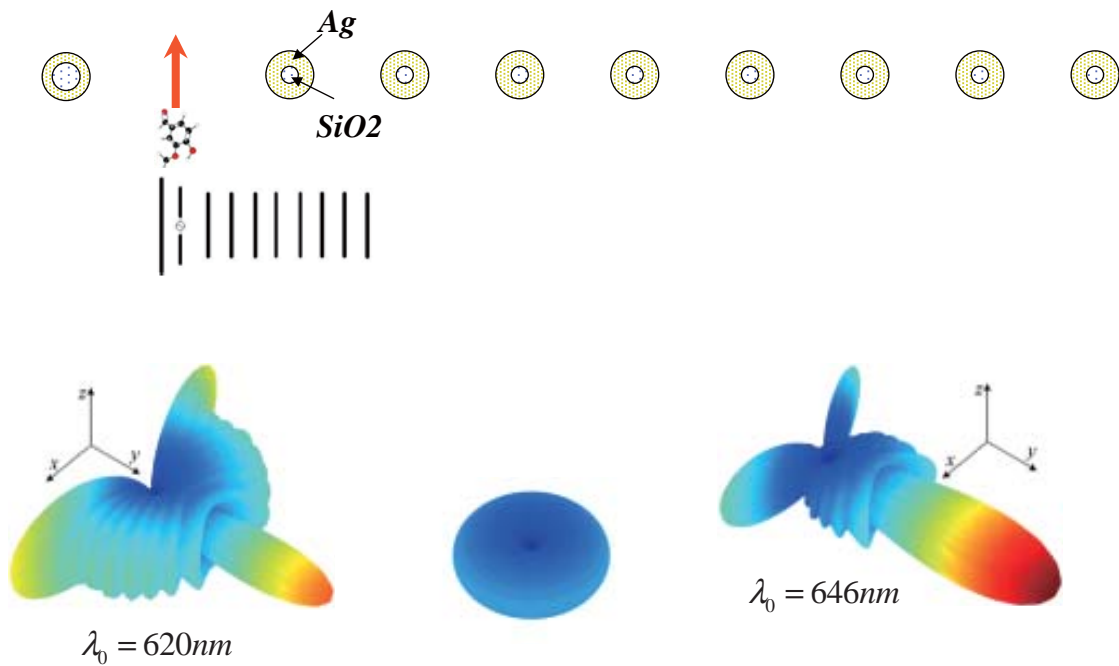


[Picasaweb.google.com/.../YOKis5Vf7nhDG5dGAoSD0w](https://www.picasaweb.google.com/.../YOKis5Vf7nhDG5dGAoSD0w)





# Optical "Yagi-Uda" Nanoantenna

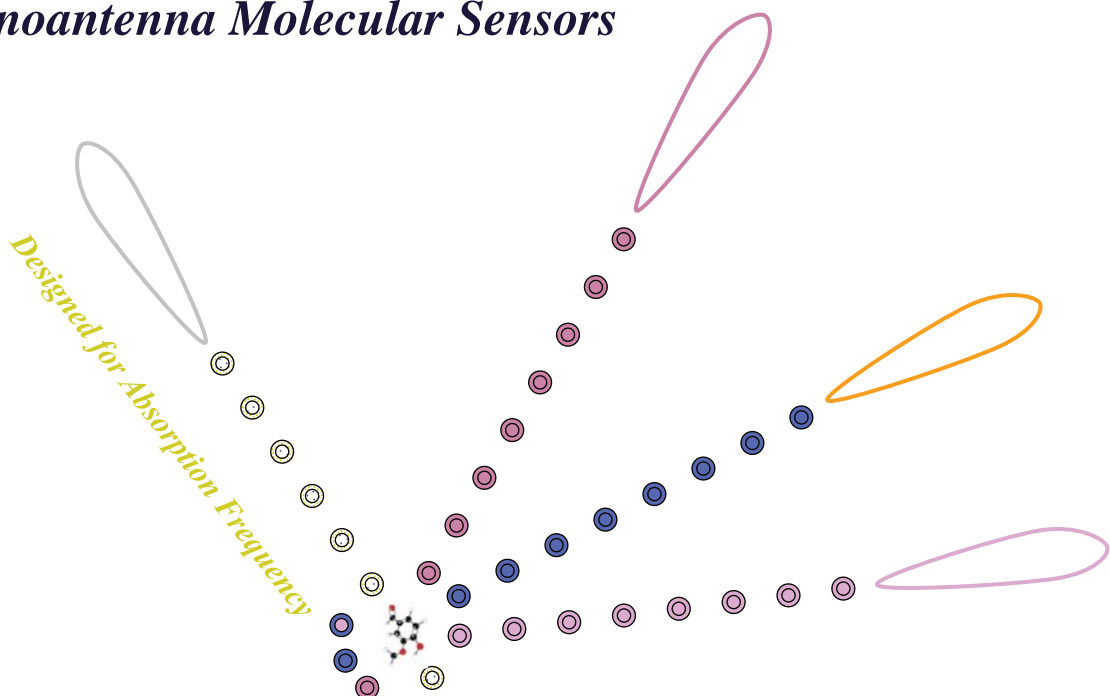


Li, Salandrino, and Engheta, *Phys. Rev. B*, 76, 245403 (2007)

# Nanoscale "Spectrometer" in Molecular Spectroscopy



## Nanoantenna Molecular Sensors



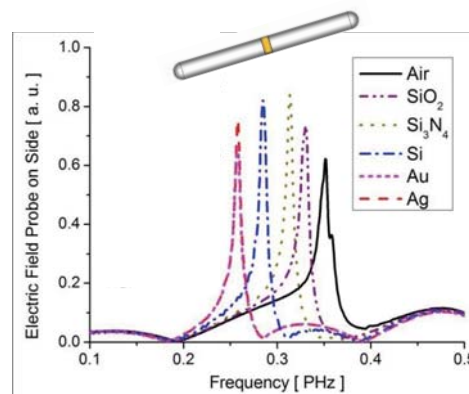
Li, Salandrino, and Engheta, *Phys. Rev. B*, 2009



# Optical Wireless Link at Nanoscales



Antennas, local oscillators, filters, switches, mixers, modulators, demodulators, etc. etc.



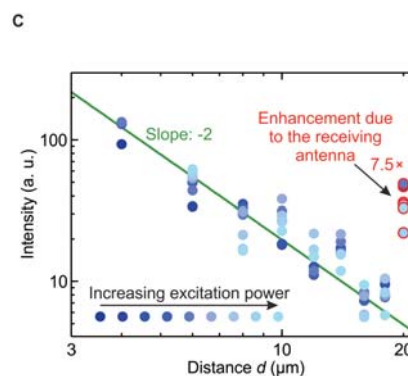
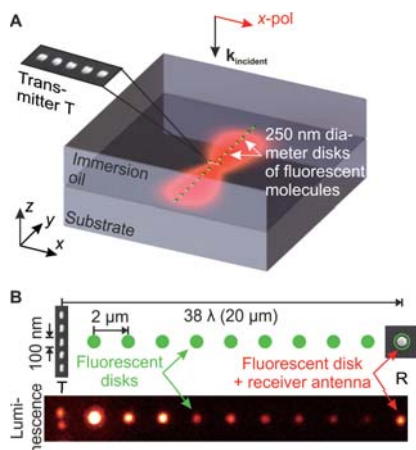
Alu and Engheta, *Phys. Rev. Lett.*, May 2010

Alu and Engheta, *Nature Photonics*, Vol. 2, May 2008



# Experimental Verification

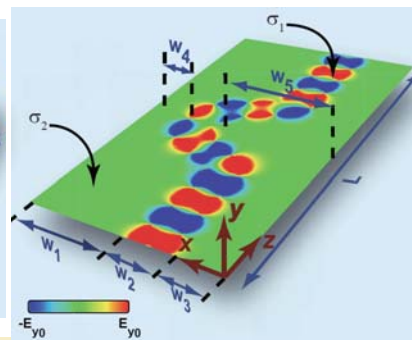
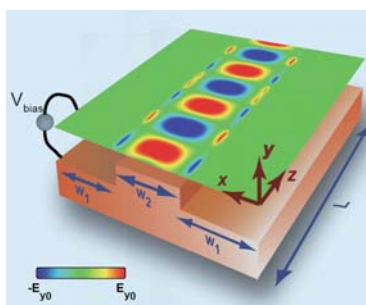
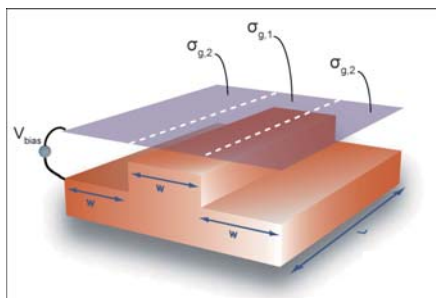
Harald Giessen's group in collaboration with my group



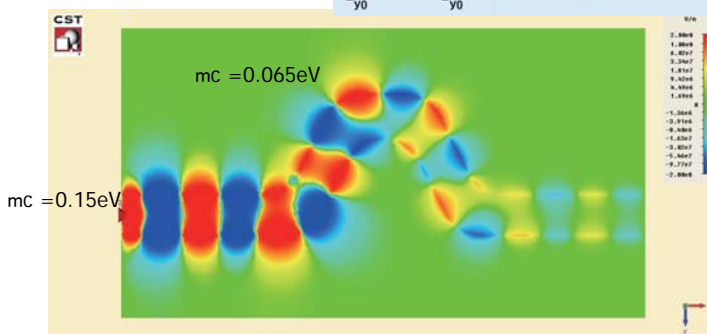
D. Dregely, K. Lindfors, M. Lippitz, N. Engheta, M. Totzeck, H. Giessen, *Nature Communications*, 2014



# One-Atom-Thick Optical Devices

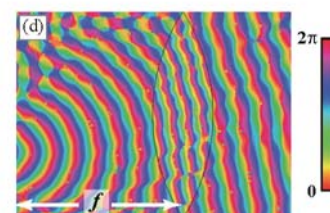
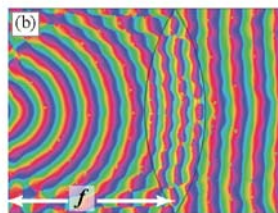
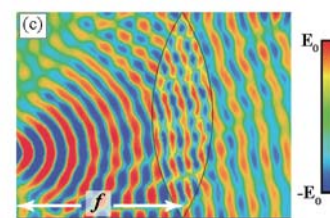
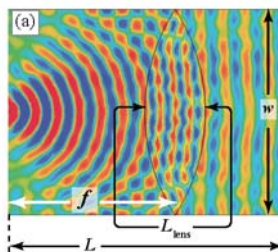
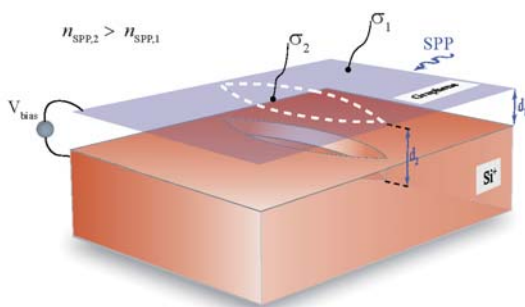


Region 1:  $\sigma_{g,i} > 0$   
 $\mu_c = 150 \text{ meV}$   
 Region 2:  $\sigma_{g,i} < 0$   
 $\mu_c = 65 \text{ meV}$



A. Vakil and N. Engheta, *Science*, 2011

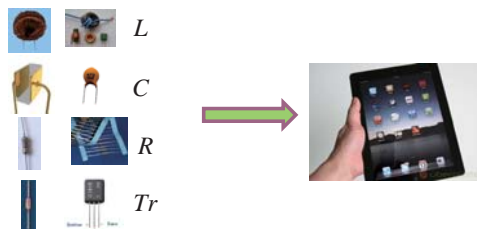
# One-Atom-Thick Signal Processing: Fourier Transform



Vakil, Engheta, *Phys. Rev. B*, (2012)



## *Metasystems*



## *Signal-Processing Metamaterials?*

## *Metamaterial Computing*

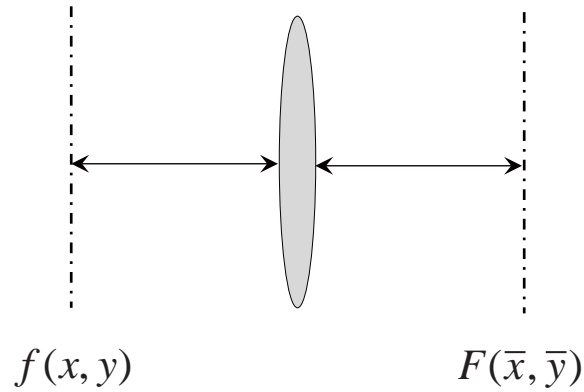


$$g(x_1, x_2, \dots) = \iiint f(u_1, u_2, \dots) k(x_1, x_2, \dots; u_1, u_2, \dots) du_1 du_2 \dots$$

## *Metamaterial Analog Computer?*



# Fourier-Transform

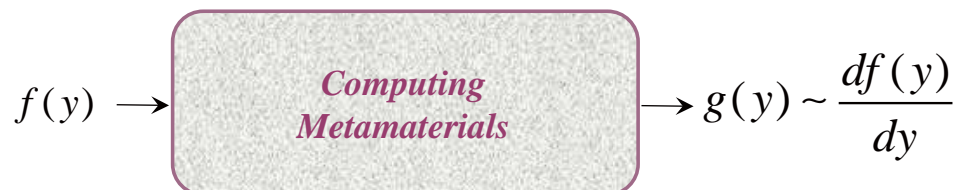


$$F(\bar{x}, \bar{y}) : \text{Fourier Transform}[f(x, y)]$$

J. Goodman, *Fourier Optics*, 1994



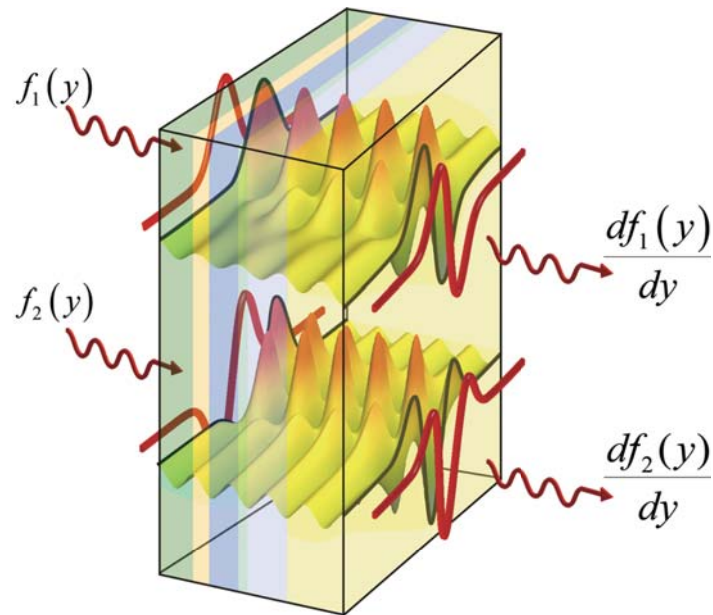
# Metamaterial Computing



**“Differentiator” Metamaterial**



# Computing Metamaterial



# “Differentiator” Metamaterial



$$g(y) \sim \frac{df(y)}{dy}$$



$$f(y) \xrightarrow{\text{Fourier}} F(\bar{y})$$

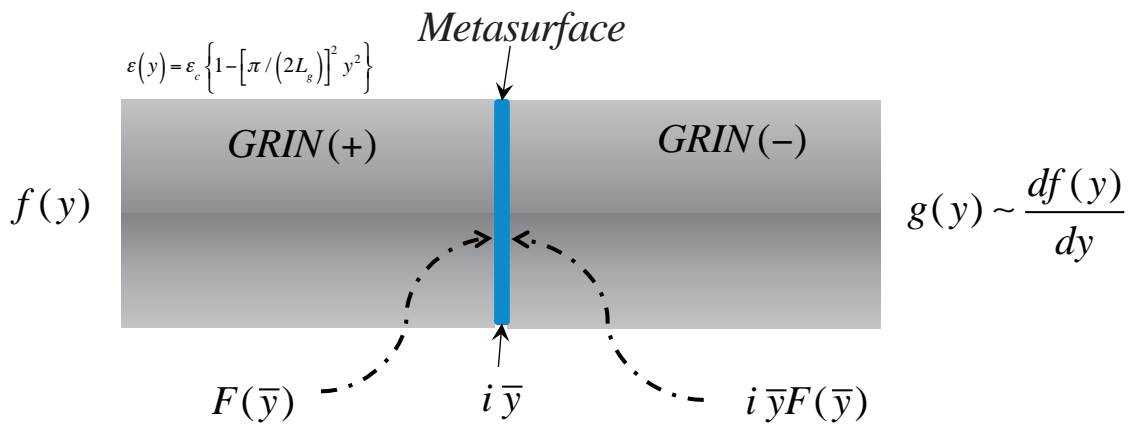
$$g(y) \xrightarrow{\text{Fourier}} G(\bar{y})$$

$$G(\bar{y}) \propto (i\bar{y}) F(\bar{y})$$





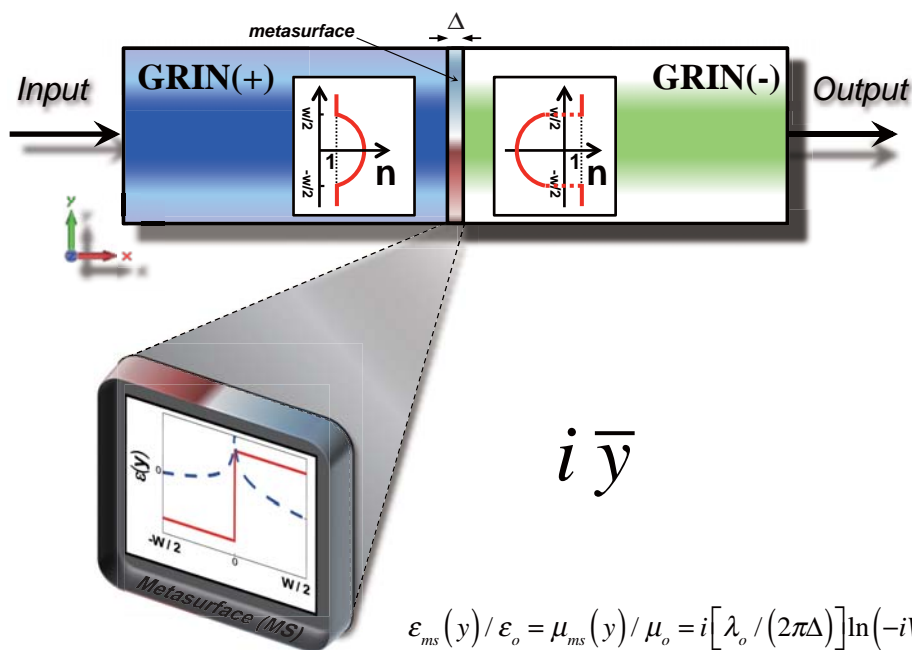
# “Differentiator” Metamaterial



A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014



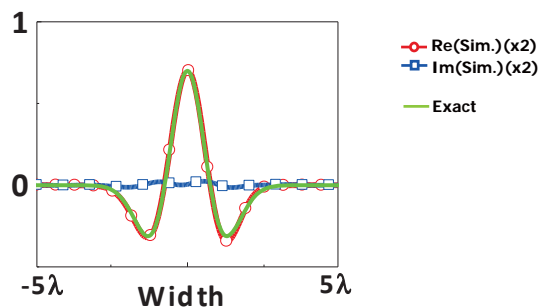
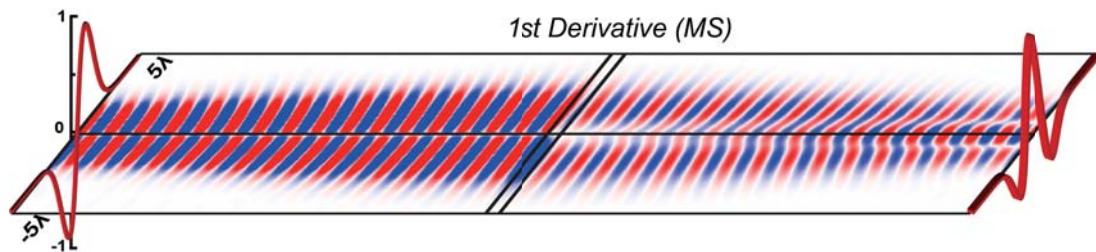
# GRIN(+) - MS - GRIN (-)



A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014



# Metamaterial as Differentiator

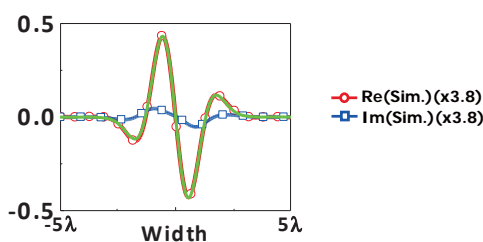
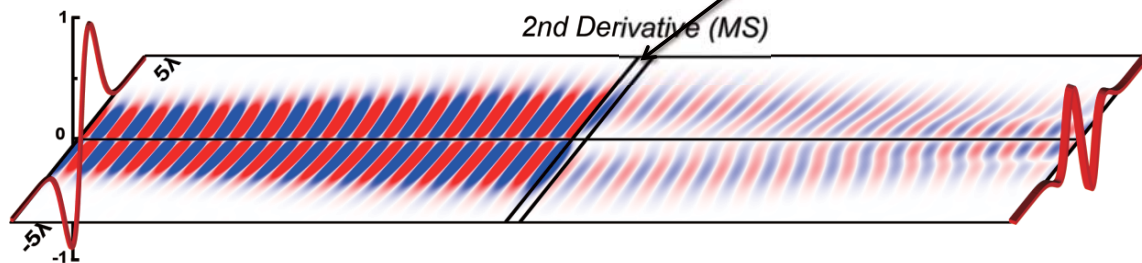


A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014



# Metamaterial as 2<sup>nd</sup> Differentiator

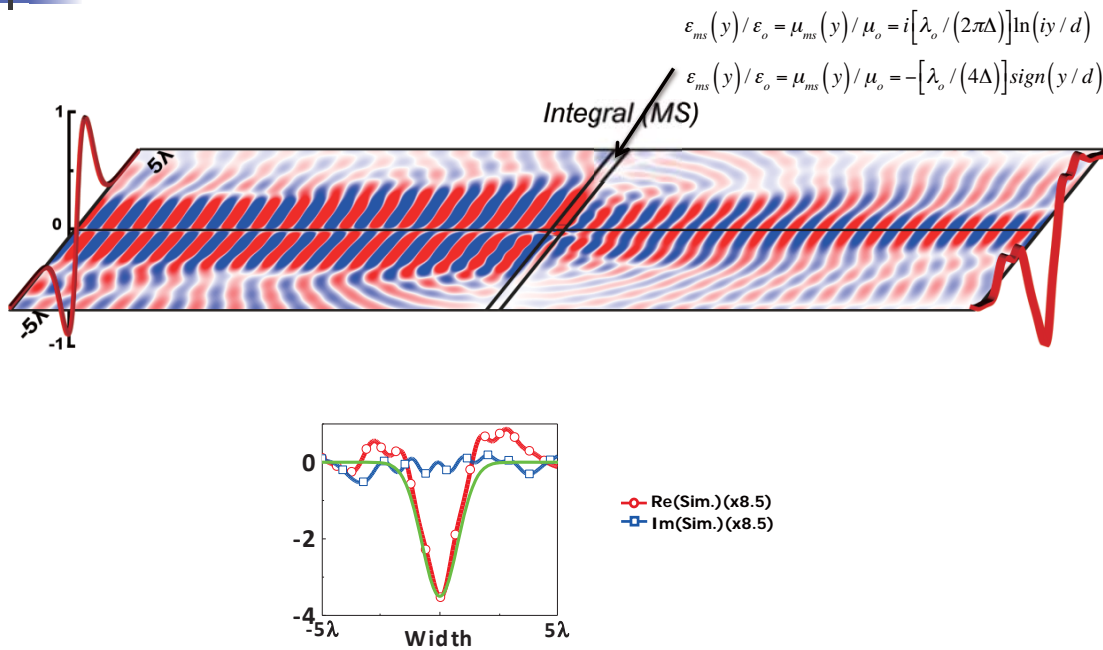
$$\epsilon_{ms}(y) / \epsilon_o = \mu_{ms}(y) / \mu_o = i2 \left[ \lambda_o / (2\pi\Delta) \right] \ln(-iW / (2y))$$



A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014



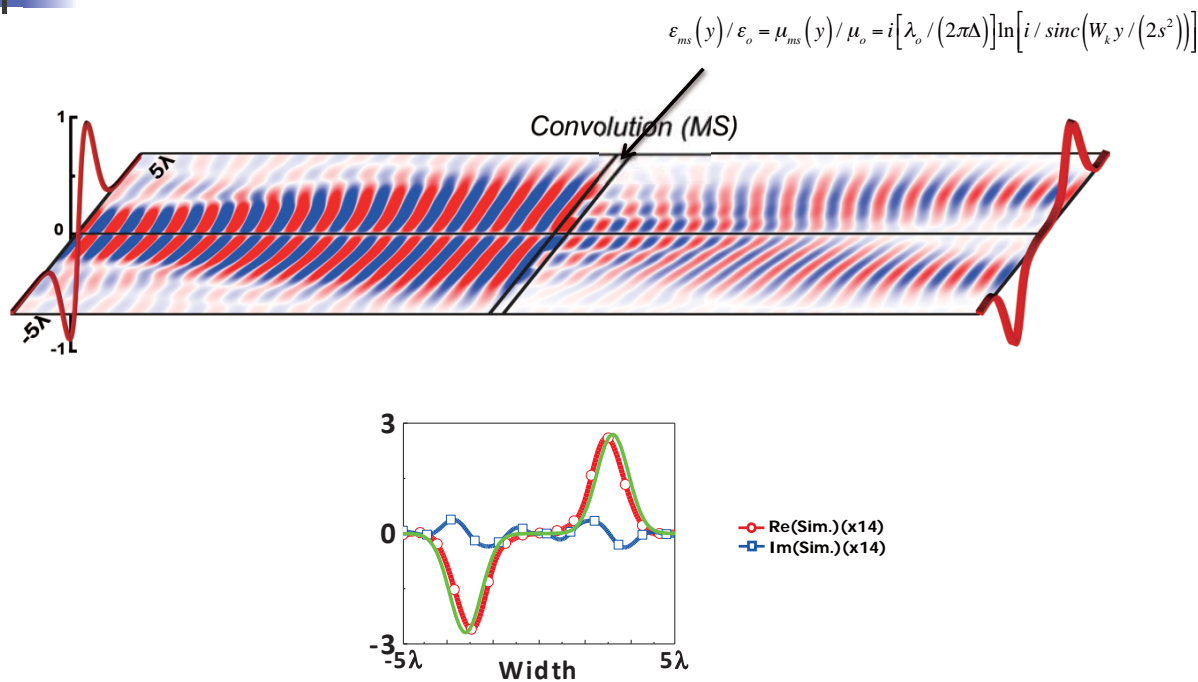
# Metamaterial as Integrator



A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014

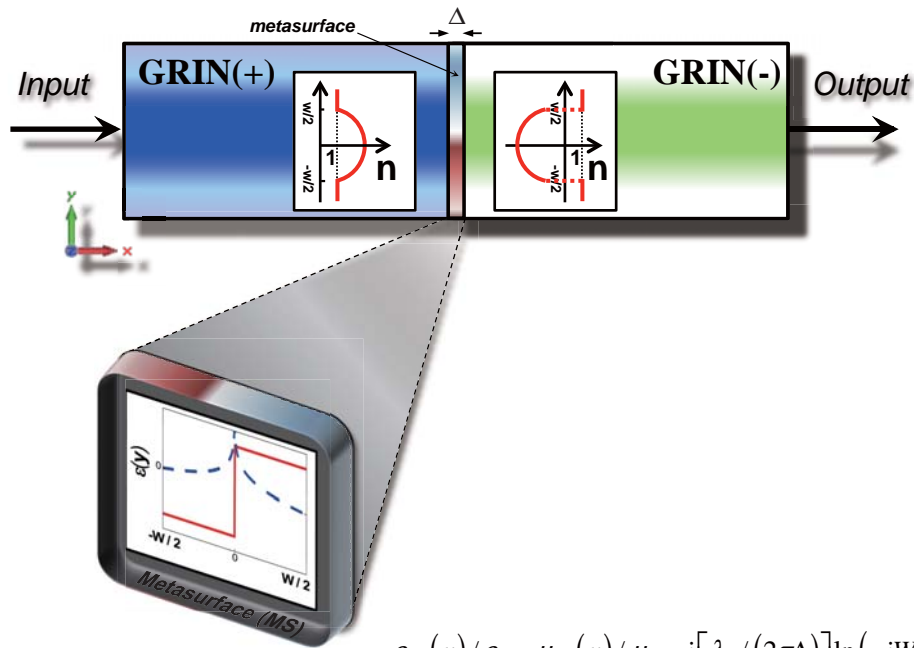


# Metamaterial as Convolver



A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014

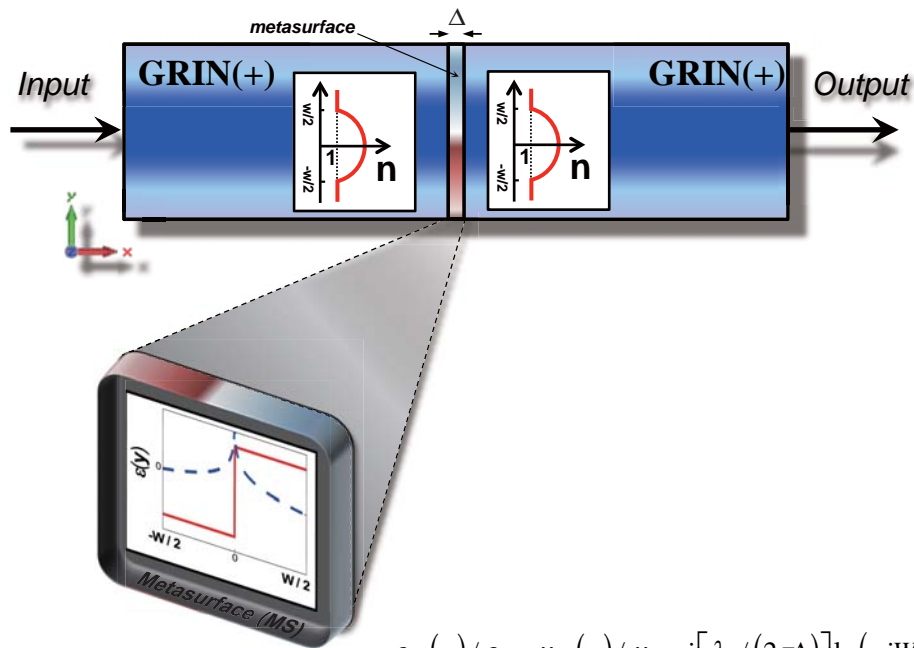
# Realistic Materials for Structures



$$\epsilon_{ms}(y) / \epsilon_o = \mu_{ms}(y) / \mu_o = i \left[ \lambda_o / (2\pi\Delta) \right] \ln(-iW / (2y))$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014

# Realistic Materials for Structures

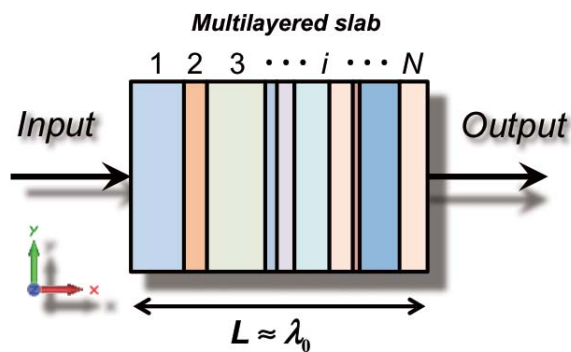


$$\epsilon_{ms}(y) / \epsilon_o = \mu_{ms}(y) / \mu_o = i \left[ \lambda_o / (2\pi\Delta) \right] \ln(-iW / (2y))$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014



# Green's Function Approach

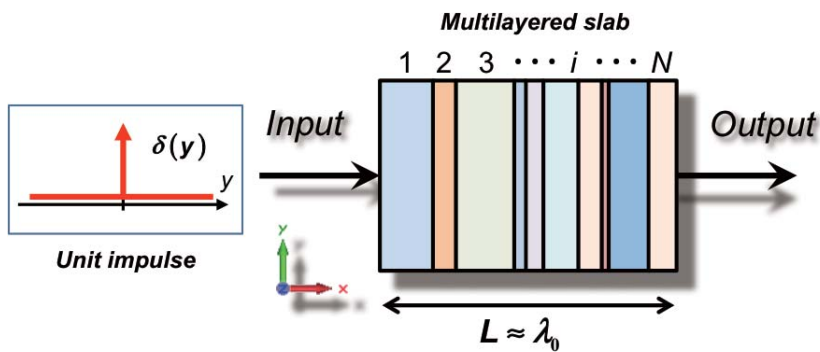


$$g(y) = \int f(y')G(y - y')dy'$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014



# Green's Function Approach

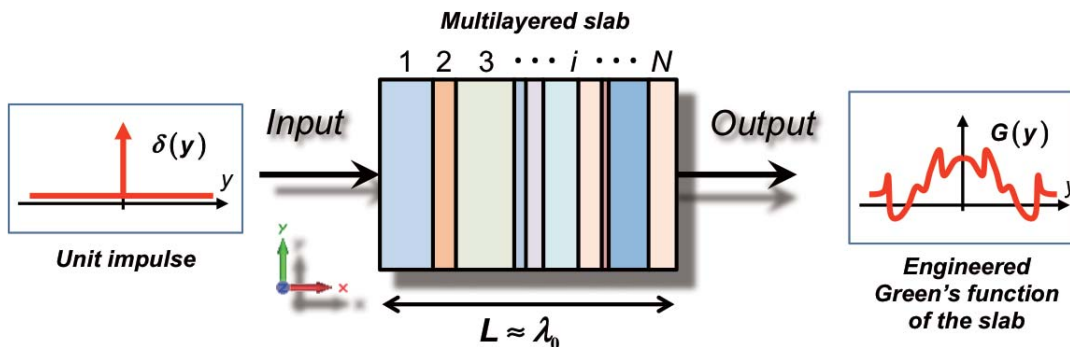


$$g(y) = \int f(y')G(y - y')dy'$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014



# Green's Function Approach

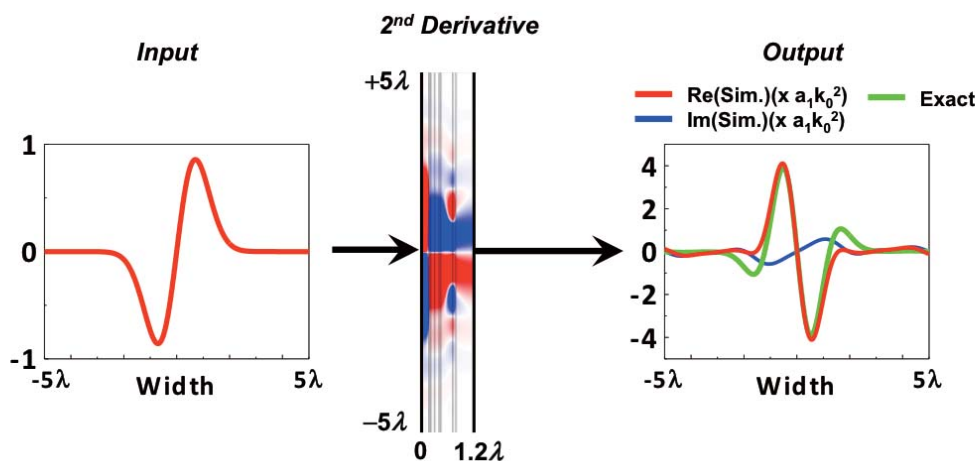


$$g(y) = \int f(y')G(y - y') dy'$$

$$\frac{d^2 f(y)}{dy^2} \propto \int f(y')\delta^{(2)}(y - y') dy'$$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014

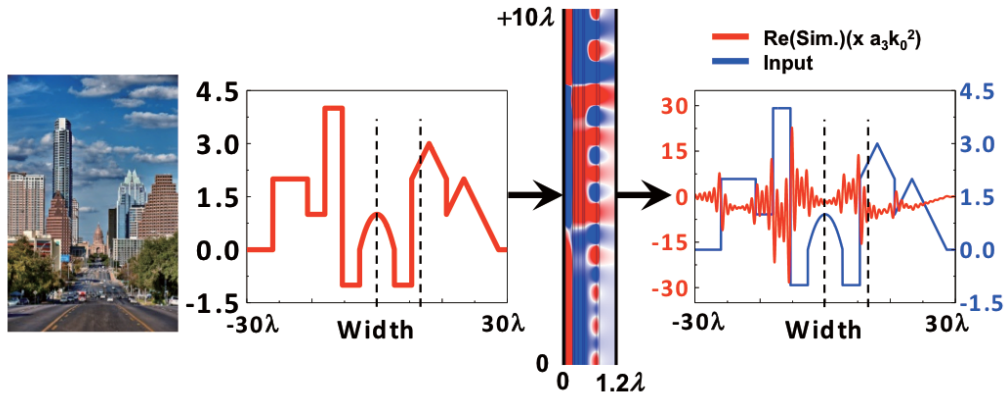
# 2<sup>nd</sup> Differentiation: Green's Function Approach



	1	2	3	4	5	6	7	8	9	10
$\epsilon_r$	13.85	5.98	4.44	0.06	0.03	0.01	-0.003	-2.12	2.30	0.08
$d$	$\lambda_0/293.4$	$\lambda_0/6.0$	$\lambda_0/212.9$	$\lambda_0/24.2$	$\lambda_0/12.1$	$\lambda_0/9.8$	$\lambda_0/25.0$	$\lambda_0/3.6$	$\lambda_0/14.5$	$\lambda_0/2.4$

A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, *Science*, Jan 2014

# 2<sup>nd</sup> Differentiation: Green's Function Approach



*A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Science, Jan 2014*





# Fields and Waves in Metamaterials

## Part 2



*Nader Engheta*

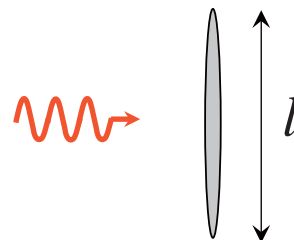
*University of Pennsylvania  
Philadelphia, PA 19104, USA*

*August 16-17, 2014*

## *Light-Matter Interaction*



$$\omega \leftrightarrow 2\pi/T$$



$$k \leftrightarrow 2\pi/\lambda$$

$$k \equiv \frac{2\pi}{\lambda} = \omega \sqrt{\epsilon\mu} \quad \text{Metamaterials}$$

*N. Engheta, Science, 340, 286 (2013)*



## What will happen, if epsilon is near zero?

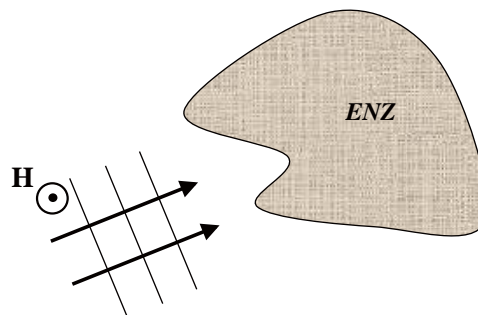
- **Maxwell Equations**  $\nabla \times \mathbf{H} = -i\omega\epsilon\mathbf{E} \longrightarrow \nabla \times \mathbf{H} = 0$

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}$$

- **2-D Scenario with TM polarization**

$$\mathbf{H} = H(x, y) \hat{\mathbf{u}}_z$$

$$\mathbf{E} = \frac{1}{-i\omega\epsilon} \nabla H(x, y) \times \hat{\mathbf{u}}_z$$



$$H = \text{const.} \quad \text{inside ENZ material.} \quad n = \sqrt{\epsilon\mu} \rightarrow 0$$

M. Silveirinha & N. Engheta, *Phys. Rev. Lett.* 97, 157403, Oct 2006

## What will happen, if epsilon is near zero?



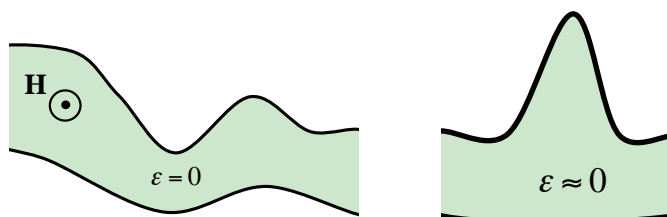
- **Maxwell Equations**  $\nabla \times \mathbf{H} = -i\omega\epsilon\mathbf{E} \longrightarrow \nabla \times \mathbf{H} = 0$

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}$$

- **2-D Scenario with TM polarization**

$$\mathbf{H} = H(x, y) \hat{\mathbf{u}}_z$$

$$\mathbf{E} = \frac{1}{-i\omega\epsilon} \nabla H(x, y) \times \hat{\mathbf{u}}_z$$

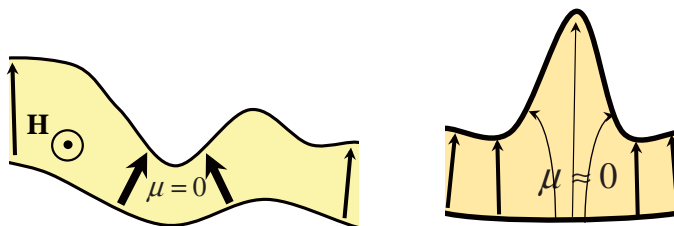


$$H = \text{const.} \quad \text{inside ENZ material.} \quad n = \sqrt{\epsilon\mu} \rightarrow 0$$



## What will happen, if $\mu$ is near zero?

- **Maxwell Equations**  $\nabla \times \mathbf{H} = -i\omega\epsilon\mathbf{E}$   
 $\nabla \times \mathbf{E} = i\omega\mu\mathbf{H} \quad \longrightarrow \quad \nabla \times \mathbf{E} = 0$
- **2-D Scenario with TM polarization**



## “ENZ Supercoupling”

*M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006*

*M. Silveirinha & N. Engheta, Phys. Rev. B., 76, 245109 (2007)*

*B. Edwards, A. Alu, M. Young, M. Silveirinha, N. Engheta, Phys. Rev. Lett., 100, 033903, 245109 (2008)*

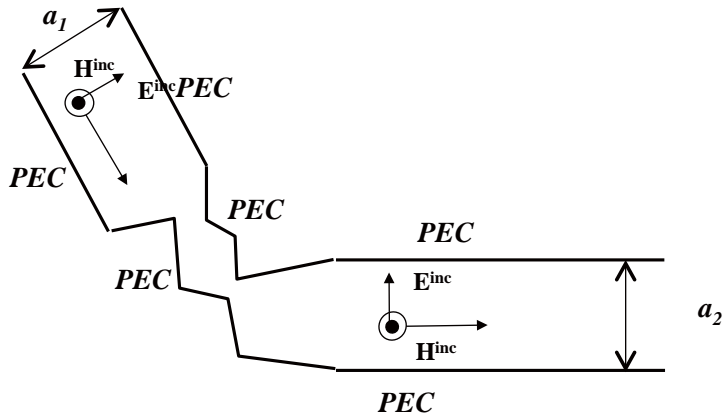
*A. Alu, M. Silveirinha, N. Engheta, Phys. Rev. E., 78, 016604 (2008)*

*A. Alu, N. Engheta, Phys. Rev. B., 78, 045102 (2008)*

*A. Alu, N. Engheta, Phys. Rev. B., 78, 035440 (2008)*



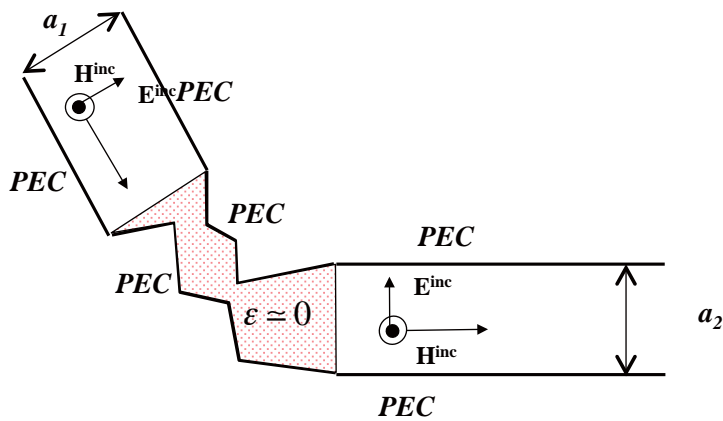
## “Supercoupling” in Sub- $\lambda$ Channels



*M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006*

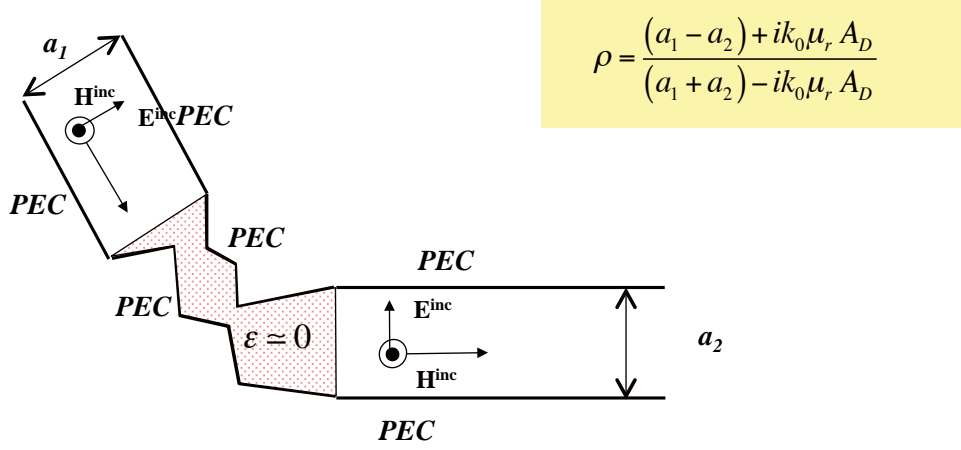


## “Supercoupling” in Sub- $l$ Channels



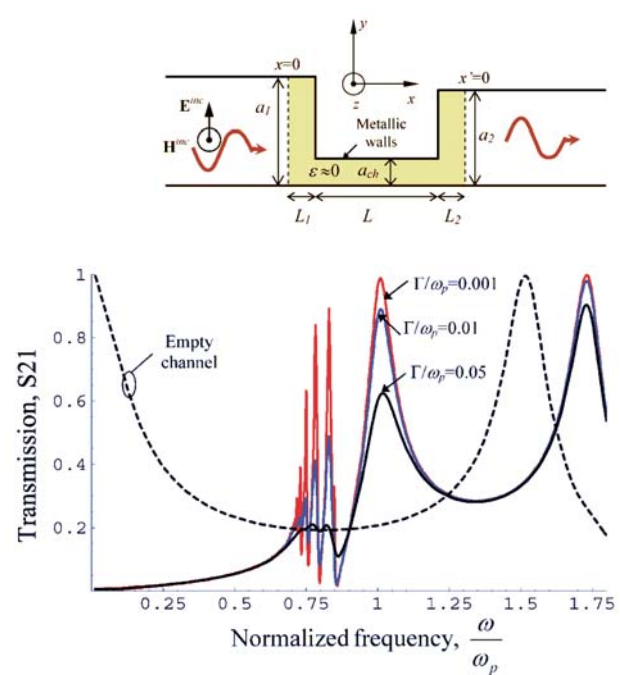
*M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006*

# “Supercoupling” in Sub- $l$ Channels



*M. Silveirinha & N. Engheta, Phys. Rev. Lett. 97, 157403, Oct 2006*

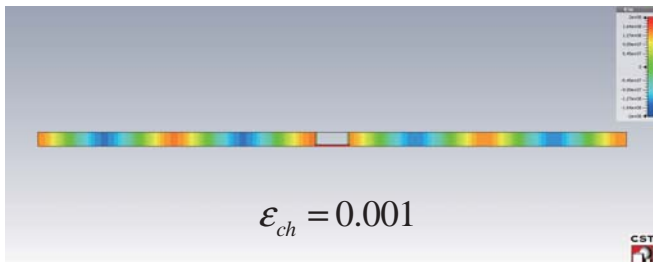
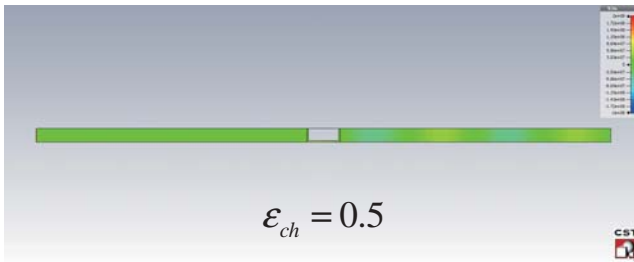
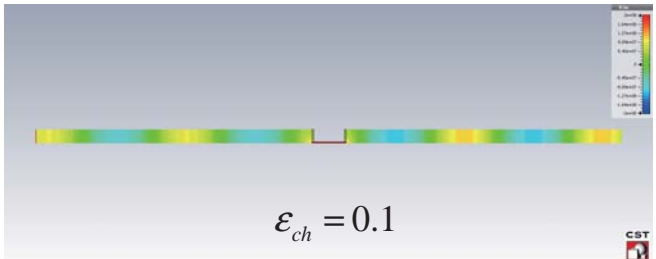
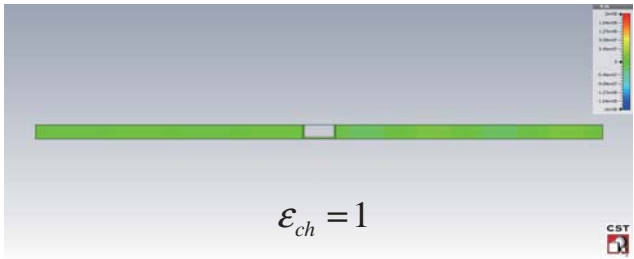
# U-shaped Waveguide Transition & Supercoupling (cont'd)



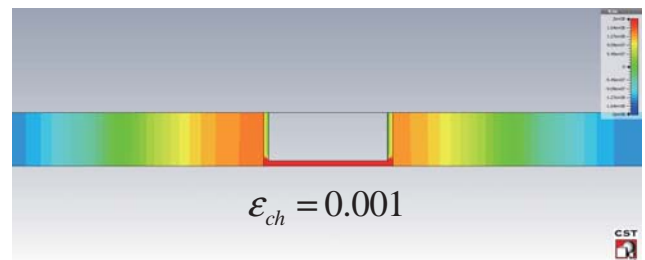
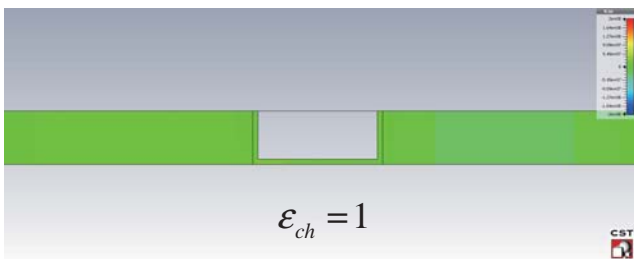
*M. Silveirinha & N. Engheta, Phys. Rev. B., 76, 245109 (2007)*



## Simulation Results: 2D scenario

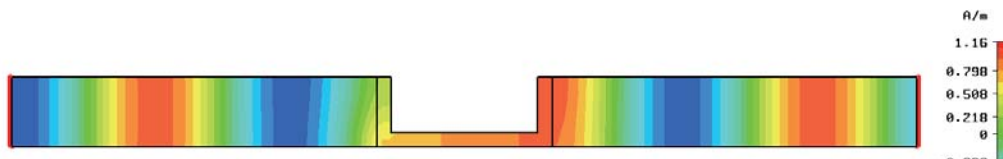


## Simulation Results: 2D scenario





## Intuitive Explanation



$$Z_{wg} = \frac{V}{I} = \frac{E \cdot h}{H \cdot W} = \frac{E}{H} \frac{h}{W} = \sqrt{\frac{\mu}{\epsilon}} \frac{h}{W}$$

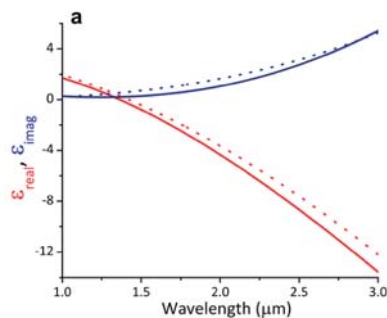
$$Z_{wg1} = Z_{wg2}$$

$$\sqrt{\frac{\mu_1}{\epsilon_1}} \frac{h_1}{W_1} = \sqrt{\frac{\mu_2}{\epsilon_2}} \frac{h_2}{W_2}$$

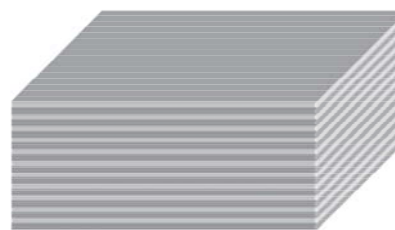
B. Edwards, A. Alu, M. Young, M. Silveirinha, N. Engheta, *Phys. Rev. Lett.*, 100, 033903, 245109 (2008)



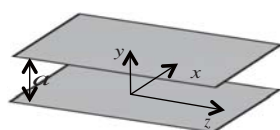
## ENZ Structures



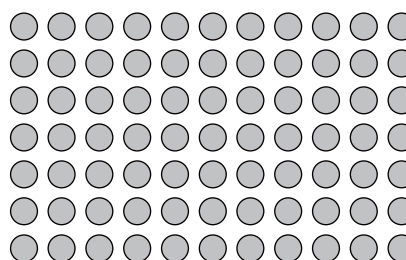
ITO



$\text{Re}(\epsilon) \equiv 0$



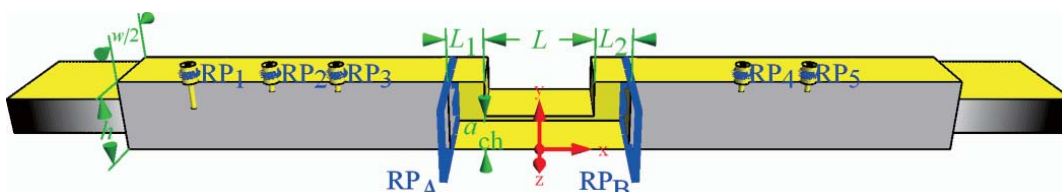
$$k_z = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r - \frac{1}{\omega^2 \mu_0 \epsilon_0} \left(\frac{\pi}{a}\right)^2}$$



$\text{Re}(\epsilon) \equiv 0$



## Experimental Verification



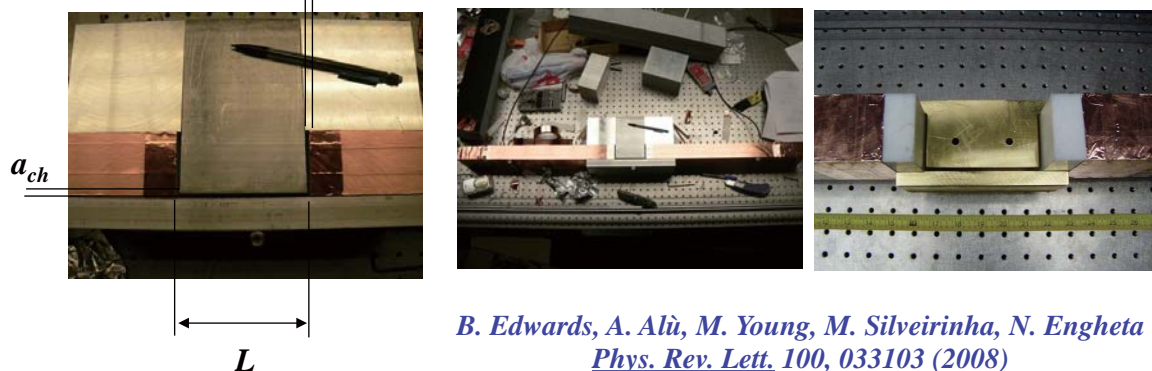
$$\beta_z = \omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r - c^2 / (4 f^2 w^2)} \rightarrow \epsilon_{eff} / \epsilon_0 = \epsilon_r - c^2 / (4 f^2 w^2)$$



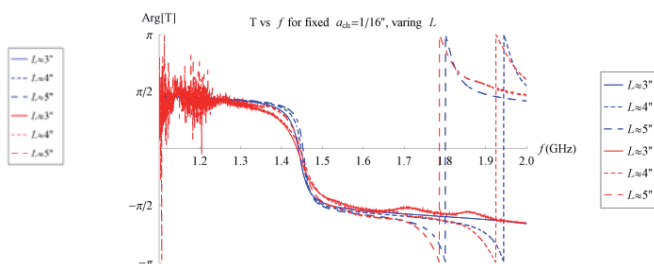
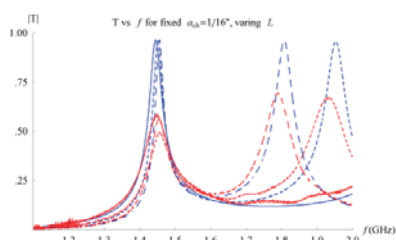
B. Edwards, A. Alu, M. Young, M. Silveirinha, N. Engheta, *Phys. Rev. Lett.*, 100, 033903, 245109 (2008)



## Experimental Verification

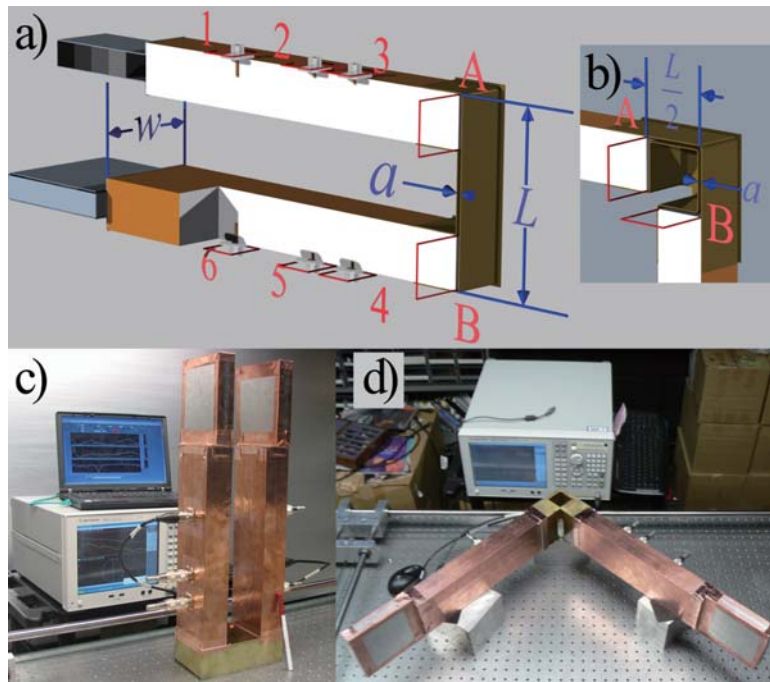


B. Edwards, A. Alù, M. Young, M. Silveirinha, N. Engheta  
*Phys. Rev. Lett.* 100, 033103 (2008)

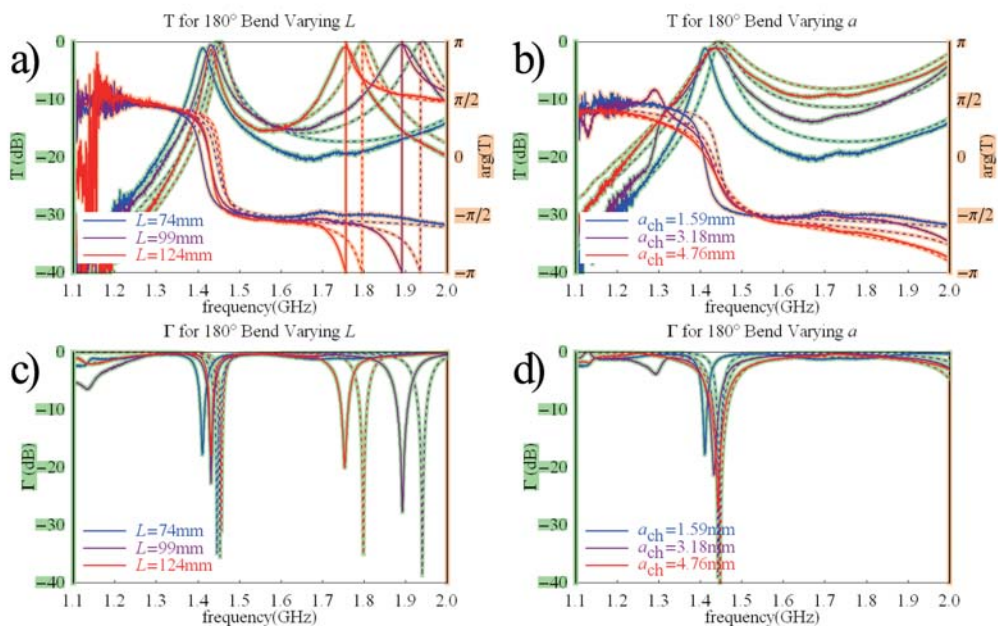




# Waveguide Bends with Narrow Channels

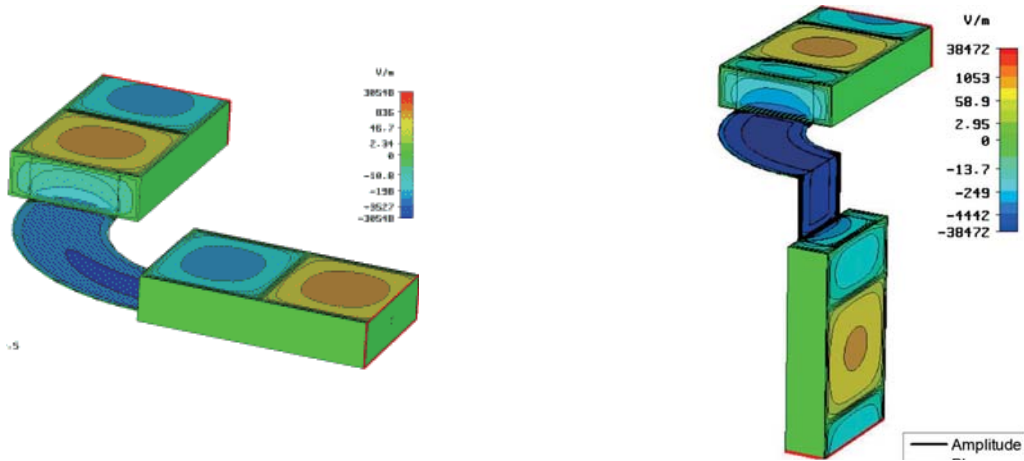


# 180-degree Waveguide Bends



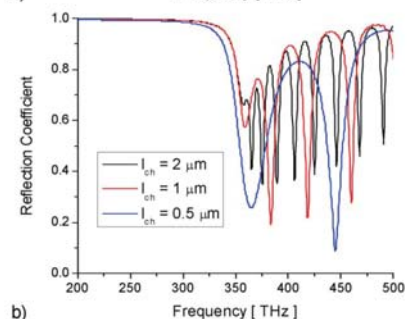
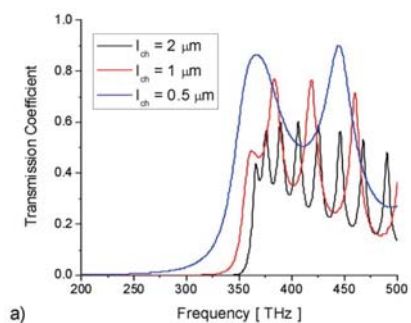
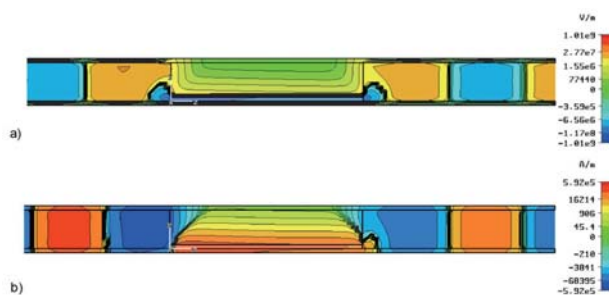
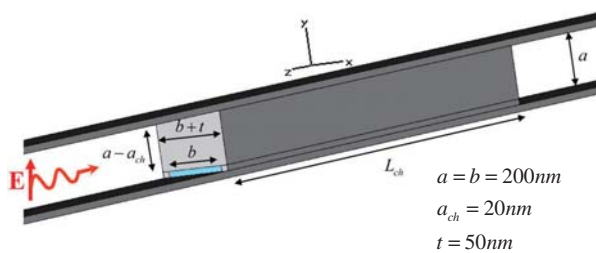
*B. Edwards, A. Alù, M. Silveirinha, N. Engheta  
Journal of Applied Physics, 2009*

# Waveguide Bends with Narrow Channels



A. Alu, M. Silveirinha, N. Engheta, *Phys. Rev. E.*, 78, 016604 (2008)

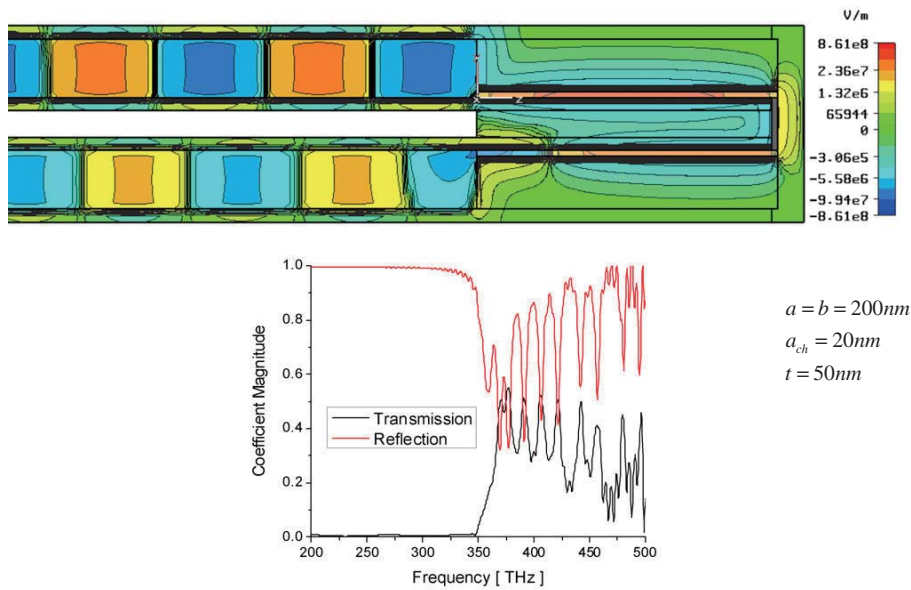
# Plasmonic Channels and ENZ Tunneling



A. Alù and N. Engheta  
*Phys. Rev. B.*, 78, 2008



## *Plasmonic Channels and ENZ Tunneling*



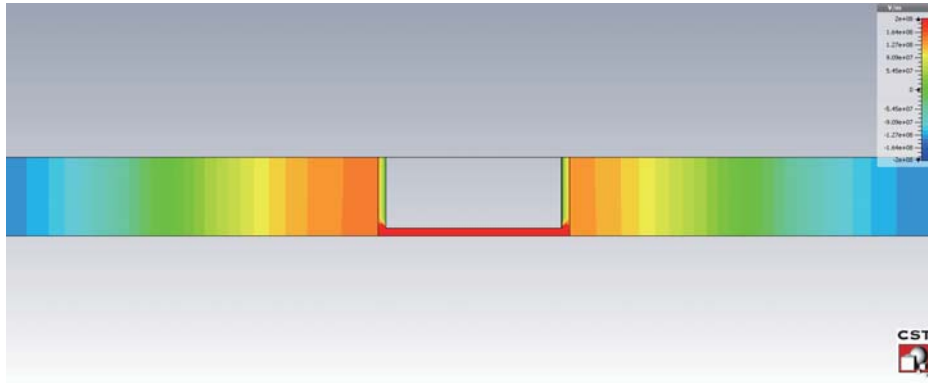
*A. Alù and N. Engheta*  
*Phys. Rev. B.* **78**, 2008



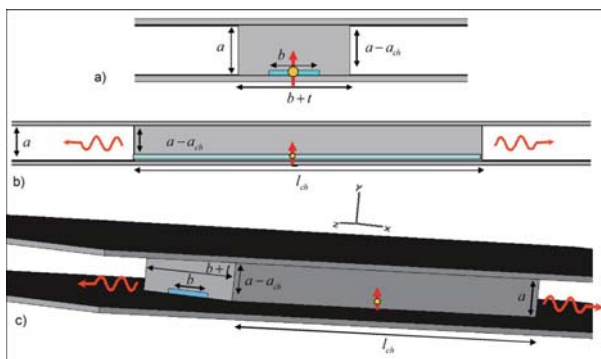
## *ENZ and Spontaneous Emission Rate of Optical Emitters*

*A. Alu and N. Engheta, Phys. Rev. Lett.* **103**, 043902 (2009)

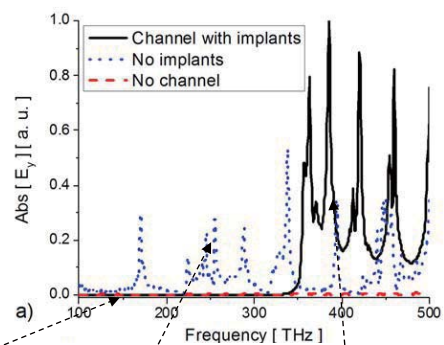
# Field Enhancement Using ENZ



# Enhancement of Optical Emitters

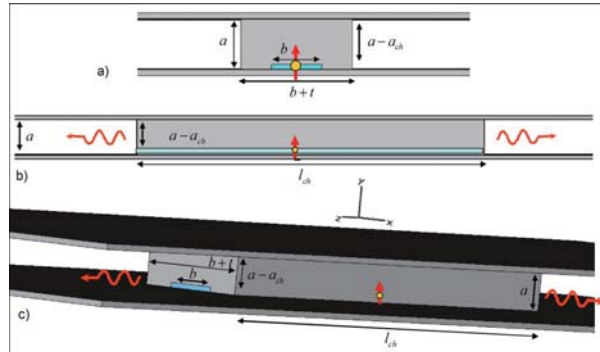


$a = b = 200\text{nm}$   
 $a_{ch} = 20\text{nm}$   
 $t = 300\text{nm}$

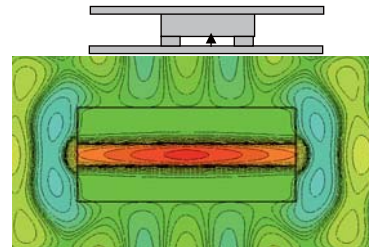
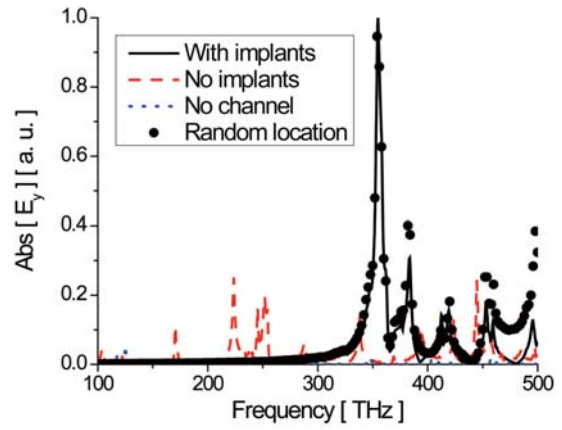


A. Alù and N. Engheta  
*Phys. Rev. Lett.* 2009

# Enhancement of Optical Emitters

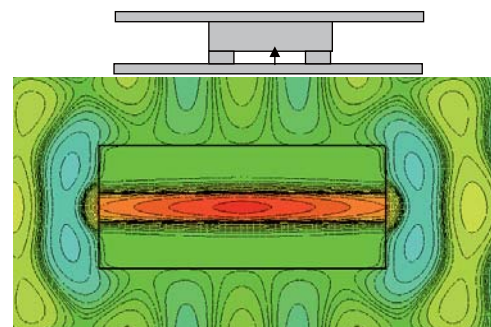
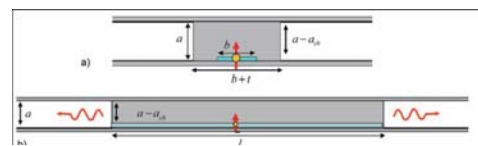
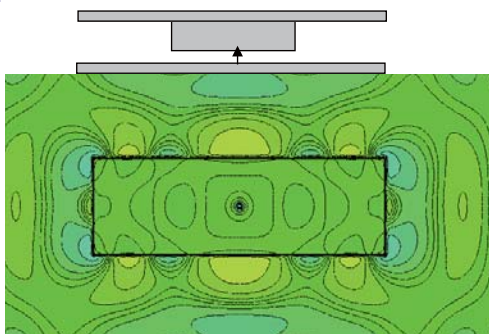
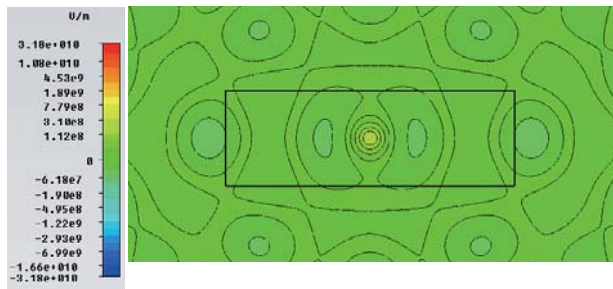


$a = b = 200\text{nm}$   
 $a_{ch} = 20\text{nm}$   
 $t = 50\text{nm}$



A. Alù and N. Engheta  
*Phys. Rev. Lett.* 103, 043902 (2009)

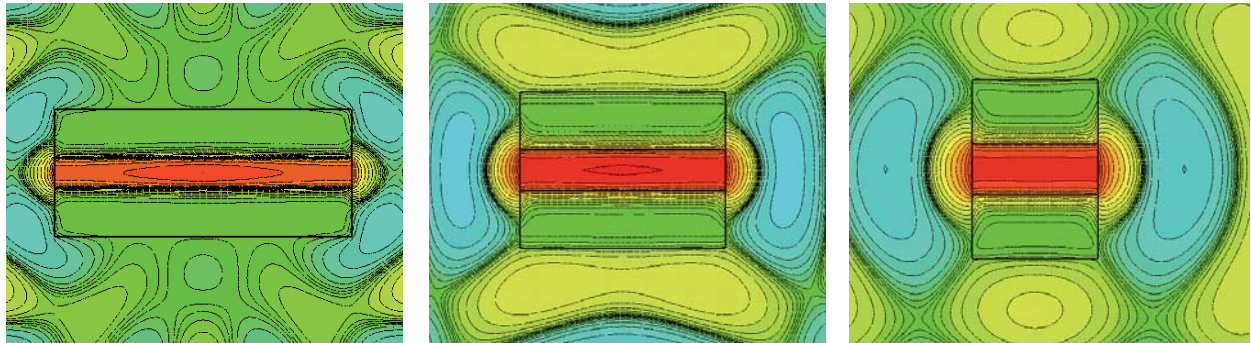
# ENZ and Purcell Effects



A. Alù and N. Engheta  
*Phys. Rev. Lett.* 2009



## ENZ and Purcell Effects

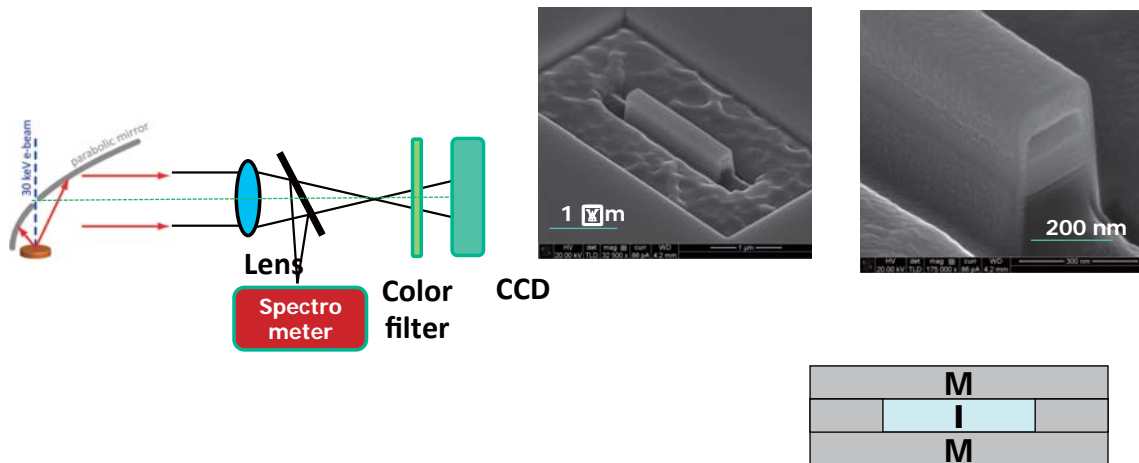


*A. Alù and N. Engheta  
Phys. Rev. Lett. 2009*



## Experimental Verification Using CL Spectroscopy

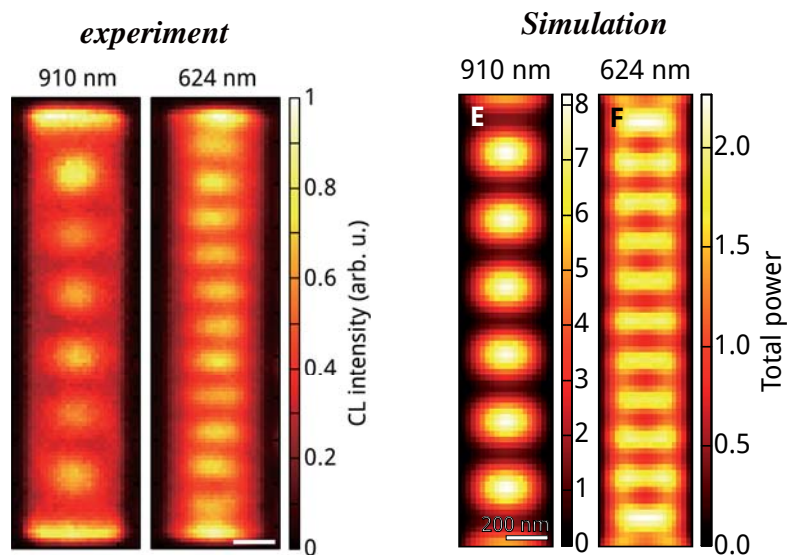
*Collaboration with Albert Polman's Group in AMOLF*



*E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman Phys. Rev. Lett., 110, 013902 (2013)*

## Experimental Verification Using CL Spectroscopy

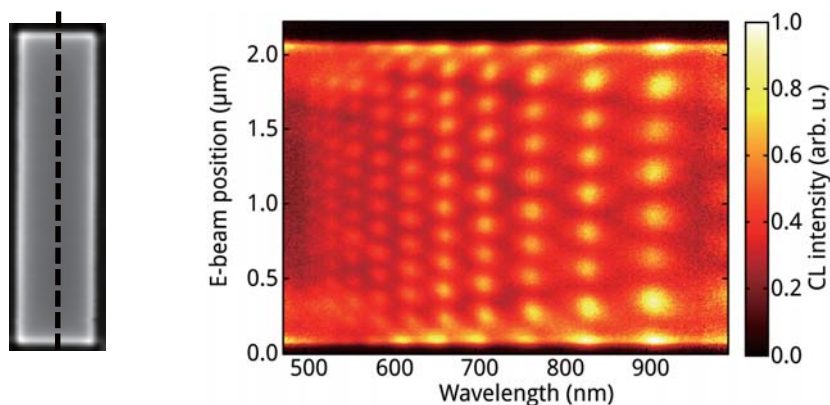
### Collaboration with Albert Polman's Group in AMOLF



*E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman Phys. Rev. Lett., 110, 013902 (2013)*

## Experimental Verification Using CL Spectroscopy

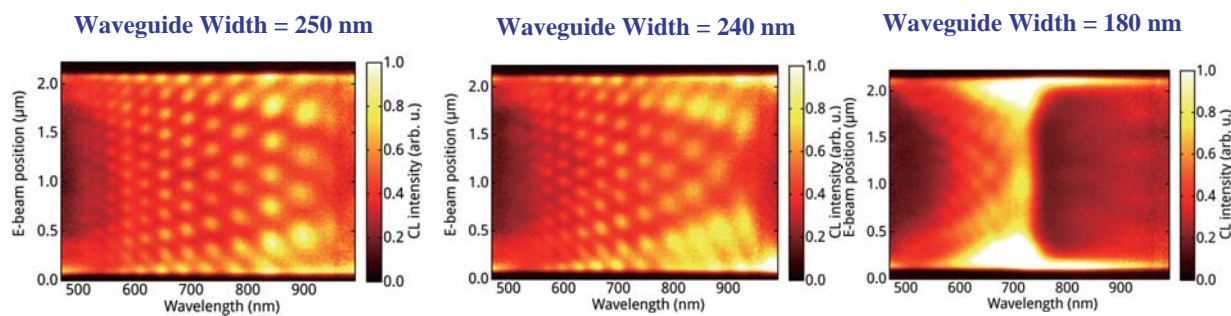
### Collaboration with Albert Polman's Group in AMOLF



*E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman Phys. Rev. Lett., (2013)*

# Experimental Verification Using CL Spectroscopy

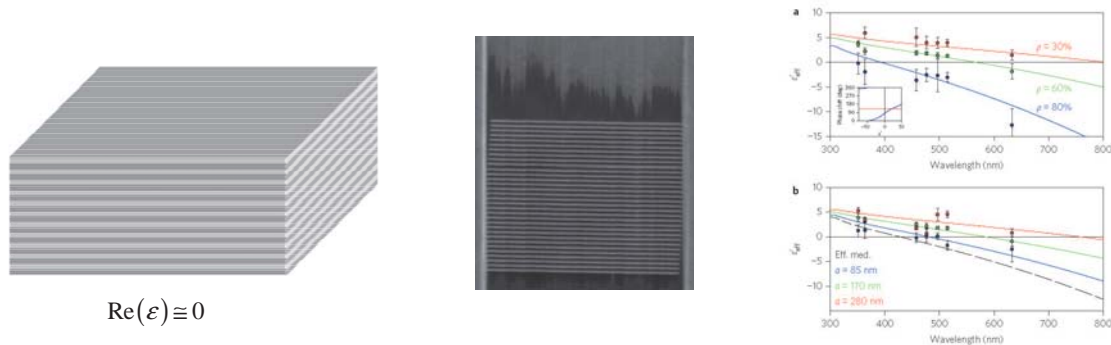
Collaboration with Albert Polman's Group in AMOLF



*E. J. Vesseur, T. Coenen, H. Caglayan, N. Engheta, A. Polman Phys. Rev. Lett., 110, 013902 (2013)*

# Experimental Verification ENZ Stack

Collaboration with Albert Polman's Group in AMOLF



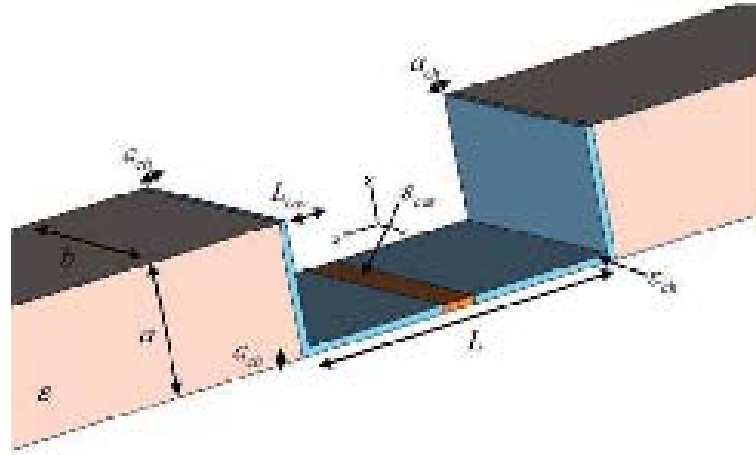
$\text{Re}(\epsilon) \equiv 0$

*R. Maas, J. Parsons, N. Engheta, A. Polman Nature Photonics, 7(11), 907-912 (2013)*





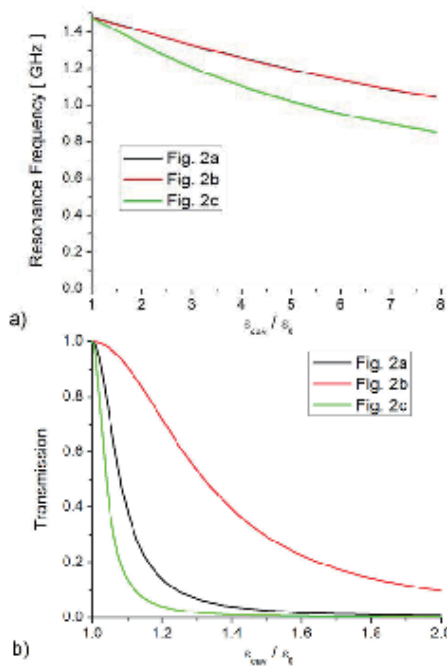
# Dielectric Sensing



A. Alù and N. Engheta, *Phys. Rev. B.*, 78, July 2008



# Dielectric Sensing



$$L_{cav} = L/10$$

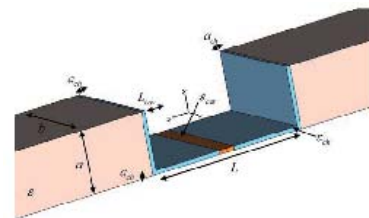
$$a_{ch} = a/64$$

$$L_{cav} = L/10$$

$$a_{ch} = a/16$$

$$L_{cav} = L/5$$

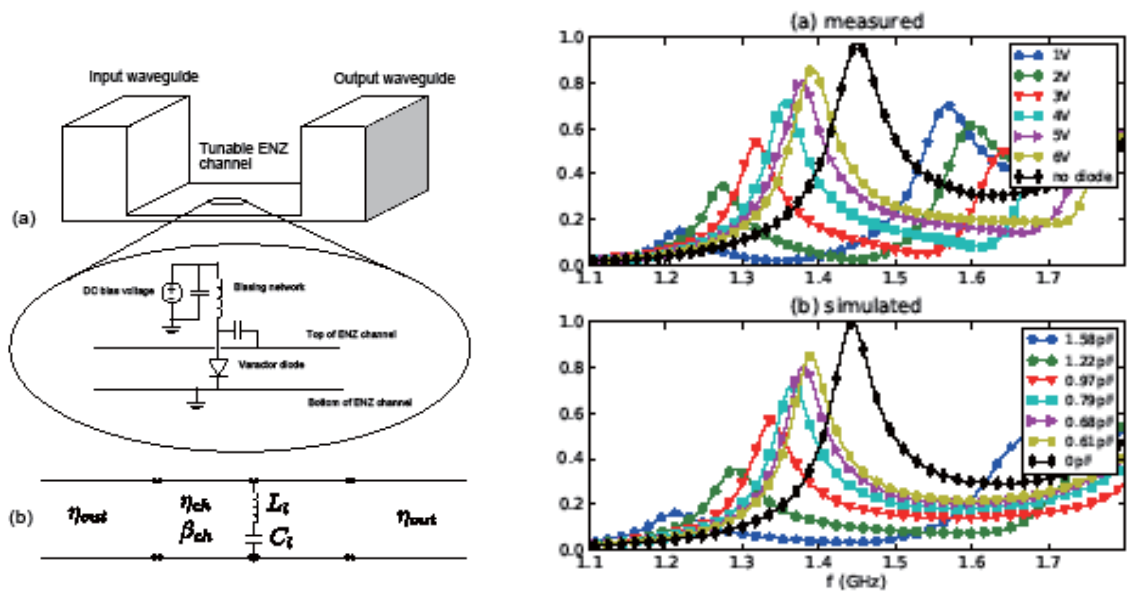
$$a_{ch} = a/64$$



A. Alù and N. Engheta, *Phys. Rev. B.*, 78, July 2008



# Nonlinearity in ENZ Channels



*D. Powell, A. Alù, B. Edwards, A. Vakil, Y. Kivshar, and N. Engheta, Phys. Rev. B. 2009.*



## Fields and Waves in Metamaterials Part 3



*Nader Engheta*

*University of Pennsylvania  
Philadelphia, PA 19104, USA*

*August 16-17, 2014*



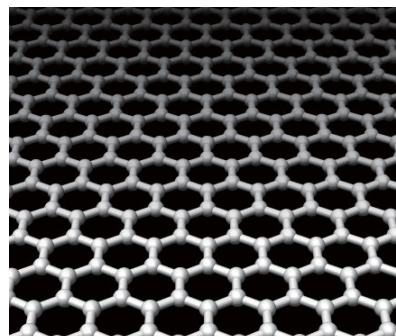
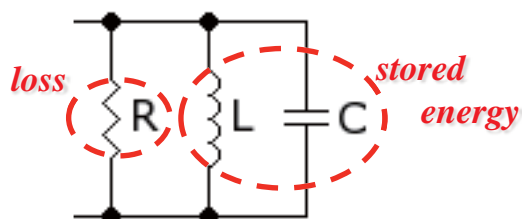
# Graphene

$$J_s = \sigma_g E$$

$$I = \sigma_g V = Y \cdot V$$

$$\sigma_g = \overset{>0}{\sigma_{g,r}} + i \overset{>0 \text{ or } <0}{\sigma_{g,i}}$$

$$Y = G + iB$$



<http://math.ucr.edu/home/baez/graphene.jpg>

# Graphene Conductivity



$$\sigma_g(\omega, \mu_c, \Gamma, T) = \frac{-ie^2(\omega + i2\Gamma)}{\pi h^2} \left[ \frac{1}{(\omega + i2\Gamma)^2} \int_0^\infty \Omega \left( \frac{\partial f_d(\Omega)}{\partial \Omega} - \frac{\partial f_d(-\Omega)}{\partial \Omega} \right) d\Omega - \int_0^\infty \frac{f_d(-\Omega) - f_d(\Omega)}{(\omega + i2\Gamma)^2 - 4(\Omega/h)^2} \Omega \right]$$

$$\sigma_g = \sigma_{\text{interband}} + \sigma_{\text{intradband}}$$

$$f_d(\Omega) \equiv \left[ e^{(\Omega - \mu_c)/k_B T} + 1 \right]^{-1}$$

$$\sigma_{\text{interband}} \approx \frac{ie^2}{4\pi h} \ln \left[ \frac{2|\mu_c| - (\omega + i2\Gamma)h}{2|\mu_c| + (\omega + i2\Gamma)h} \right]$$

$$k_B T \ll |\mu_c|$$

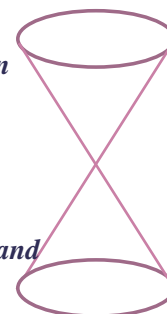
Conduction Band

$$\text{Im}(\sigma_{\text{interband}}) < 0$$

Valence Band

$$\sigma_{\text{intradband}} = \frac{ie^2 k_B T}{\pi h^2 (\omega + i2\Gamma)} \left[ \frac{\mu_c}{k_B T} + 2 \ln \left( e^{-\mu_c/k_B T} + 1 \right) \right]$$

$$\text{Im}(\sigma_{\text{intradband}}) > 0$$





# Graphene Conductivity

$$\sigma_g = \sigma_{\text{interband}} + \sigma_{\text{intra-band}}$$

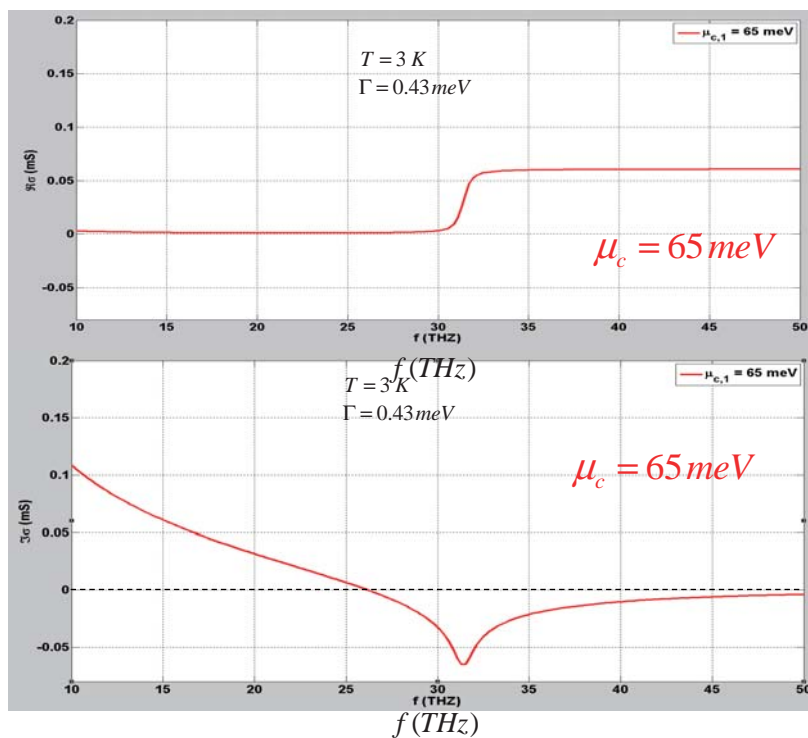
Re( $\sigma_g$ )

$$\sigma_g = \sigma_{g,r} + i\sigma_{g,i}$$

$$\sigma_{g,r} = f_1(\omega, \mu_c, \Gamma, T)$$

$$\sigma_{g,i} = f_2(\omega, \mu_c, \Gamma, T)$$

Im( $\sigma_g$ )



P. Gusynin et al., *J. Phys: Condens. Matter*, 19 (2007)

G. Hanson, *J. Appl. Phys.* 103, 064302 (2008)



# Graphene Conductivity

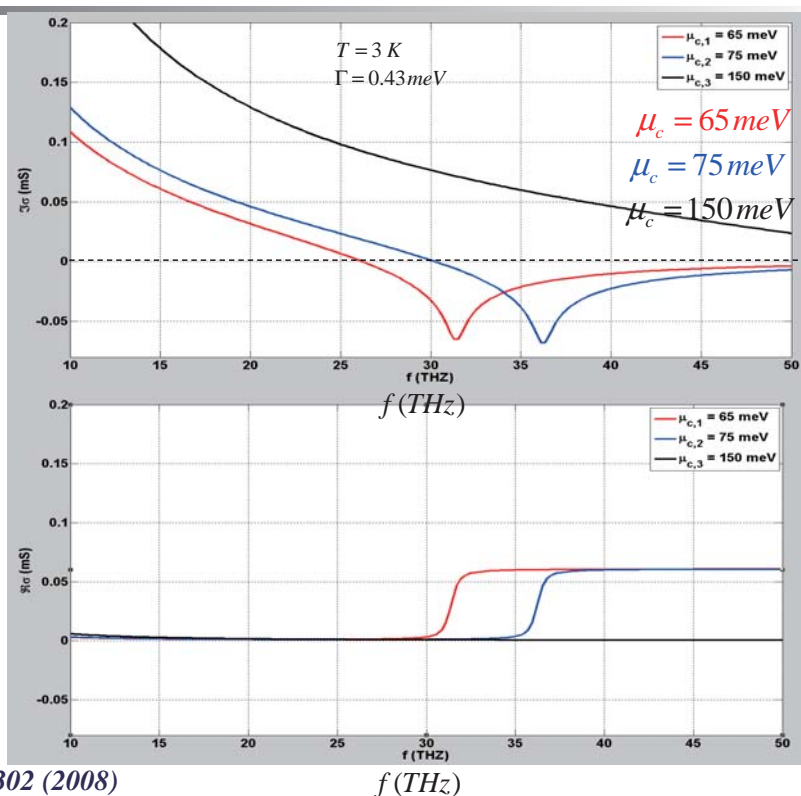


$$\sigma_g = \sigma_{g,r} + i\sigma_{g,i}$$

$$\sigma_{g,r} = f_1(\omega, \mu_c, \Gamma, T)$$

$$\sigma_{g,i} = f_2(\omega, \mu_c, \Gamma, T)$$

Im( $\sigma_g$ )

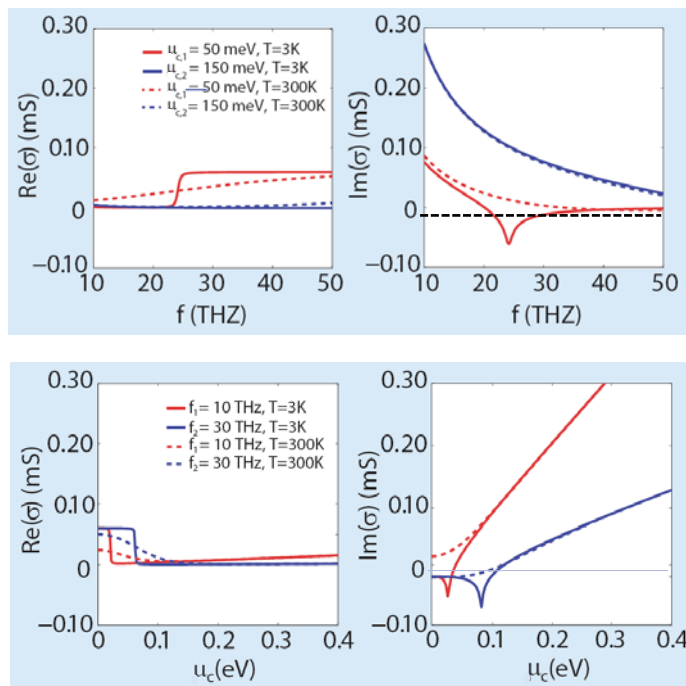


G. Hanson, *J. Appl. Phys.* 103, 064302 (2008)

f (THz)



# Graphene Conductivity



$\Gamma = 0.43 \text{ meV}$

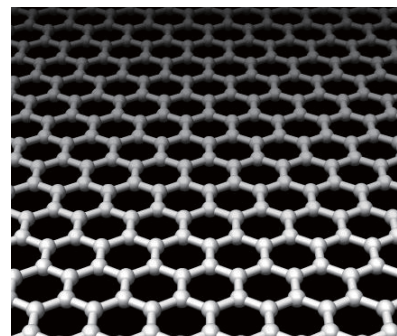
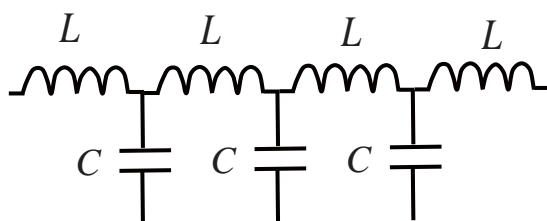
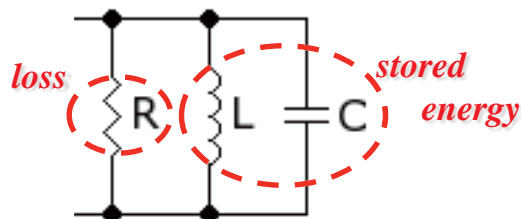
$$\sigma_g = \sigma_r + i\sigma_i$$

G. Hansen, *J. Appl. Phys.* 103, 064302 (2008)



# From Transmission Line to Graphene

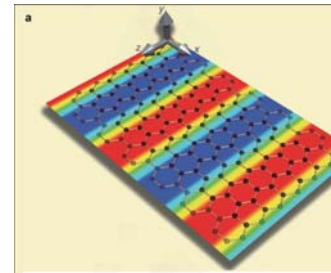
$$\sigma_g = \sigma_{g,r} + i\sigma_{g,i}$$



<http://math.ucr.edu/home/baez/graphene.jpg>



# SPP along Graphene

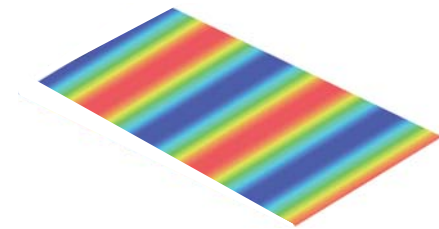


$$\beta_{SPP} = \omega \sqrt{\epsilon_0 \mu_0} \sqrt{1 - \left( \frac{2}{\sigma_g \sqrt{\mu_0 / \epsilon_0}} \right)^2} \quad \sigma_{g,i} > 0$$

$$\beta_{SPP} \gg \omega \sqrt{\epsilon_0 \mu_0}$$

$$\lambda_{SPP} \ll \lambda_{free-space} \quad \beta_{SPP} = n_{SPP} k_0$$

$$\lambda_{SPP} \approx \frac{\lambda_0}{70} \approx 144 \text{ nm} \quad \beta_{SPP} \approx 70 k_0$$



S. A. Mikhailov, K. Ziegler, *Phys. Rev. Lett.* **99**, 016803 (2007)

G. Hanson, *J. Appl. Phys.* **103**, 064302 (2008)

M. Jablan, H. Buljan, M. Soljacic, *Phys. Rev. B.*, **80**, 245435 (2010)

# Tailoring Conductivity and SPP



$$\sigma_{g,i} = f(\omega, \mu_c, \Gamma, T)$$

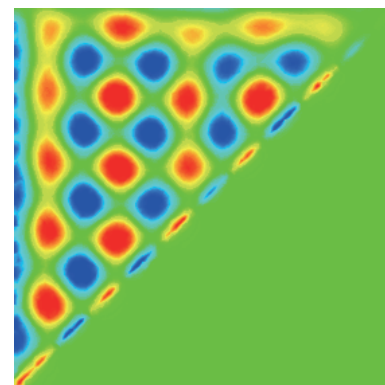
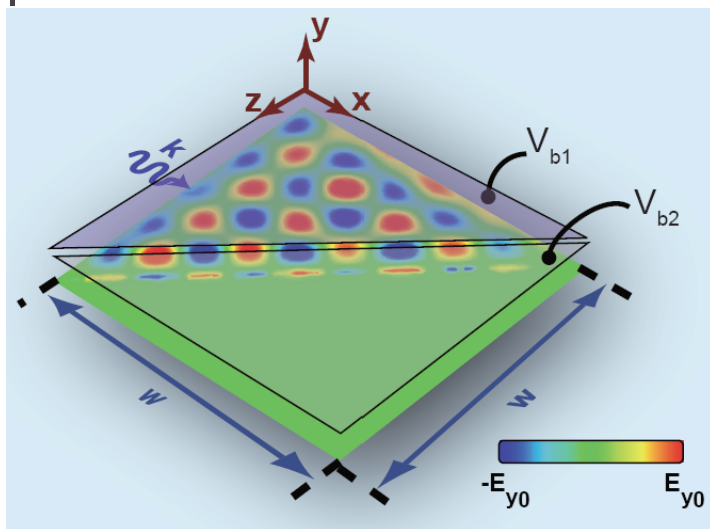
cnst
cnst cnst

$$n_{SPP} = \frac{\beta_{SPP}}{k_0} \propto \frac{1}{\sigma_{g,i}}$$

$\sigma_{g,i} ? \sigma_{g,r}$



# Fresnel Reflection



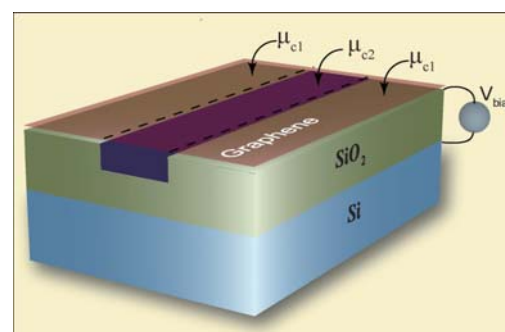
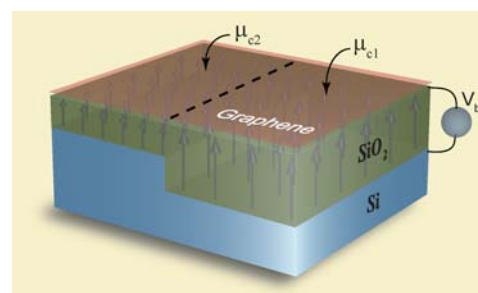
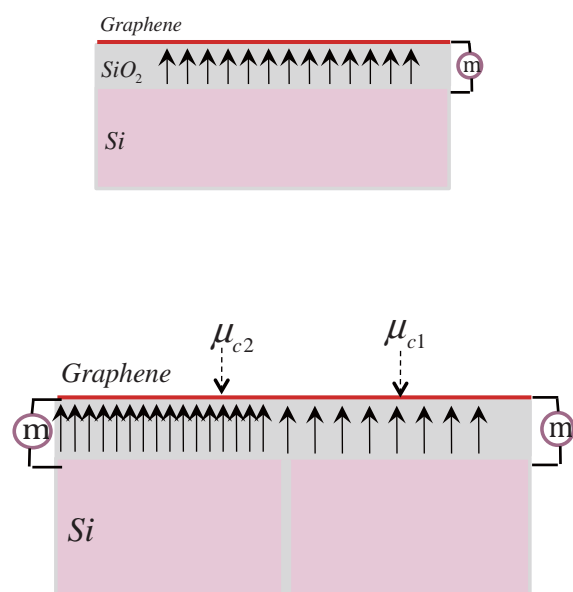
$w = 800 \text{ nm}$   
 $T = 3 \text{ K}, \Gamma = 0.43 \text{ meV}$

$$m_{c,1} = 150 \text{ meV} \rightarrow \sigma_{g1} = 0.0009 \text{ } \ominus / 0.0765 \text{ mS}$$

$$m_{c,2} = 6.5 \text{ meV} \rightarrow \sigma_{g2} = 0.0039 \text{ } \ominus / 0.0324 \text{ mS}$$

Vakil, Engheta, *Science* 332, 1291 (2011)

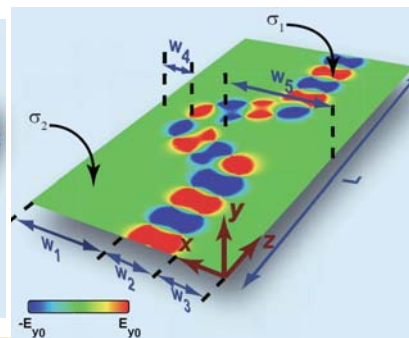
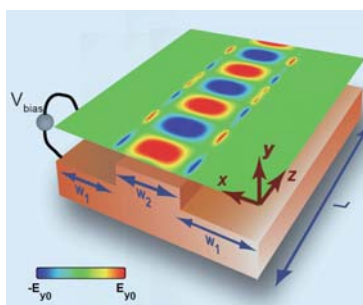
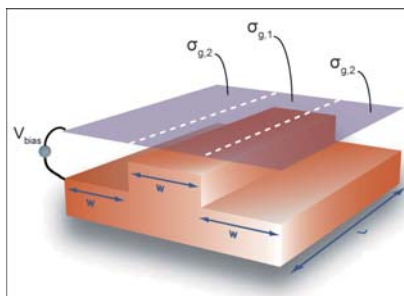
# Inhomogeneous Conductivity across Graphene



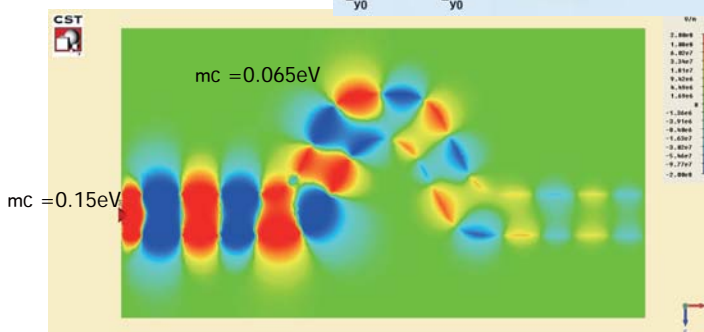
Vakil, Engheta, *Science* 332, 1291 (2011)



# One-Atom-Thick Waveguides



Region 1:  $\sigma_{g,i} > 0$   
 $\mu_c = 150 \text{ meV}$   
 Region 2:  $\sigma_{g,i} < 0$   
 $\mu_c = 65 \text{ meV}$

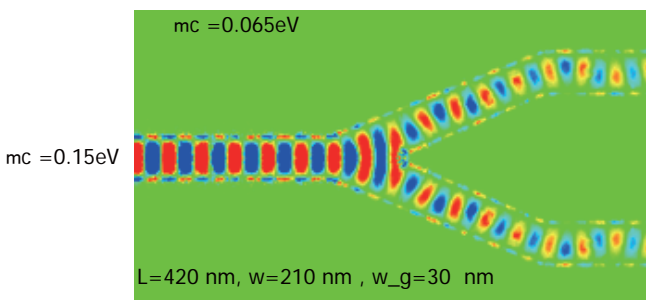
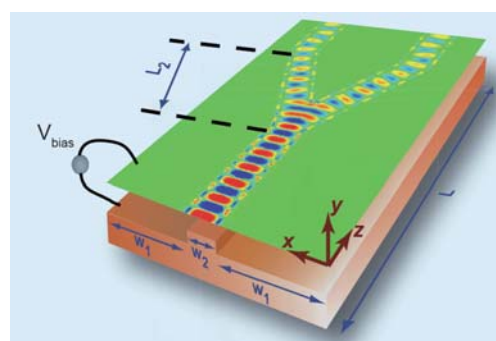


# One-Atom-Thick IR Splitter



Region 1:  $\sigma_{g,i} > 0$   
 $\mu_c = 0.15 \text{ eV}$

Region 2:  $\sigma_{g,i} < 0$   
 $\mu_c = 0.065 \text{ eV}$







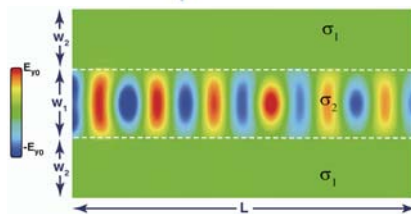
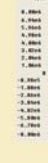
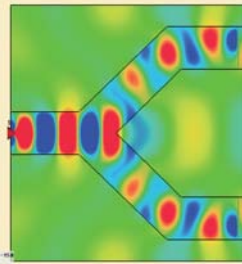
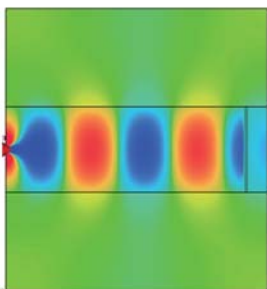
# One-Atom-Thick Optical "Fiber"

Region 1:  $\sigma_{g,i} > 0$

$\mu_c = 150 \text{ meV}$   
 $\text{Re}(\beta_{SPP}) ; 70k_o$

Region 2:  $\sigma_{g,i} > 0$

$\mu_c = 300 \text{ meV}$   
 $\text{Re}(\beta_{SPP}) ; 30k_o$



## Guiding Waves on one-atom-thick Platform



	3D component	One-Atom-Thick Version
Waveguide		
Bent Waveguide		
Splitter/Divider		
Optical Fiber/ Dielectric Slab waveguide		



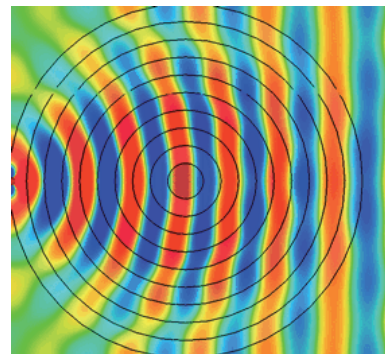
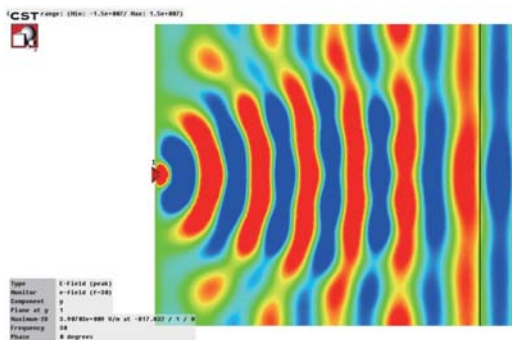
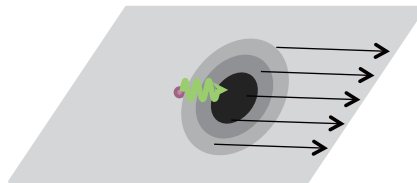
# Graphene SPP Lens



# One-Atom-Thick TO: Lens

$$\beta_{SPP} \equiv n_{SPP} k_o \gg k_o$$

$$\lambda_{SPP} \ll \lambda_{free-space}$$



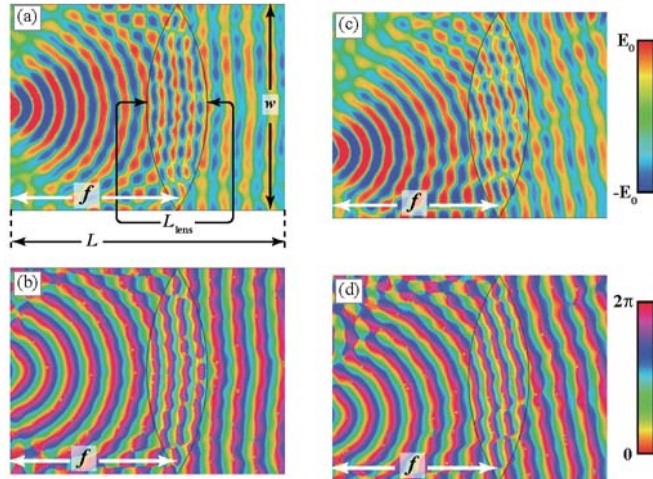
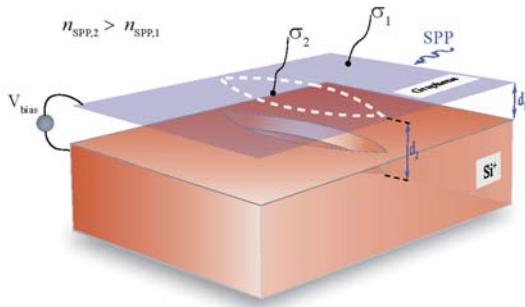
$D = 1.5 \mu\text{m}$

$L = 1.6 \mu\text{m}$

$W = 75 \text{ nm}$

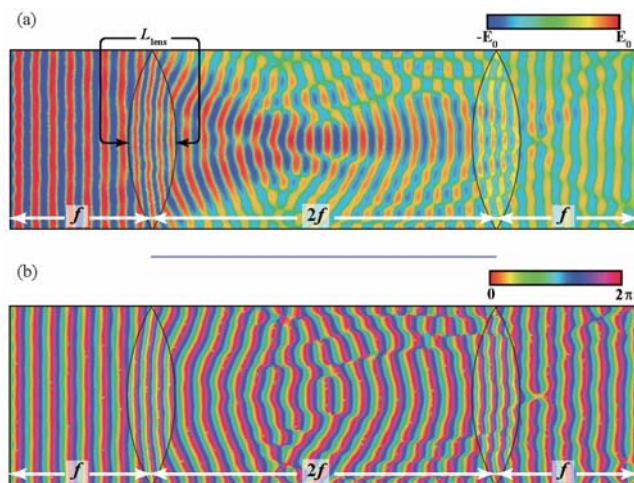
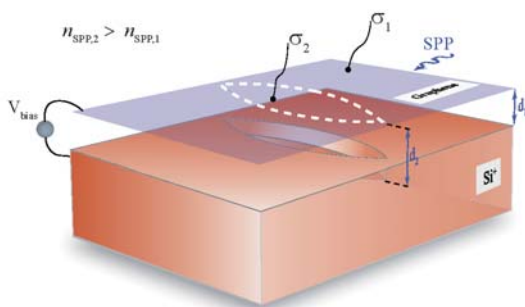
Vakil, Engheta, *Science* 332, 1291 (2011)

# One-Atom-Thick Signal Processing: Fourier Transform



Vakil, Engheta, *Phys. Rev. B*, (2012)

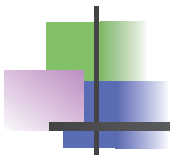
# Graphene Fourier Optics



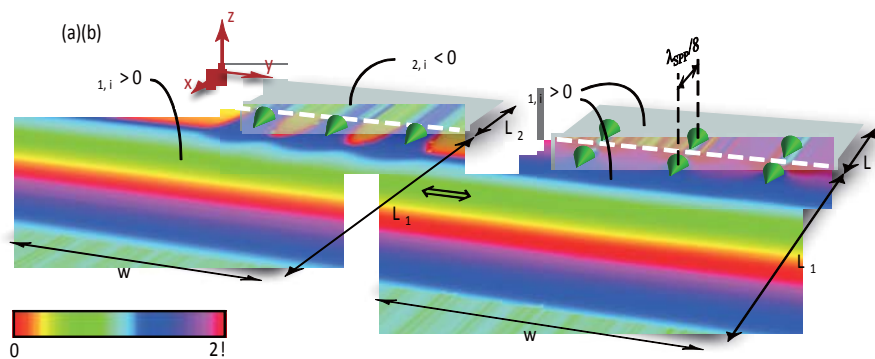
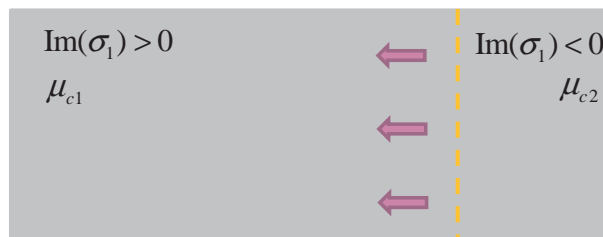
Vakil and Engheta, *Phys. Rev. B* (2012)



## Graphene SPP Mirror



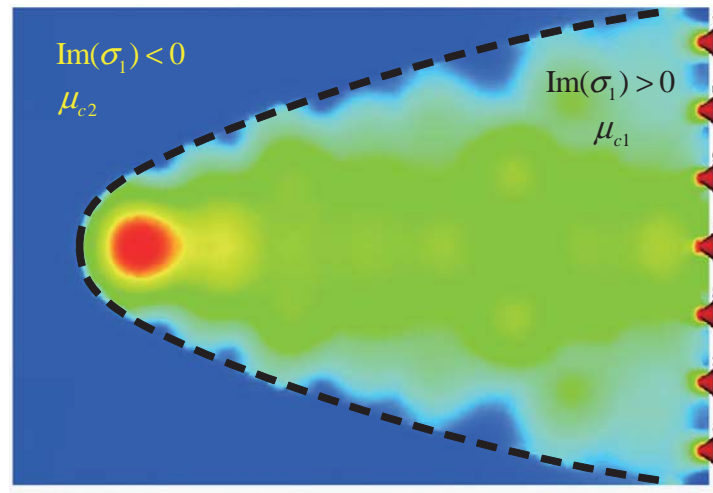
## One-Atom-Thick SPP Reflector



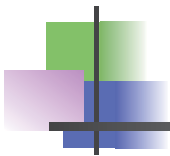
Vakil and Engheta, *Optics Communications*, (2012)



## *One-Atom-Thick SPP Reflector*



*Vakil and Engheta, Optics Communications (2012)*



## *Graphene Metamaterials*



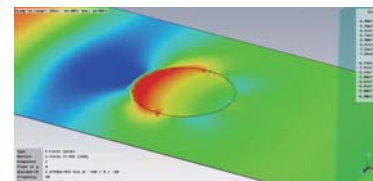
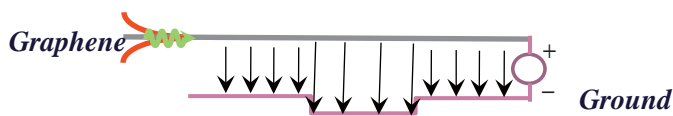
# One-Atom-Thick Scatterer

Region 1:  $\sigma_{g,i} > 0$

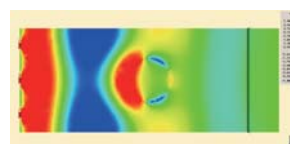
$$\mu_c = 150 \text{ meV}$$

Region 2:  $\sigma_{g,i} < 0$

$$\mu_c = 65 \text{ meV}$$

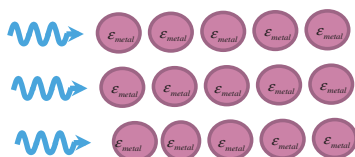


w=120nm  
r=25nm



w=55 nm  
D = 30 nm

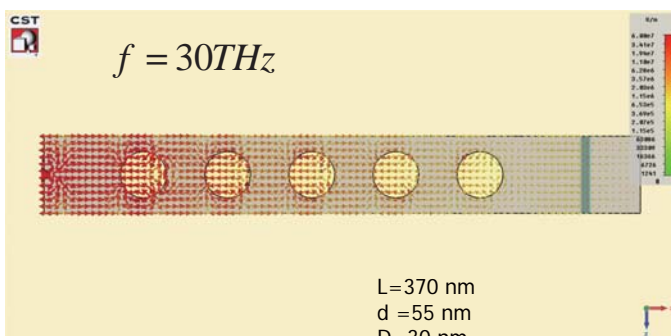
# One-Atom-Thick Metamaterials



Region 1:  $\sigma_{g,i} > 0$     Region 2:  $\sigma_{g,i} < 0$

$$\mu_c = 150 \text{ meV}$$

$$\mu_c = 65 \text{ meV}$$



L=370 nm  
d = 55 nm  
D=30 nm  
w=55 nm

